Homework Assignment 2

Due: Tuesday, May 31, 2016

Problem 1: A cannon can fire a cannonball at a speed of 40 m/s. You're trying to take down a castle with the cannon from a distance of 80 m. The cannonball will be fired from the same height as the base of the castle. What is the range of angles (measured from the horizontal) at which you can fire the cannonball in order to achieve this? Find the minimum and maximum allowed angles. Notice that if the angle is too small or too large, the shot won't reach the castle. The castle is tall enough that you don't have worry about the cannonball going over the it. You may find the trignometric identity "$2\sin\theta\cos\theta = \sin(2\theta)$", and the graph of sine curve useful.

Solution

Let's first find the distance the cannonball can reach when fired at some angle θ. To do this we want the horizontal displacement Δx when the cannonball gets back to the ground, i.e., $\Delta y = 0$. Of course, before it gets back to the ground, it might hit the castle, but we'll get to that in a bit.

Relevant equations are

$$\Delta y = v_{i,y}\Delta t - \frac{1}{2}g(\Delta t)^2$$
$$\Delta x = v_x \Delta t$$

We first solve the y-equation for Δt using $\Delta y = 0$. The two possible solutions are $\Delta t = 0$ or $2v_{i,y}/g$. 0 just corresponds to the time when the cannon is fired (when the cannonball is obviously on the ground). Th other solution corresponds to the time the cannonball gets back to the ground. Plugging that into the x-equation, we get

$$\Delta x = \frac{2v_x v_{i,y}}{g} = \frac{2v_i\cos\theta v_i\sin\theta}{g} = \frac{2v_i^2\cos\theta\sin\theta}{g}$$

where we've used $v_x = v_i\cos\theta$ and $v_{i,y} = v_i\sin\theta$.

The cannonball will strike the castle if this distance is greater than the distance between the cannon and the castle, 80 m, which we'll refer to as D. So we want

$$\frac{2v_i^2\cos\theta\sin\theta}{g} > D$$

or

$$2\cos\theta\sin\theta > \frac{gD}{v_i^2}$$

If you plot the left hand side of this inequality as a function of theta, you'll find that it first increases, reahes a maximum at 45°, and then decreases. We're looking for the two angles at which it equals the right hand side exactly (the cannonball just reaches the wall). All angles inbetween will hit the wall. You can do this this graphically, or use the formula given in the hint. When the cannonball just reaches the wall

$$\sin(2\theta) = \frac{gD}{v_i^2} = \frac{1}{2}$$

One solution is easy. Since $\sin 30° = \frac{1}{2}$, $2\theta = 30°$ or $\theta = 15°$ is a solution. The other solution comes from $\sin 150° = \frac{1}{2}$, which gives $\theta = 75°$ as the second solution. We don't normally see this, since we only look at angles between 0° and 90° in right-triangle trignometry. But since θ can be anywhere from 0° to 90°, 2θ can be from 0° to 180°. Looking at a graph of $\sin\theta$ of $\sin 2\theta$ online would've helped.

So, **if the cannonball is fired at an angle between 15° and 75°, it would've reached the castle**, else it would've dropped short.

Another interesting thing is, the total horizontal distance covered by the projectile (also called the *range* of the projectile) is the same for complementary angles (angles that add up to 90°) of launch. Meaning if you fire the projectile at 10° and 80°, it'll reach the same distance, and the same for 15°, 75°; 30°, 60°; etc. The maximum range happens when $\theta = 45°$, and the complementary angle of is 45° itself.

So you can imagine the range increasing as the angle goes up from 0° to 45° and then decreasing from 45° to 90°.

Problem 2: In the same situation as the previous problem, you decide to fire at an angle of 30° above the horizontal. [30° lies in the allowed range of angles.]

(a) At what height above the ground does the cannonball strike the castle wall?

(b) When it strikes the wall, is the cannonball on its way up, or on its way down?

Solution

This problem is more straight forward to solve than the previous problem, though it uses the same setup.

(a) We need to find Δy when $\Delta x = D$, where D is the distance between the cannon and the castle. The releant equations stay the same as before. We solve the x-equation for Δt to find that $\Delta t = D/v_x$. Then Δy becomes

$$\Delta y = v_{i,y}\left(\frac{D}{v_x}\right) - \frac{1}{2}g\left(\frac{D}{v_x}\right)^2$$

where $v_{i,y} = v_i \sin 30°$ and $v_x = v_i \cos 30°$. Plugging all the numbers in we get $\Delta y \approx$ **19.52 m**

(b) Like most problems in physics 1, there's more than one way to solve this problem.

One way is noticing that if a projectile goes up and makes it all the way back to the ground, then the first half of it will be spent moving up and the second half will be spent coming down. We already have the expression for *range* from the previous solution. Range $= \frac{2v_i^2 \cos\theta \sin\theta}{g} = \frac{v_i^2 \sin 2\theta}{g}$. This distance is 138.6 m for the numbers involved. Then the question is, is 80 m in the first half of 138.6 m or the second? From this it is easy to see that the cannonball is **on it's way down** when it hits the castle.

Another way is to attack the question directly, by looking at the sign of v_y when it hits the castle. If the sign is positive, it's on it's way up, if the sign is negative, it is on it's way down.

$$v_{f,y} = v_{i,y} - g\Delta t = v_i \sin\theta - g\left(\frac{D}{v\cos\theta}\right) = -3.09 \text{ m/s}$$

This also says that the cannonball was **on its way down** when it hits the castle.

Problem 3: Two airplanes take off from an airport at the same time. One flies at 800 km/h directly north and the other flies at 1000 km/h 30° west of south, both velocities measured with respect to the ground.

(a) After 5 hours of continuous flying, find the displacements (including the direction) of both the airplanes. Using this, find the distance between them.

(b) What is the speed of the second airplane relative to the first? Use this relative speed to find the distance between them after 5 hours of flying (since take off). Compare with your answer to (a).

Ignore the effects of earth's curvature. [We go back to the good old days when the earth was flat.]

Solution

(a) After five hours of continuous flying, plane 1 travels $800 \times 5 = 4000$ km and plane 2 travels $1000 \times 5 = 5000$ km. So, if we take north as the $+y$-direction and east as the $+x$-direction

$$\begin{aligned}
\Delta y_1 &= y_{1,f} - y_{1,i} = 4000 \text{ km} \\
\Delta x_1 &= x_{1,f} - x_{1,i} = 0 \\
\Delta y_2 &= y_{2,f} - y_{2,i} = -5000 \cos 30^\circ = -4330.13 \text{ km} \\
\Delta x_2 &= x_{2,f} - x_{2,i} = -5000 \sin 30^\circ = -2500 \text{ km}
\end{aligned}$$

The distance between the two airplanes is

$$D = \sqrt{\left(x_{2,f} - x_{1,f}\right)^2 + \left(y_{2,f} - y_{1,f}\right)^2}$$

Since the starting points of the two airplanes is the same, this can be written as

$$\begin{aligned}
D &= \sqrt{\left(\Delta x_2 - \Delta x_1\right)^2 + \left(\Delta y_2 - \Delta y_1\right)^2} \\
&= \sqrt{(-4330.13 - 4000)^2 + (-2500)^2} \text{ km} = \mathbf{8697.19 \text{ km}}
\end{aligned}$$

(b)

$$\begin{aligned}
\vec{v}_1 &= 800\ \hat{y} \text{ km/h} \\
\vec{v}_2 &= [-1000 \sin 30^\circ\ \hat{x} - 1000 \cos 30^\circ \hat{y}]\ \text{ km/h} \\
&= [-500\ \hat{x} - 866.03\ \hat{y}]\ \text{ km/h}
\end{aligned}$$

The velocity of plane 2 with respect to plane 1 is given by

$$\vec{v}_{21} = \vec{v}_2 - \vec{v}_1 = [-500\ \hat{x} - 1666.03\ \hat{y}]\ \text{ km/h}$$

The relative speed is

$$v_{21} = \sqrt{(-500)^2 + (-1666.03)^2} \text{ km/h} = 1739.44 \text{ km/h}$$

Traveling for 5 hours will cause a separation of $1739.44 \times 5 =$ **8697.2 km** between the planes, which is what we got earlier.

Problem 4: You find yourself in a car chase in an action film! You are chasing the bad guys. Towards the end of the chase, there's a 10 m gap between them and you. You're travelling at a nerve-wracking 50 m/s while the bad guys are getting away at 55 m/s. As a last resort, you decide to fire a hook-gun at the other car. If the hook latches onto the other car, you can use lazy writing to quantum-electrically fry them. The hook-gun can shoot the hook at 45 m/s **(relative to the shooter)**. But the wire connecting the gun and the hook has a maximum length of 13 m, i.e., if the gap between the two cars goes beyond 13 m before the hook gets them, they'll get away.

In the course of answering this question, you'll see how this nailbiter ends. Ignore the size of the cars, i.e., don't worry about the length of your arm, the length of your car's hood, etc.

(a) What is the speed of the hook as seen by the bad guys?

(b) How long will it take the hook to reach the bad guys? [Don't worry about the length of the wire for this part.]

(c) When the hook reaches the bad guys according to (b), what is the distance between the two cars?

(d) Did you get them? [Use the length of the wire now.]

Solution

This problem is all conceptual, the math itself is simple. And again there's multiple ways of solving the problem. We'll look at a few. Lets say the chase is happening from left to right, i.e., in the positive x-direction. First step is to put all the information given to us into equations. And since this is a relative velocity problem, we should be clear about what the different frames of reference are, and use appropriate indices. I'll use g for the ground, 1 for the chasing car, 2 for the bad guys' car, and h for the hook. v_{ab} is the velocity of a with respect to b. Then we have

$$v_{1g} = 50 \text{ m/s}$$

$$v_{2g} = 55 \text{ m/s}$$

$$v_{h1} = 45 \text{ m/s}$$

(a) The first part is to find v_{h2}, the velocity of the hook as seen by the bad guys. One way is to find the velocity of the second car with respect to the first,

$$v_{21} = v_{2g} - v_{1g} = 5 \text{ m/s}$$

This is the speed at which the bad guys are getting away from you. Now using this, we can write

$$v_{h2} = v_{h1} + v_{12} = v_{h1} - v_{21} = \mathbf{40\ m/s}$$

Alternatively, we can first find the velocity of the hook with respect to the ground

$$v_{hg} = v_{h1} + v_{1g} = 95 \text{ m/s}$$

Using this you, we can write

$$v_{h2} = v_{hg} - v_{2g} = \mathbf{40\ m/s}$$

It's all about seeing how to contruct v_{h2} from the three numbers given in the problem using relative velocity formulas. Whichtever way you solve it in, the math is effectively the same, i.e., $45-55+50 = 40$ in some order.

(b) This part is simpler. The hook is traveling towards the bad guys at 40 m/s, from their point of view. The original separation when the hook is fired is 110 m. How long will it take the hook to cover this gap? Ans: $10/40 = \mathbf{0.25\ s}$

The problem can also be solved from the point of view of someone at rest on the ground. In this frame, both the hook and the bad guys are moving. We'll have to set their positions as equal and solve for time. But the problem becomes so much easier in the frame of car 2 since in this frame, car 2 is as rest and only the hook is moving. This is the power of switching referece frames in problem solving.

(c) Again, there're multiple ways of solving this. One is to find how much distance each of the cars travel in 0.25 s and then find the separation between them using that. That's similar to Problem 3(a). But in the spirit of relative velocities (and Problem 3b), we use the velocity of car 2 with respect to car 1. That is the speed at which car 2 is getting away from car 1. We calculated this to be 5 m/s in part (a). In 0.25 s the separation will increase by $5 \times .25 = 1.25$ m. So the separation when the hook gets to the second car is $10{+}1.25 =$ **11.25 m**.

(d) Since the separation is less than 13 m, **you got them**. Congratulations!

If the separation was greater, it means that the hook will never have gotten to the second car. Another way of answering part (d) without answering part (c), is to find the time it takes the separation to become 13 m. This time will be $(13 - 10)/5 = 0.6$ s. So, it'll take 0.6 s for the separation to exceed 13 m, but the hook gets there in 0.25 s. Which means **you'll get them**.

Problem 5: You're walking two dogs. A person on the other side of the street says "Who's a good boy?" (with the voice appropriate for the phrase) and one of your dogs loses it and tries to dart towards him. The other dog, decides to run straight down the road because he's not very smart. You hold your ground, so neither of the dogs can get away. If the first dog is exerting a force of 100 N on you in a direction 30° west of north, and the second dog is exerting 60 N directly north, what is the net force you're exerting on the dogs, i.e., the total force on both dogs combined? Give the magnitude and direction.

Solution

Choosing the x and y directions in the usual way

$$\vec{F}_1 = [-100 \sin 30^\circ \ \hat{x} + 100 \cos 30^\circ \ \hat{y}] \ \text{N} = [-50 \ \hat{x} + 86.6 \ \hat{y}] \ \text{N}$$
$$\vec{F}_2 = 60 \ \vec{y} \ \text{N}$$

The force exerted by you on the dogs is the negative of the force exerted by the dogs on you, by Newton's third law. So, the force exerted by you, $\vec{F}_{\text{ans}}$ is given by

$$\vec{F}_{\text{ans}} = -(\vec{F}_1 + \vec{F}_2) = [50 \ \hat{x} - 146.6 \ \hat{y}] \ \text{N}$$

Magnitude: **154.89 N**. Direction: $\mathbf{-71.17^\circ}$ or $\mathbf{71.17^\circ}$ **SoE** or $\mathbf{18.83^\circ}$ **EoS**

Problem 6: You want to throw a 1 kg object **vertically upwards**. You want to accelerate it from rest to 16 m/s within a distance of 1 m (after which the object will leave your hand). Assuming that the acceleration of the object is uniform, how much force are you exerting on the object during the throw? [Hint: Make sure you account for *all* the forces acting on the object.]

Solution

This is an "$F = ma$" problem. First step, identify all the forces on the object. The forces are the upward force applied by you, and the downward force of gravity. Before we apply Newton's second law, to find the upward force, we need to calculate the upward acceleration, given $v_i = 0$, $v_f = +16$ m/s, $\Delta y = +1$ m. We can use

$$v_f^2 - v_i^2 = 2a\Delta y$$

to find $a = 128$ m/s^2. Then using Newton's second law

$$F_{\text{up}} - mg = ma$$

which gives $F_{\text{up}} = m(a + g) = 1 \times (10 + 128) =$ **138 N**.

Problem 7: Two teams are playing tug of war. Team A has a combined mass of 300 kg and the team B has 320 kg. Assume that the rope is massless. Despite having a lesser combined weight, the team A is winning; the rope (and the two teams) are accelerating at 1 m/s^2 in team A's direction. If team A is exerting a force of 5000 N on team B (through the rope), how much force does team B exert on team A? How do you know this?

Solution

From **Newton's third law**, team B exerts a force equal in magnitude (**5000 N**) in the opposite direction on team A.

Problem 8: In the previous problem,

(a) Find the horizontal component of the contact force between the ground and team A.

(b) Find the horizontal component of the contact force between the ground and team B.

(c) Find the horizotal component of the total contact force between the ground and all the players (both teams combined), by adding the two forces together.

Solution

If both teams are exerting the same force on each other, then how come team A is winning? That's what we seek to answer in this question. The key lies in the force exerted by the ground on the two teams. If not for this force, the two teams will simply move towards each other, like on ice.

I'm not going to draw a diagram, but imagine that team A is on the left, and team B on the right. [We're only interested in horizontal forces here].

(a) Forces on A: Force from B (F_{AB}) and force from the ground (F_{Ag}). Acceleration of team A: -1 m/s. So,

$$F_{\text{Ag}} + 5000 \text{ N} = 300 \times (-1) \text{ N}$$

which gives $F_{\text{Ag}} = -$**5300 N**. The ground pushes team A to the left.

(b) Forces on B: Force from A (F_{BA}) and force from the ground (F_{Bg}). Acceleration of team B: -1 m/s. So,

$$F_{\text{Bg}} - 5000 \text{ N} = 320 \times -1 \text{ N}$$

which gives $F_{\text{Bg}} =$ **4680 N**. The ground pushes team B to the right.

(c) If you're careful with the signs, you'll get the sum of F_{Ag} and F_{Bg} to be **-620 N**.

Problem 9: Now treat all the players involved (teams A and B combined) as one system. Applying Newton's 2$^{\text{nd}}$ law to this system, find the net horizontal force acting on it. Compare with your answer to Problem 8 (c). [Hint: Recall what the acceleration of this sytem is.]

Solution

We the two teams are treated as one system, the forces they exert on each other don't count as external forces. For this system, the mass is 300+320 = 620 kg, and the acceleration is -1 m/s. The net external horizontal forces that causes this acceleration is $m \times a =$ **-620 N** as expected from Problem 8(c).

Problem 10: An object of mass 10 kg is at rest on a rough horizontal surface while being pulled horizontally by a force of 10 N. ~~The coefficient of static friction μ_s between the surfaces in contact is 0.2, and the coefficient of kinetic friction μ_k is 0.18.~~ What is the magnitude of the frictional force in play? ~~Is it static or kinetic?~~

Solution

The object is at rest, which means that the net (horizontal) forces much add upto 0. So the magnitude of the frictional force is **10 N** to cancel the applied force of 10 N.