Exam 2

Thursday, June 16, 2016
5:00-6:15 p.m.

Name: KEY

There are seven problems on this exam. You must show all your work to get full credit.
Please box your final answers.

Problem 1: A table has x- and y-coordinates drawn on it. Three balls sit on the table; in terms of these coordinates, ball A of mass 5.0 kg sits at (0.5 m, 1.5 m), ball B of mass 3.0 kg sits at (−1.0 m, −3.0 m), and ball C of mass 2.0 kg sits at the origin. Find the position vector (both direction and magnitude) of the center-of-mass of the three balls in terms of the coordinates on the table.

The three balls have coordinates:

$$\vec{r}_A = (0.5\,m, 1.5\,m) \; ; \; \vec{r}_B = (-1.0\,m, -3.0\,m) \; ; \; \vec{r}_C = (0, 0).$$

he x-comp. of the CoM is:

$$x_{cm} = \frac{m_A r_{Ax} + m_B r_{Bx} + m_C r_{Cx}}{m_A + m_B + m_C} = \frac{5(0.5) + 3(-1) + 2(0)}{10} = -0.05\,m$$

he y-comp. is:

$$y_{cm} = \frac{m_A r_{Ay} + m_B r_{By} + m_C r_{Cy}}{m_A + m_B + m_C} = \frac{5(1.5) + 3(-3) + 2(0)}{10} = -0.15\,m$$

thus

$$r_{cm} = \sqrt{x_{cm}^2 + y_{cm}^2} = 0.158\,m \qquad \theta = \arctan\left(\frac{-0.15}{-0.05}\right) = 71.6^\circ$$

as both x- and y-comps. are negative, we are in quadrant 3. So add 180° to θ

$$\boxed{\begin{aligned} r_{cm} &= 0.158\,m \\ \theta &= 251.6^\circ \end{aligned}}$$

Problem 2: A DVD player takes 1.2 s to bring a DVD disk from rest up to play speed of 3000 rad/s uniformly. How long does it take for the DVD to undergo 100 rotations when starting from rest? Assume that the required number of rotations is reached before 1.2 s.

The disk requires $\Delta t = 1.2s$ to ~~perform~~ reach $\omega_f = 3000$ rad/s, starting from rest.
This allows us to calculate the angular acceleration:

$\Delta t = 1.2s$

$\omega_f = 3000$ rad/s

$\omega_i = 0$

$\alpha = ?$

$$\omega_f = \omega_i + \alpha \Delta t$$

$$\rightarrow \alpha = \frac{\omega_f}{\Delta t} = 2500 \text{ rad/s}^2.$$

Now we may use α to find Δt for when it has completed 100 rotations.

100 rotations $\longrightarrow$ $\Delta\theta = 200\pi$ rad

$\omega_i = 0$

$\alpha = 2500$ rad/s²

$\Delta t = ?$

We use

$$\Delta\theta = \cancel{\omega_i}^{\,0} \Delta t + \frac{1}{2}\alpha(\Delta t)^2$$

$$= \frac{1}{2}\alpha(\Delta t)^2$$

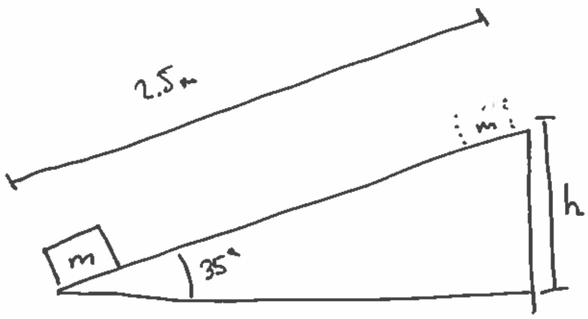
Thus $$\boxed{\Delta t = \sqrt{\frac{2\,\Delta\theta}{\alpha}} = 0.71s}$$

Problem 3: A group of movers uses a ramp to load boxes into the back of a truck. Assume the ramp is inclined at an angle of 35° above the horizontal and is 2.5 m long. A 35 kg box of books is slid from the base of the ramp into the truck. The coefficient of kinetic friction between the block and the ramp is $\mu_k = 0.3$.

(a) What is the work done by gravity on the box while it slides up the ramp?

(b) How much work does *a mover* do on the box in sliding it up the ramp at constant speed?
[Hint: Consider every force doing work on the box and the value of the total work done on the box.]

a)



$m = 35\ kg$ $\quad h = (2.5)\sin 35° = 1.43\ m$

$W_{grav} = F_g \Delta y \cos\theta = (mg)(h)\cos(180°)$

$\boxed{W_{grav} = -500.5\ J}$

(b) As the speed is constant, $W_{tot} = \Delta KE = 0$. To find the total work, we calculate the work done by each force separately:

$$W_{tot} = W_{grav} + W_{friction} + W_{mover} = 0.$$

So $\quad W_{mover} = -\left(W_{grav} + W_{friction}\right).$

$W_{grav} = -500.5\ J$ from (a).

slides up the ramp, but friction points down the ramp

$$W_{fric} = \underbrace{(\mu_k mg\cos 35°)}_{f_k}\underbrace{(2.5\ m)}_{\Delta x}\cos(180°)$$

$W_{fric} = -215.0\ J$

thus

$$W_{mover} = -\left(-500.5 - 215.0\right)J$$

$$\boxed{W_{mover} = 715.5\ J}$$

Problem 4: Two blocks of clay, A and B, slide along a frictionless, horizontal surface. Block A has an initial velocity $\vec{v}_A = 3.0\,\hat{x}$ m/s. The two blocks collide, stick together, and travel off with a final velocity $\vec{v}_{A+B} = (5.5\,\hat{x} - 2.5\,\hat{y})$ m/s. If the masses of the two blocks are $m_A = 1.0$ kg and $m_B = 2.0$ kg, what are the x- and y-components of block B's initial velocity before the collision?

From the picture, we imagine that B is moving to the right and down.

Use momentum conservation for the perfectly inelastic collision:

$$\vec{P}_{tot\,i} = \vec{P}_{tot\,f} \longrightarrow \vec{P}_{Ai} + \vec{P}_{Bi} = \vec{P}_{Af} + \vec{P}_{Bf}$$

Use components:

<u>in x-dir:</u>

$$P_{Ai_x} + P_{Bi_x} = m_A(3\,\text{m/s}) + m_B v_{Bi_x} = (m_A + m_B)(5.5\,\text{m/s}).$$

$$\longrightarrow (1\,\text{kg})(3\,\text{m/s}) + 2\,\text{kg}\, v_{Bi_x} = (3\,\text{kg})(5.5\,\text{m/s})$$

$$\text{so} \quad v_{Bi_x} = 6.75\,\text{m/s}.$$

<u>in y-dir:</u>

$$P_{Ai_y} + P_{Bi_y} = m_A(0) + m_B v_{Bi_y} = (m_A + m_B)(-2.5\,\text{m/s})$$

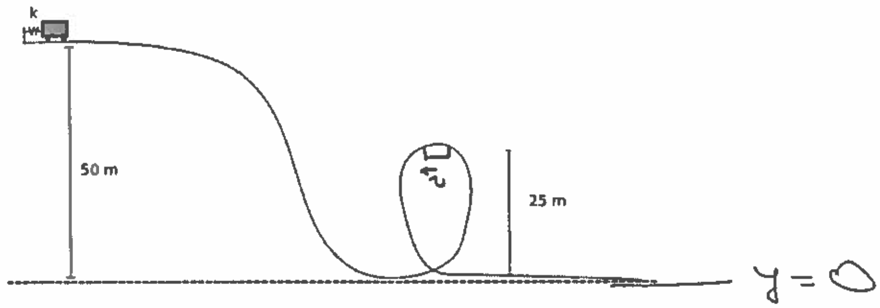
$$\longrightarrow 2\,\text{kg}\, v_{Bi_y} = 3\,\text{kg}\,(-2.5\,\text{m/s})$$

$$\text{So} \quad v_{Bi_y} = -3.75\,\text{m/s}$$

Thus

$$\boxed{v_{Bi_x} = 6.75\,\text{m/s}, \quad v_{Bi_y} = -3.75\,\text{m/s}}$$

Problem 5: A 1500 kg cart sits on a track, initially 50 m above the ground. It is loaded into a spring-launch system, with spring constant $k = 2000$ N/m, compressing the spring by 2.0 m. When the brakes are released, the cart shoots down the track and enters a loop, as shown in the figure below. If the top of the loop is 25 m above the ground, what is the speed of the cart as it passes through the top of the loop? Assume the track is frictionless.



Initially, the cart is at rest at the top of the track:

$$E_i = PE_g + PE_s = mgh + \frac{1}{2}k(\Delta L)^2$$

$$= (1500)(10)(50) + \frac{1}{2}(2000)(2)^2$$

$$= 754{,}000 \text{ J}$$

Finally, the cart is on the top of the loop and moving with some speed v:

$$E_f = \frac{1}{2}mv^2 + mgh$$

$$= \frac{1}{2}(1500)v^2 + (1500)(10)(25)$$

From energy conservation, $E_i = E_f$:

$$\frac{1}{2}(1500)v^2 + (1500)(10)(25) = 754{,}000$$

$$\longrightarrow \boxed{v = 22.5 \text{ m/s}}$$

Problem 6: Your weight on the surface of Planet X is 600 N, and on the surface of Planet Y it is 1200 N. The radius of Planet Y is twice that of Planet X. If the mass of Planet X is 6.0×10^{25} kg, find the mass of Planet Y.

We take the ratio of the two weights to get our answer:

$$W_x = 600\,N = G\frac{mM_x}{R_x^2} \quad ; \quad W_y = 1200\,N = G\frac{mM_y}{R_y^2}$$

Given $R_y = 2R_x$ and $M_x = 6.0 \times 10^{25}$ kg,

$$\frac{W_y}{W_x} = \frac{1200}{600} = 2 \quad \text{and} \quad \frac{W_y}{W_x} = \frac{\left(G\frac{mM_y}{R_y^2}\right)}{\left(G\frac{mM_x}{R_x^2}\right)} = \frac{M_y}{R_y^2}\frac{R_x^2}{M_x} = \frac{M_y}{M_x}\left(\frac{R_x}{R_y}\right)^2$$

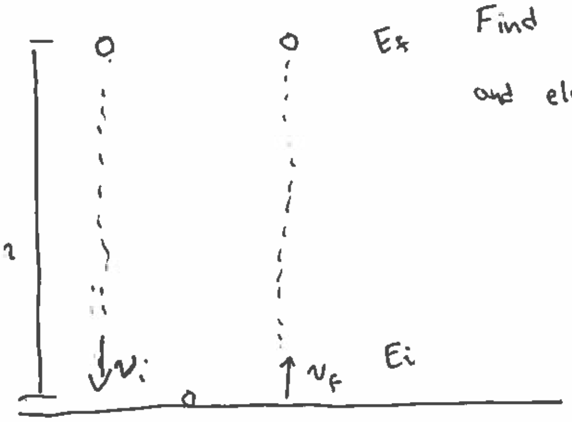
Setting these equal gives

$$\frac{M_y}{M_x}\left(\frac{R_x}{R_y}\right)^2 = 2 \quad \text{and} \quad \frac{R_x}{R_y} = \frac{1}{2}$$

$$\rightarrow \quad \frac{M_y}{M_x}\left(\frac{1}{2}\right)^2 = 2 \longrightarrow M_y = 8M_x$$

$$\boxed{M_y = 4.8 \times 10^{26}\ \text{kg}}$$

Problem 7: A bouncy ball is dropped from rest onto a floor from an unknown height h. It rebounds off the floor with the same speed it had just before hitting the floor. If the mass of the ball is 5.0 grams and the impulse provided by the floor was measured to be 0.2 N s, find the height h from which the ball was dropped.



Find h, given the impulse $\Delta\vec{p} = 0.2$ Ns up, mass $m = 0.005$ kg, and elastic collision:

$\vec{v}_f = -\vec{v}_i$ (one is up, one is down)

From the impulse,

$$\Delta\vec{p} = \underset{\text{up}}{0.2\text{ Ns}} = m\vec{v}_f - m\vec{v}_i$$

$$= m(2v_f).$$

Thus we get $v_f = \dfrac{0.2}{(0.005)(2)} = 20$ m/s.

We can use energy to connect v_f to h:

$$E_i = \frac{1}{2}mv_f^2 \qquad E_f = mgh$$

$$\longrightarrow \quad \frac{1}{2}mv_f^2 = mgh \quad \text{or} \quad \frac{1}{2}v_f^2 = gh$$

So $\boxed{h = \dfrac{v_f^2}{2g} = 20\text{ m}}$