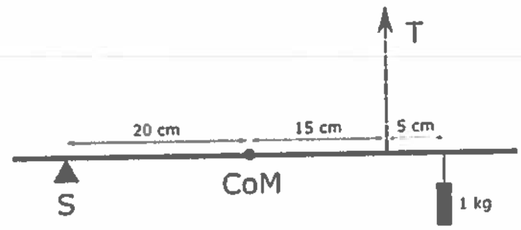
Exam 3

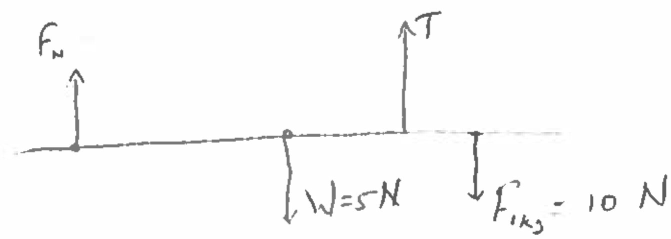
Thursday, July 14, 2016
5:00-6:15 p.m.

Name: KEY

There are seven problems on this exam. You must show all your work to get full credit.
Please box your final answers.

Problem 1: A stick of mass 0.5 kg is suspended by a string (the dashed line) as shown. S is a supporting platform under the stick and CoM is the center-of-mass of the stick. Calculate the tension T exerted by the string if the setup is in equilibrium.





Picking the pivot to be at S, the torques due to the ~~other~~ forces are

$\tau_W = -5\ N \times 0.2\ m = -1\ N.m$ (clockwise)

$\tau_T = T \times (0.2 + 0.15)\ m = T \times 0.35\ m$ (counter CW)

$\tau_{F_{1kg}} = -10\ N \times (0.2 + 0.15 + 0.05)\ m$

$= -4\ N.m$ (Clockwise).

$\tau_{F_N} = 0$

$\Sigma \tau = 0$

$\Rightarrow -1\ Nm + T \times 0.35\ m - 4\ Nm = 0.$

$T = \frac{5\ Nm}{0.35\ m} = 14.29\ N$

$\boxed{T = 14.29\ N}$

Problem 2: A small child holds a party balloon attached to an ideal string. The balloon has a mass of 2.0 g when empty and is filled with helium to a volume of $V_{\text{balloon}} = 0.0072\ \text{m}^3$. If the balloon is held at rest by the string and kept from floating away, what is the tension in the string? Ignore the thickness of the balloon. The following densities are important: $\rho_{\text{He}} = 0.18\ \text{kg/m}^3$, $\rho_{\text{air}} = 1.20\ \text{kg/m}^3$. Hint: consider all of the forces acting on the balloon, both causing it to float and causing it to remain stationary.

The balloon experiences 3 forces

1) It's own weight downwards (which includes the weight of empty balloon and the helium).

2) Buoyant force upwards (due to ~~displ~~ air)

3) Tension. ~~~~ downwards.

Since the balloon is held at rest

$$-W + F_B - T = 0$$

or

$$T = -W + F_B$$

$$W = \underbrace{\left(0.002\ \text{kg} \times 10\ \text{m/s}^2\right)}_{\text{empty balloon}} + \underbrace{\left(\overbrace{0.0072 \times 0.18}^{m_{He}} \times 10\right)}_{\text{Helium}}\ \text{N}$$

$$W = 0.033\ \text{N}.$$

$$|F_B| = |\text{Weight of displaced air}|$$

$$= \underbrace{0.0072 \times 1.20\ \text{kg}}_{m_{\text{displaced air}}} \times 10\ \text{m/s}^2 = 0.086\ \text{N}$$

$$T = -0.033\ \text{N} + 0.086\ \text{N}$$

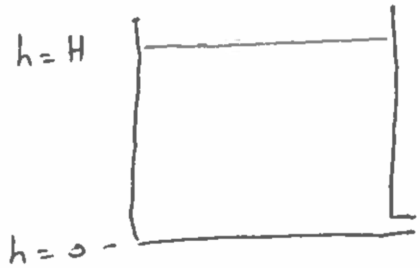
or

$$\boxed{T = 0.053\ \text{N}}$$

Problem 3: A sports drink dispenser is used on the sidelines of some sporting games to deliver cold beverages to the players. The dispenser is constructed with a small circular valve at its base which allows the liquid to pour out when the valve is opened; assume that the valve has a diameter of 0.8 cm when fully opened. It takes the dispenser 5 s to fully fill an empty 0.001 m^3 bottle when the valve is fully opened. The top of the dispenser is open to the air (P_{ATM} = 100 kPa) and the level of the fluid in the dispenser is approximately constant when filling the bottle.

(a) How high is the surface of the sports drink in the dispenser above the valve?

(b) If the valve of the dispenser is only opened to a diameter of 0.5 cm, how long does it take to fill up the same volume of bottle above?



$P_1 = P_{atm}$
$y_1 = H$
$v_1 = 0$

$P_2 = P_{atm}$
$y_2 = 0$
$v_2 = v_2$.

Bernoulli's eqn

$$\cancel{P_1} + \rho g y_1 + \frac{1}{2}\rho v_1^2 = \cancel{P_2} + \rho g y_2 + \frac{1}{2}\rho v_2^2.$$

since $P_1 = P_2 = P_{atm}$.

$$\rho g H + \cancel{0} = \cancel{0} + \frac{1}{2}\rho v_2^2.$$

(or)

$$v_2^2 = 2gH \quad \text{or} \quad v_2 = \sqrt{2gH}$$

Given v_2 we can find H.

To find v_2: Volume flow rate $= A_2 v_2 = \dfrac{0.001\ m^3}{5\ s} = 0.0002\ m^3/s$.

$$A_2 = \frac{1}{4}\pi d_2^2 = \frac{1}{4}\times\pi\times(0.008\ m)^2$$

$$= 5.03\times10^{-5}\ m^2.$$

$$v_2 = \frac{\text{Volume flow rate}}{A_2} = 3.98\ m/s.$$

(continued at the end).

Problem 4: Calculate the period of oscillation of an object exhibiting simple harmonic motion, if its maximum displacement from the equilibrium point is 20 cm and its maximum speed during the oscillation is 1 m/s.

Amplitude $A = 20 \text{ cm} = 0.2 \text{ m}$

Velocity Amplitude $A_v = 1 \text{ m/s}$

$$A_v = A\omega$$

$$\Rightarrow \omega = \frac{A_v}{A} = \frac{1 \text{ m/s}}{0.2 \text{ m}} = 5 \text{ rad/s}$$

Also $\omega = \frac{2\pi}{T}$ where T is the period

So, $T = \frac{2\pi}{\omega} = \frac{2\pi}{5} \text{ s} = \boxed{1.26 \text{ s}}$

Problem 5: A cyclist coasts down a hill of height 4 m. The mass of the cyclist and bicycle combined (including wheels) is 90 kg. The two wheels weigh 1 kg *each*. The cyclist starts from rest at the top. If there are no dissipative forces acting, find the speed of the bike at the bottom of the hill.

The mass distribution on the wheels is such that the moment of inertia of a wheel is given by $I = 0.9mr^2$, where m is the mass of the wheel, and r is it's radius. The wheels roll without slipping on the road. You can ignore the rotational energy stored in the pedals, chain, crankset, derailleur gears, etc.

Energy conservation.

$M = 90\ kg$

$m = 1\ kg.$

We'll solve this problem using conservation of energy

At the top of the hill.

$K.E. = \emptyset$

$P.E = Mg \times 4\ m = (90 \times 10 \times 4)\ J = 3600\ J$

At the bottom of the hill.

$P.E = \emptyset$

$K.E = K.E._{rot} + KE_{trans}$

$= \underset{\uparrow}{2} \times \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2$

for 2 wheels

$= \left[\cancel{2} \times \frac{1}{\cancel{2}} (0.9\ m \cancel{r^2}) \frac{v^2}{\cancel{r^2}}\right] + \frac{1}{2} M v^2$

$= 0.9\ kg\ v^2 + 45\ kg\ v^2$ $= (45.9\ kg)\ v^2.$ ~~$= (91.8\ kg)\ v^2$~~

From conservation of energy.

$3600\ J = (45.9\ kg)\ v^2$

$v^2 = 78.43\ m^2/s^2.$

or $\boxed{v = 8.86\ m/s}.$

Problem 6: A crane slowly lifts a palafitte (stilt house) from where it was constructed up onto the pilings that form its foundation. The house weighs 10^8 N.

(a) The crane uses a steel cable ($Y = 200$ GPa) with cross-sectional area 0.1 m^2 to lift the house at a constant speed. If the cable is 90 m long, how much does the cable stretch in lifting the house?

(b) The house is placed onto several wooden stilts of 3 m length and cross-sectional area 0.04 m^2. How many stilts must be used to support the weight of the house if each stilt may only compress by 1% its total length? You may assume that the weight of the house will be distributed equally over the stilts. The Young's modulus for wood is 10 GPa. Round your answer up to the nearest whole number.

Acceleration = 0, so Tension = Weight of the house 10^8 N

a) In part a you're given the force / tension in the cable, and you're looking for ΔL

$$\frac{\Delta F}{A} = Y \frac{\Delta L}{L}$$

$\Delta F = 10^8$ N

$A = 0.1\ m^2$

$Y = 200$ GPa $= 200 \times 10^9$ Pa

$L = 90$ m.

$$\Delta L = \frac{\Delta F}{A} \times \frac{L}{Y} = \frac{10^8 \times 90}{0.1 \times 200 \times 10^9} \text{ m} = 0.45 \text{ m}$$

or $\boxed{\Delta L = 45 \text{ cm}}$

b) Let's deal with one stilt. How much weight does it take to compress a stilt by 1% of its length? Here $\frac{\Delta L}{L}$ is given, we're looking for ΔF

$$\Delta L = \frac{1}{100} L \quad \text{or} \quad \frac{\Delta L}{L} = 0.01$$

$$\Delta F = Y A \frac{\Delta L}{L} = 10 \times 10^9 \text{ Pa} \times 0.04 \text{ m}^2 \times 0.01 = 4 \times 10^6 \text{ N}$$

(6 continued at the end).

Problem 7: A figure skater spins with his arms outstretched with an angular speed of 2.0 rad/s. He tucks his arms in to increase his angular speed. His rotational inertia in the outstretched and tucked positions are 1.8 kg.m^2 and 1.5 kg.m^2 respectively. Find the rotational kinetic energy of the skater in the outstretched and tucked positions. No external torques act on the skater in this problem.

Conservation of angular momentum.

Outstretched ≡ subscript 1

Tucked ≡ subscript 2.

$$I_1 \omega_1 = I_2 \omega_2$$

$$1.8 \text{ kg m}^2 \times 2.0 \text{ rad/s} = 1.5 \text{ kg m}^2 \times \omega_2$$

or

$$\omega_2 = \frac{1.8 \times 2.0}{1.5} \text{ rad/s} = 2.4 \text{ rad/s}.$$

Rotational KE

$$KE_1^{rot} = \frac{1}{2} I_1 \omega_1^2 = \frac{1}{2} \times 1.8 \times (2.0)^2 \text{ J}$$

$$\boxed{KE_1^{rot} = 3.6 \text{ J}}$$

$$KE_2^{rot} = \frac{1}{2} I_2 \omega_2^2 = \frac{1}{2} \times 1.5 \times (2.4)^2$$

$$\boxed{KE_2^{rot} = 4.32 \text{ J}}$$

Energy increases!

This is a case of the person doing work on themselves, i.e., ~~part of the~~ other forms of the skater's energy gets converted into mechanical energy.

3) continued

We have $V_2^2 = 2gH$

and $V_2 = 3.98 \text{ m/s}$.

$$\Rightarrow \quad H = \frac{V_2^2}{2g} = 0.79 \text{ m}$$

or

$$\boxed{H = 79 \text{ cm}}$$

b) Note that V_2 stays the same, irrespective of the change in diameter.

The expression $V_2 = \sqrt{2gH}$ in independent of d_2 or A_2.

But volume flow rate changes

$$\hookrightarrow \text{V.F.R}_{new} = A_{new} \times \overset{\text{same } V_2}{V_2}$$

$$= \frac{1}{4}\pi (0.005)^2 \text{ m}^2 \times 3.98 \text{ m/s}$$

$$= 0.000078 \text{ m}^3\text{/s}.$$

So, time to fill = ~~VFR~~ $\dfrac{\text{Volume to fill}}{\text{VFR}}$

$$= \frac{0.0001}{0.000078} \simeq \boxed{13 \text{ s.}}$$

(You can also see this as 'Time to fill is inversely proportional to area.')

So, $\dfrac{T}{5\,s} = \dfrac{\frac{1}{4}\pi(0.008)^2 \text{ m}^2}{\frac{1}{4}\pi(0.005)^2 \text{ m}^2} = \dfrac{8^2}{5^2}$ (or) $T = \dfrac{64\,s}{5} \simeq 13$ s

6) continued..

$$\Delta F = 4 \times 10^{6} \text{ N}$$

What this means is that if a load of 4×10^{6} N is placed on ~~the~~ a stilt, it will compress by 1% of its length. So, we should make sure that no stilt takes more ~~that~~ than 4×10^{6} N of the load.

Total weight of the house $= 10^{8}$ N.

$$\frac{10^{8}}{4 \times 10^{6}} = \boxed{25}$$

So we need atleast 25 stilts to support the weight of the house, if none of them is to compress by more that 1% of its length.