Final Exam

Wednesday, August 3, 2016
5:00-6:15 p.m.

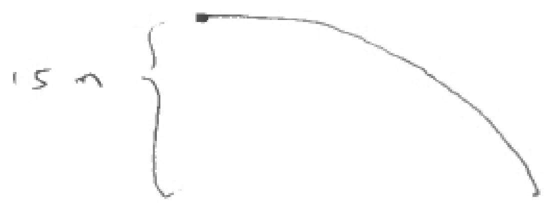
Name:______________________________

There are seven problems on this exam. You must show all your work to get full credit.
Please box your final answers.

Problem 1: A ball is thrown horizontally off a bridge from a height of 15 m above a river. The ball's initial speed is 5.0 m/s.

(a) Find the magnitude of the ball's displacement 1.3 s after it is thrown.

(b) At what horizontal distance away from the bridge does the ball hit the water?



a) $V_x = 5\ m/s$ $\qquad$ $V_{i,y} = 0$

$a = -g$

$\Delta t = 1.3\ s$

$\Delta x = V_x\, \Delta t = 5 \times 1.3 = 6.5\ m$

$\cancel{V_i = 5\ m/s}$ $\qquad$ $\cancel{V_{i,y} = 0}$

$\cancel{\Delta x}$ $\qquad$ $\cancel{a = -g}$

$\cancel{\Delta y = -15\ m}$

$$\Delta y = \cancel{V_{i,y}\, \Delta t} + \frac{1}{2} a (\Delta t)^2$$

$$= -\frac{10}{2} \times (1.3)^2 = -8.45\ m$$

$$|\text{Displacement}| = \sqrt{(\Delta x)^2 + (\Delta y)^2} = 10.66\ m$$

b) $V_x = 5\ m/s$ $\qquad$ $V_{i,y} = 0$

$a = -g$

$\Delta x = ?$

$\Delta y = -15\ m$

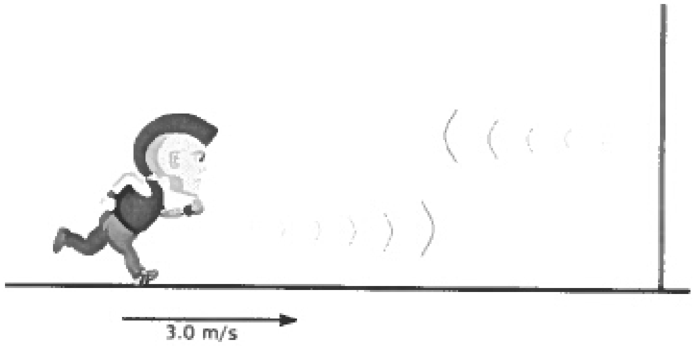
$$\Delta y = \cancel{V_{i,y} \Delta t} + \frac{1}{2} a (\Delta t)^2 .$$

$$(\Delta t)^2 = \frac{-2 \Delta y}{g} = \frac{-2 \times (-15)}{10} s^2 . = 3 s^2 .$$

$$\Delta t = 1.73 s$$

$$\Delta x = V_x \Delta t = 8.66 m$$

Problem 2: You run toward a wall at a speed of 3.0 m/s while continuously yelling at the top of your lungs. You shout at a frequency of 500 Hz. What is the beat frequency between your shout and the sound you hear reflected off of the wall? The speed of sound in air is $v_{sound} = 340$ m/s.



W = wall
Y = you

$$f_o^W = f_s^Y \left(\frac{V_{sound}}{V_{sound} - V_{you}} \right)$$

$$f_o^Y = f_s^W \left(\frac{V_{sound} + V_{you}}{V_{sound}} \right)$$

Also, $f_o^W = f_s^W$

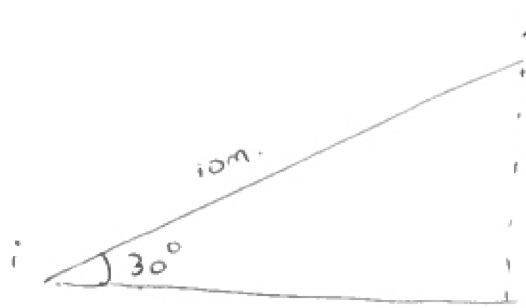
So, $$f_o^Y = f_s^Y \left(\frac{\cancel{V_{sound}}}{V_{sound} - V_{you}} \right) \left(\frac{V_{sound} + V_{you}}{\cancel{V_{sound}}} \right)$$

$$f_s^Y = 500 \text{ Hz}$$

$$f_o^Y = 500 \text{ Hz} \left(\frac{340 + 3}{340 - 3} \right) = 509 \text{ Hz}$$

Beat frequency between your shout and echo $= |509 - 500|$ Hz $= 9$ Hz

Problem 3: A uniform solid sphere is launched up an inclined plane of angle 30° above the horizontal. Calculate the speed with which it was launched if it travels 10 m along the incline before coming to a momentary stop. Assume that throughout its motion the sphere rolls without slipping. The rotational inertia for a solid sphere of mass m and radius r is given by $I_{\text{sphere}} = \frac{2}{5}mr^2$.



$10\,m \times \sin 30^\circ = 5\,m.$

$PE_i = \phi$

$KE_i = \frac{1}{2} m v_i^2 + \frac{1}{2} I \omega_i^2$

$= \frac{1}{2} m v_i^2 + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{5} m \cancel{r^2}\right)\frac{v_i^2}{\cancel{r^2}}$

$= \frac{7}{10} m v_i^2$

$PE_f = m g h_f$

$KE_f = \phi$

Since the sphere rolls without slipping, friction does not work

So, energy is conserved

$\frac{7}{10} \cancel{m} v_i^2 = \cancel{m} g h_f$

$v_i^2 = (10 \times 5) \times \frac{10}{7}\ m^2/s^2$

or

$\boxed{v_i = 8.45\ m/s}$

Problem 4: A guitar string when plucked produces a sound at a fundamental frequency of 196 Hz. What will be the fundamental frequency of sound produced when the string is plucked with a finger placed half-way along the string and holding the string in place [to reduce the length of the vibrating section of the string to half the original length]?

$$\lambda_1 = \frac{2L}{1}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

Given: f_1 for full string is 196 Hz.

To find: f_1 for half string

$$f_1^{\text{half string}} = \frac{v}{2\left(\frac{L}{2}\right)} = 2\left(\frac{v}{2L}\right) = 2 \times 196 \text{ Hz}$$

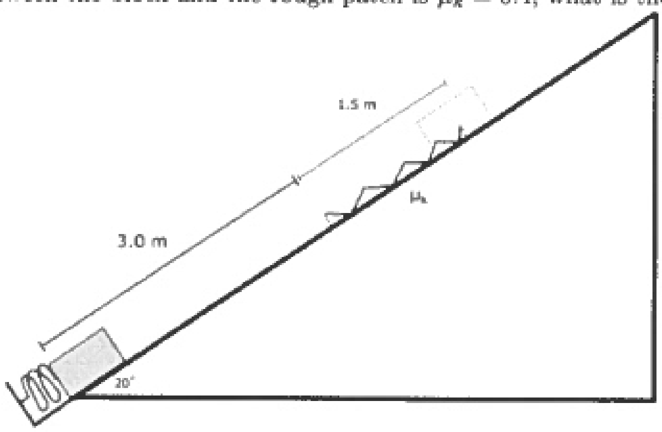
$$\boxed{= 392 \text{ Hz}}$$

(or)

$$f_1 \propto \frac{1}{L}$$

If L is halved, f_1 is doubled

Problem 5: A block of mass m sits at the base of a frictionless ramp inclined at 20° above the horizontal, pressed against a spring with a spring constant of 800 N/m. The spring is initially compressed by 0.38 m. When the block is released, it travels up the ramp until it encounters a rough patch located 3.0 m up the ramp. The block stops moving after sliding 1.5 m along the rough patch. If the coefficient of kinetic friction between the block and the rough patch is $\mu_k = 0.4$, what is the mass m of the block?



Initial energy : Gravitational $PE_i = \phi$

Elastic $PE_i = \frac{1}{2} k (\Delta x)^2$

$= \frac{1}{2} \times 800 \times (0.38)^2$ J

$= 57.76$ J

$KE_i = \phi$

Final energy : Gravitational $PE_f = \cancel{\phi}\ mgh_f = \underset{\text{mass}}{m} g \underset{\text{meter}}{(4.5\,m)} \sin 20°$

$KE_f = \phi$

Elastic $PE_f = \phi$

Work done by friction : $F_N = mg \cos 20°$

$|f_k| = 0.4\, mg \cos 20°$

Work $= -|f_k| \times 1.5$ m

$= -\underset{\text{meter}}{(0.6\,m)}\ \underset{\text{mass}}{m} g \cos 20°$

Using work energy theorem

$$ME_f = ME_i + Work$$

(Ignoring units)

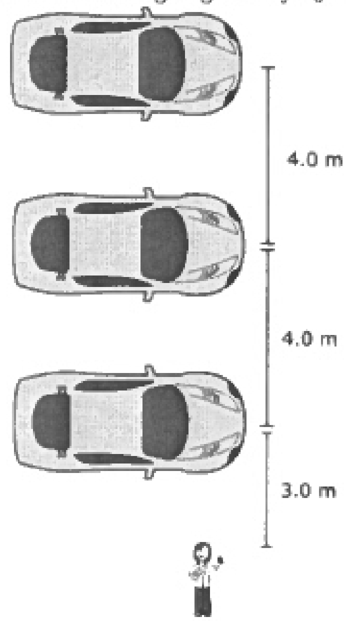
$$mg(4.5\sin 20^\circ) = 57.76 - 0.6\,mg\cos 20^\circ$$

$$mg\left[4.5\sin 20^\circ + 0.6\cos 20^\circ\right] = 57.76$$

or $$mg = \frac{57.76}{2.10} = 27.5$$

or $$\boxed{m = 2.75\ kg}$$

Problem 6: A race car engine produces sound with a power of $P_{\text{sound}} = 470$ W in all directions at once. You stand at one end of a line of three race cars, each producing sound P_{sound}, sitting ready to begin a race. You stand 3.0 m away from the nearest race car, and each further car down the line is 4.0 m away from the next car. What is the *intensity level* of the sound that you hear from the line of race cars? The threshold of human hearing is given by $I_0 = 10^{-12}$ W/m^2.



$P = 470$ W

$I = \dfrac{P}{4\pi d^2}$

where d is the distance from the source of sound.

$$I_1 = \frac{P}{4\pi (3\,\text{m})^2}$$

$$I_2 = \frac{P}{4\pi (4+3)^2\text{m}^2} = \frac{P}{4\pi (7\,\text{m})^2}$$

$$I_3 = \frac{P}{4\pi (11\,\text{m})^2}$$

Since the cars are incoherent sources of sound

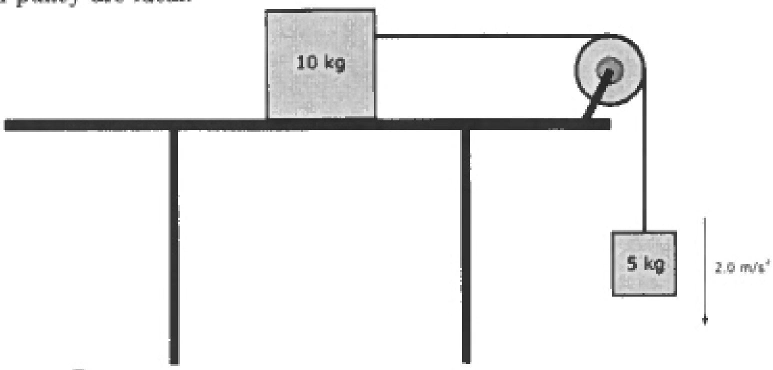
$$I_{tot} = I_1 + I_2 + I_3 = 5.23\ \text{W/m}^2$$

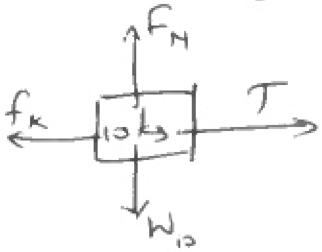
$$\beta = 10\,\text{dB} \log_{10}\left(\frac{I_{tot}}{I_0}\right)$$

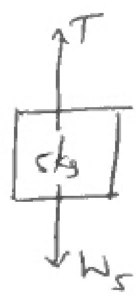
$$= 10\,\text{dB} \log_{10}\left(5.23 \times 10^{12}\right)$$

$$= 127\ \text{dB}$$

Problem 7: A 10 kg block sits on a table with a rough surface. A 5.0 kg block is attached to the 10 kg block by a string over a pulley, as shown. Calculate the coefficient of kinetic friction between the 10 kg block and the table if the hanging block is observed to accelerate downwards at 2.0 m/s^2. Assume that the rope and pulley are ideal.







For 5kg block:

$$T - W_5 = 5\ kg \times (-2\ m/s^2)$$

$$T - 50\ N = -10\ N$$

$$T = 40\ N$$

For 10 kg block:

$$T - f_K = 10\ kg \times (2\ m/s^2) = 20\ N$$

$$f_k = T - 20\ N = 20\ N$$

Also $F_N = 10\ kg \times g = 100\ N$.

Since $f_K = \mu_K F_N$, $\boxed{\mu_K = \frac{f_K}{F_N} = 0.2}$