(Practice) Final Exam

Wednesday, August 3, 2016
5:00-6:15 p.m.

Name: KEY

There are seven problems on this exam. You must show all your work to get full credit.
Please box your final answers.

Problem 1: A ball is launched with some initial speed at an initial angle above the horizontal. When the ball reaches its maximum height, it just clears a 7 m tall net. The ball hits the ground a horizontal distance of 44 m from where it was first launched.

(a) What was the ball's initial speed?

(b) What was the initial launch angle of the ball?

Let the ball be lauched at an angle θ above the ground with speed v.

$V_x = V\cos\theta$ $V_{i,y} = V\sin\theta$

$a = -g$

Max height

$V_{f,y} = 0$

$\Delta y = 7\text{ m}$

$V_{f,y}^2 - V_{i,y}^2 = 2a\Delta y$

$\cancel{0} - v^2\sin^2\theta = -20\times 7\ m^2/s^2 = -140\ m^2/s^2$.

Or $v\sin\theta = \sqrt{140}\ m/s \simeq 11.83\ m/s$ ①

Horizontal distance

$\Delta y = 0$

$\Delta y = V_{i,y}\Delta t + \frac{1}{2}a\Delta t^2$

$\cancel{0} = v\sin\theta\,\Delta t - \frac{g}{2}\Delta t^2$.

Or $\Delta t = \dfrac{2v\sin\theta}{g}$

$\Delta x = V_x\Delta t = \dfrac{2v^2\sin\theta\cos\theta}{g} = 44\text{ m}$ ②

Plugging ① in ② $v\cos\theta = \left[\dfrac{44\text{ m}\times g}{2}\times\dfrac{1}{v\sin\theta}\right] \simeq 18.59\ m/s$. ③

Now from (1) $\quad V\sin\theta = 11.83\ m/s$

from (3) $\quad V\cos\theta = 18.54\ m/s$

Dividing (1) by (3)

$$\frac{V\sin\theta}{V\cos\theta} = \frac{11.83}{18.59}$$

or

$$\tan\theta = 0.64$$

$$\theta = \arctan(0.64) \approx 32.5^\circ$$

Plugging this back into (1)

$$V = \frac{11.83\ m/s}{\sin\theta} = 22.04\ m/s.$$

$$\boxed{\begin{aligned} V &= 22.04\ m/s \\ \theta &= 32.5^\circ \end{aligned}}$$

Problem 2: Two coherent sound waves of the same frequency interfere constructively in a region of space. If each of them has a sound intensity level of 70 dB at a point in the absence of the other, calculate the sound intensity level of the resultant wave at that point.

$$A_{resultant} = A_1 + A_2$$

or

$$\sqrt{I_{res}} = \sqrt{I_1} + \sqrt{I_2}$$

for constructive interference of coherent waves

Also $I_1 = I_2$ and correspond to $\beta = 70\,dB$.

$$70\,dB = 10\,dB \log_{10}\left(\frac{I_1}{I_0}\right)$$

$$\Rightarrow \cancel{I_1 = I_0 10}$$

$$\Rightarrow \log_{10}\left(\frac{I_1}{I_0}\right) = 7$$

$$\Rightarrow I_1 = I_0 10^7 = 10^{-12}\,W/m^2 \times 10^7 = 10^{-5}\,W/m^2$$

$$\sqrt{I_{res}} = \left[\sqrt{10^{-5}} + \sqrt{10^{-5}}\right]\sqrt{W/m}$$

or

$$I_{res} = 4\times10^{-5}\,W/m^2$$

$$\beta_{res} = 10\,dB \log_{10}\left(\frac{4\times10^{-5}}{10^{-12}}\right) = 76\,dB.$$

Problem 3: A wave travels leftward along a string of mass per unit length $\mu = 9.8$ g/m being held taut with a tension $T = 350$ N. The distance between the height of a crest and height of a trough of the wave is 8.0 cm. It takes 0.25 s for any piece of the string to pass from crest to crest of the wave.

(a) Find the maximum velocity and maximum acceleration of the string.

(b) Write the equation for the wave in the form $y(x,t) = A\cos(kx \pm \omega t)$.

Tension $T = 350$ N $\qquad \mu = 9.8$ g/m

$A = \frac{8\text{ cm}}{2} = 0.04$ m $\qquad = 0.0098$ kg/m

time period $t = 0.25$ s

a) $V_{max} = \omega A = \frac{2\pi}{t} A = \frac{2\pi}{0.25} \times 0.04$ m/s $\simeq 1.0$ m/s

$a_{max} = \omega^2 A = \left(\frac{2\pi}{t}\right)^2 A \simeq 25.3$ m/s².

These are max velocity and acceleration of particles on the string.

b) $\omega = \frac{2\pi}{t} = 25.13$ rad/s

Speed of the wave $= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{350}{0.0098}}$ m/s

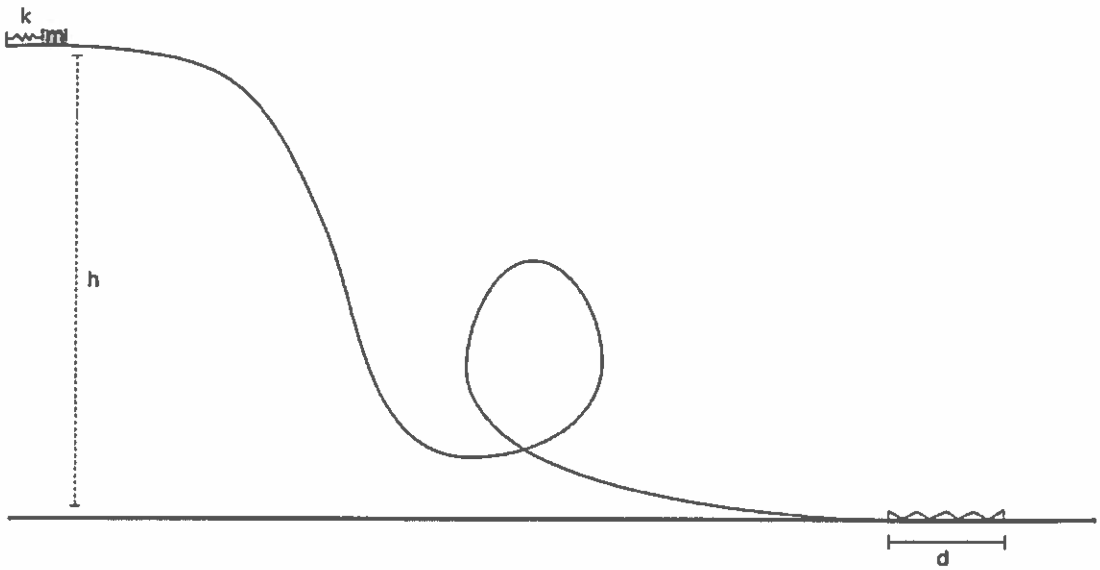
$= 188.98$ m/s.

$k = \frac{\omega}{v} = 0.13$ rad/m.

So ~~y = 0.04 m cos([0.13 ...]~~

$y = 0.04\text{m} \cos\left[\left(0.13\text{ rad/m}\right]x + \left(25.13\text{ rad/s}\right)t\right]$

Problem 4: A block of mass $m = 8.0$ kg sits at the top of a hill, $h = 15$ m, pressed against a spring with a spring constant of $k = 600$ N/m. The spring is initially compressed by 0.42 m. When the block is released, it travels down a frictionless hill and through a loop. After the loop, the ball encounters a rough patch on the ground with coefficient of kinetic friction $\mu_k = 0.4$. If the rough patch is $d = 10$ m long, what is the speed of the block after it slides completely over the rough patch?



Energy at the top $= mgh + \frac{1}{2}k\Delta x^2$

$$= \left[(8\times 10\times 15) + \frac{600}{2}\times(0.42)^2\right] \text{ J}$$

$$= 1252.92 \text{ J}$$

Work done by friction $= -f_k \times 10$ m

$$= -(\mu_k F_N)\times 10 \text{ m}$$

$$= -(0.4\times 8\times 10\times 10) \text{ J}$$

$$= -320 \text{ J}$$

Energy of block after rough ~~path~~ patch

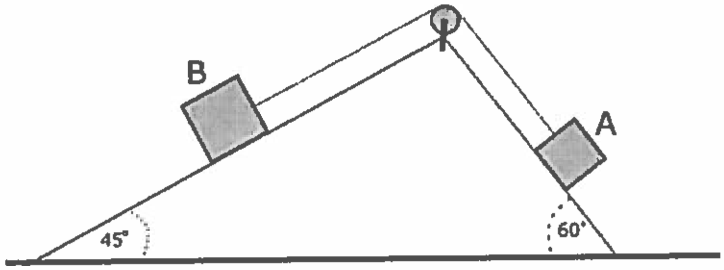
$$= 1252.92 \text{ J} - 320 \text{ J}$$

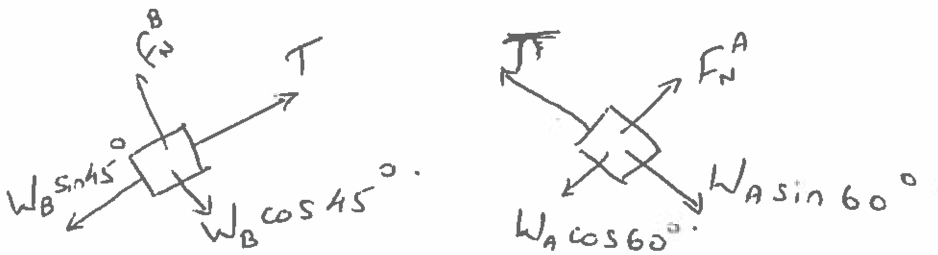
$$= 932.92 \text{ J}$$

$$\frac{1}{2}mv_f^2 = 932.92 \text{ J}$$

$$\Rightarrow \boxed{v_f = 15.3 \text{ m/s}}$$

Problem 5: The system depicted in the diagram below is in equilibrium. Both inclines are frictionless. If the mass of block A is 30 kg, find the mass of block B. Assume that the rope and pulley are ideal.





For equilibrium of B $\quad T - W_B \sin 45° = 0$

of A $\quad T - W_A \sin 60° = 0$.

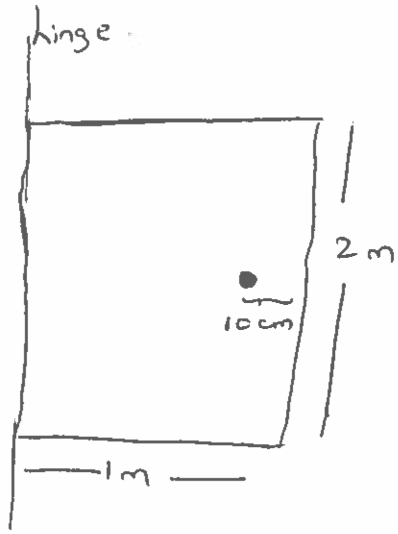
So, $\quad T = W_B \sin 45° = W_A \sin 60°$.

$$m_B \cancel{g} \sin 45° = m_A \cancel{g} \sin 60°.$$

$$m_B = m_A \frac{\sin 60°}{\sin 45°}$$

$$\boxed{m_B = 36.7 \text{ kg}}$$

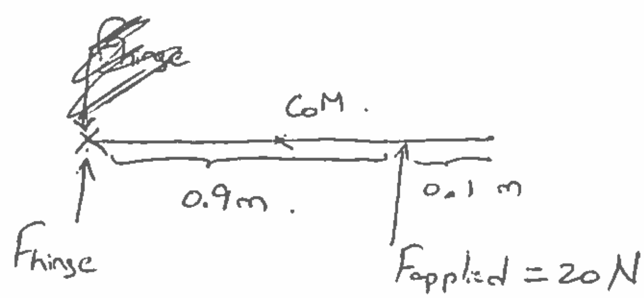
Problem 6: A uniform rectangular door of height 2.0 m, width 1.0 m, and thickness 3.0 cm, is hinged on one end along it's height and is completely free to rotate about the hinge. A doorknob is placed 10 cm from the other long end of the door. Calculate the force exerted by the hinge when you apply a force of 20 N on the doorknob perpendicular to the plane of the door.



The two forces in the horizontal direction are the applied force and force from hinge.

Top view.

Choose pivot at hinge



$\tau_{hinge} = 0$; $\tau_{applied} = 20\ N \times 0.9\ m = 18\ Nm$

$\tau = I\alpha$ $\quad I = \frac{ML^2}{3}$ The relevant L is 1m

$I = \frac{M}{3}(1\ m^2)$

$\Rightarrow \alpha = \frac{3 \times 18\ Nm}{M\ m^2} = \frac{54}{M}\ N/m$

Also $\Sigma F = Ma_{CoM} = M(\alpha\ 0.5\ m) = 27\ N$

So, $\boxed{F_{hinge} = 27\ N - F_{applied} = 7\ N}$

Problem 7: A speaker is strapped onto a car and produces sound at a certain frequency. When the car is driven directly towards you at 50 m/s, you hear the frequency of the sound to be 100 kHz. If instead the speaker is kept at rest and you drive towards the speaker in a (different) car at 50 m/s, what is the frequency of sound that you'll observe? The speed of sound in air is $v_{\text{sound}} = 340$ m/s.

Part 1 : Source moves

$$f_o^1 = f_S \left(\frac{340}{340 - 50} \right)$$

$V_D = 0$
$V_S = 50$ m/s towards

Or $f_S = f_o^1 \dfrac{290}{340}$

Part 2 : Detector moves

$$f_o^2 = f_S \left(\frac{340 + 50}{340} \right)$$

$V_D = 50$ m/s towards
$V_S = 0$

$$= f_S \left(\frac{390}{340} \right)$$

$$= f_o^1 \left(\frac{290}{340} \right) \left(\frac{390}{340} \right)$$

~~= 100 kHz~~

$$= 97.84 \text{ kHz}$$

$$\boxed{f_o^2 = 97.84 \text{ kHz}}$$

Whether is the source of detector that moves, makes a difference!