Practice Exam 3

Monday, July 11, 2016

Name:______________________________________

There are seven problems on this exam. You must show all your work to get full credit.
Please box your final answers.

Problem 1: A boat anchor is lowered over the side of a boat from a chain into the ocean (in this problem, the chain has no mass or volume!). The anchor is made of iron and has a volume of $V_{\text{anchor}} = 1.2\ \text{m}^3$. Assume the following values for these densities: $\rho_{\text{ocean}} = 1025\ \text{kg/m}^3$, $\rho_{\text{iron}} = 7860\ \text{kg/m}^3$.

(a) The anchor is fully submerged in the ocean and hangs from the chain at rest. What is the tension in the anchor chain?

(b) Assume the anchor chain is cut. What is the acceleration in the anchor?

(a) FBD of chain: The anchor is stationary, so

$$T + F_B - mg = 0$$

From Archimedes:

$$F_B = \rho_{ocean}\, g\, V_{anchor}$$
$$= (1025)(10)(1.2)$$
$$= 12.3\ \text{kN}$$

Weight of anchor:

$$mg = \rho_{anchor} V_{anchor}\, g$$
$$= (7860)(1.2)\, g$$
$$= 94.3\ \text{kN}$$

$$\rightarrow \quad T = mg - F_B = 94.3\ \text{kN} - 12.3\ \text{kN}$$

$$\boxed{T = 82\ \text{kN}}$$

(b) When the ~~anchor~~ chain is cut, we accelerate down ↓

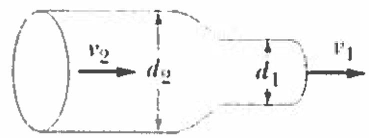
$$F_B - mg = -ma$$

from (a)

$$\rightarrow \quad a = \frac{mg - F_B}{m} = \frac{T}{m}$$

$$= \frac{82\ \text{kN}}{\rho_{iron} V_{anchor}} = \boxed{8.7\ \text{m/s}^2\ \text{down}}$$

Problem 2: Water flows through a horizontal pipe and then out into the atmosphere (P_{ATM} = 100 kPa) at a speed v_1 = 15.0 m/s. The diameters of the left and right sections of the pipe are 5.0 cm and 3.0 cm, respectively.



(a) What volume of water flows into the atmosphere during a 10 minute period?

(b) What is the pressure of the water in the left section of the pipe?

(a) The volume is given by multiplying the outflowing volume flow rate by the time interval:

$$V = \frac{\Delta V}{\Delta t}(10\,\text{min}) = (A_1 v_1)(600\,\text{s})$$

$$= \frac{\pi (0.03)^2}{4}(15)(600)$$

$$\boxed{V = 6.36\ \text{m}^3}$$

(b) Using $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$,

Assume $h_1 = h_2 = 0$, $v_2 = \frac{A_1}{A_2} v_1$, $P_1 = P_{ATM}$

$$\longrightarrow P_{ATM} + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho\left(\frac{A_1}{A_2} v_1\right)^2$$

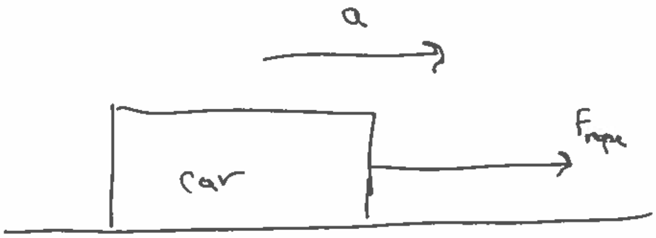
Solve for P_2:

$$P_2 = P_{ATM} + \frac{1}{2}\rho\left(1 - \left(\frac{d_1}{d_2}\right)^4\right) v_1^2$$

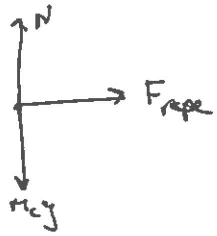
$$\boxed{\frac{A_1}{A_2} = \frac{\pi\left(\frac{d_1}{2}\right)^2}{\pi\left(\frac{d_2}{2}\right)^2} = \left(\frac{d_1}{d_2}\right)^2}$$

$$\boxed{P_2 = 198\ \text{kPa}}$$

Problem 3: A truck and a disabled car are tied together using a tow rope ($Y = 5$ GPa) with cross-sectional area 0.005 m^2. When the light at the intersection turns green, the truck accelerates forward at 3.5 m/s^2. If the maximum strain allowed in the tow rope is 0.001 before the rope breaks, what is the maximum mass of disabled car allowed for it to be pulled by the truck through the intersection? Assume the tow rope is taught between the two vehicles and that the disabled car moves without friction or air resistance.



FBD:



So $F_{rope} = m_c a$. This force stresses the rope:

$$\frac{F_{rope}}{A} = Y \frac{\Delta L}{L} \longrightarrow F_{rope} = AY\left(\frac{\Delta L}{L}\right)$$

Setting them equal and solving for m_c:

$$m_c = \left(\frac{AY}{a}\right)\left(\frac{\Delta L}{L}\right)$$

$$= \frac{(0.005)(5\times10^9)}{3.5}(0.001)$$

$$\boxed{m_c = 7{,}143 \text{ kg}}$$

Problem 4: A ceiling fan rotating with an angular speed of 10 rad/s experiences a dissipative torque of 5 Nm due to air drag. How much energy is dissipated from the ceiling fan due to this dissipative torque in 1 hour?

The energy lost due to the torque is given by the work done

$W = \tau \Delta\theta$ Here, $\Delta\theta = \omega \Delta t$

$= (10 \frac{rad}{s})(3600\ s)$

$= 36{,}000\ rad$

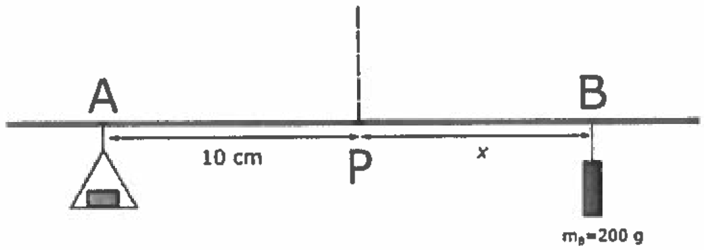
Thus

$W = \tau \Delta\theta$

$= (5\ Nm)(36{,}000)$

$= \boxed{180\ kJ}$

Problem 5: The following setup is in equilibrium. Throughout this problem, we'll choose the pivot to be the point P. We can add or remove masses from the platform A on the left. The mass B on the right can be moved along the stick.



(a) If we increase the mass on platform A by Δm, by how much does the counter-clockwise torque increase? [Hint: Look at only the torque due the added mass, all other forces (except the force from the pivot) remain the same]

(b) If we move the mass B to the right from a distance of x to a distance of $x + \Delta x$, by how much does the clockwise torque increase? [Hint: All other forces (except the force from the pivot) remain the same, only the location at which the weight of B acts changes. What is the torque due to weight of B before moving? What is it after moving?]

(c) Starting from a balanced setup, if we increase the mass on platform A by Δm, by how much should we move mass B to keep the setup balanced? [Hint: Set the increase in CCW and CW torques in parts (a) and (b) equal to each other].

This is how some weighing scales work, by relating the mass added to A to the distance B needs to be moved.

(a) $|\tau_A| = (0.1\text{ m})(\text{mass at A})g$ thus, as mass $\rightarrow$ mass + Δm

$\tau_A \longrightarrow \tau_A + (0.1\text{m})(\Delta m)g$ counterclockwise

(b) $|\tau_B| = x(0.2\text{ kg})g$ as $x \rightarrow x + \Delta x$

$\tau_B \longrightarrow \tau_B + \Delta x(0.2\text{kg})g$ clockwise.

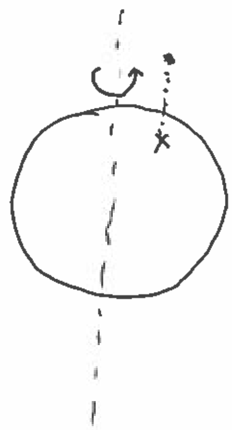
(c) A net torque = 0 we must have $\Delta\tau_A = \Delta\tau_B$, so

$(0.1\text{ m})(\Delta m)g = \Delta x(0.2\text{kg})g$

$\longrightarrow \boxed{\Delta x = 0.5\ \Delta m \text{ in meters}}$

Problem 6: A uniform sphere of mass 1 kg and radius 20 cm is rotating about an axis passing through it's center at a constant angular speed of 1 rad/s. When it is rotating, a tiny ball of molten metal of mass 0.5 kg (point mass) drips onto the sphere and sticks to it at a distance of 5 cm from the axis of rotation. Find the angular speed of the system after the 'collision'.

The point mass has no angular momentum before sticking to the sphere, and there is no external torque acting on the system in this problem. [Hint: Use conservation of momentum]



We conserve angular momentum:

$$L_i = I_{sphere}\,\omega_i$$

$$= \frac{2}{5}(1\,kg)(0.2)^2 \cdot 1\,\frac{rad}{s} = 0.016\ kg\,m^2/s$$

$$L_f = (I_{sphere} + I_{drop})\,\omega_f$$

$$= \left(\frac{2}{5}(1\,kg)(0.2)^2 + 0.5(0.05)^2\right)\omega_f$$

Setting $L_i = L_f$ gives

$$0.016 = (0.0175)\,\omega_f$$

$$\boxed{\omega_f = 0.93\ rad/s}$$

Problem 7: An ideal spring-mass system is placed on a frictionless horizontal surface as shown. When stretched and released, the system exhibits simple harmonic motion at an angular frequency of 5 rad/s. What is the magnitude of the acceleration of the mass when it's displacement from the relaxed position is 10 cm?

We are given $\omega = 5$ rad/s. We know for SHM that

$$a = -\omega^2 x, \text{ so}$$

$$|a| = \omega^2 x$$

$$= (25)(0.1)$$

$$\boxed{a = 2.5 \text{ m/s}^2}$$