# Homework Assignment 3

Due: Monday, June 6, 2016

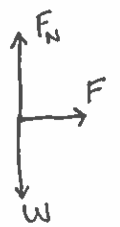
**Problem 1:** A block sits on a horizontal, frictionless table. A force of 100 N pushes horizontally on the block, to the right.

(a) Draw a free-body diagram for the block. What are the $x$- and $y$-components of the net force?

(b) If the block accelerates with $\vec{a} = 2.5\,\hat{x}$ m/s$^2$, what is the mass of the block?

(c) The force is now removed and the table is slanted. At what angle $\theta$ must the table be inclined for the block to have the same acceleration as in part (b)?

(a)



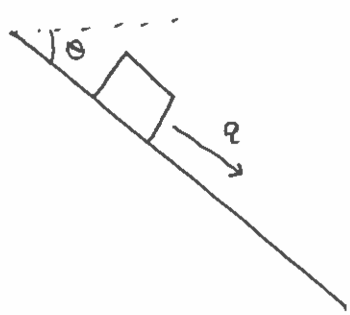

$$F_{net_x} = F$$

$$F_{net_y} = F_N - W$$

(b) Given $\vec{F} = 100\text{N}\,\hat{x}$ and $\vec{a} = 2.5\,\text{m/s}^2\,\hat{x}$, we see that

$$\boxed{m = \frac{F}{a} = \frac{100\text{N}}{2.5\,\text{m/s}^2} = 40\text{ kg}}$$

(c)





Here we see a component of the weight points in the x-dir. It is this force that will create the acceleration.

$$F_{net\,x} = W_x \qquad W_x = W\sin\theta$$

$$F_{net\,y} = F_N - W_y \qquad W_y = W\cos\theta$$

Thus we see that

$$W_x = ma$$

$$\rightarrow W\sin\theta = ma \quad \text{since } W = mg\text{, this becomes}$$

$$a = g\sin\theta \quad \text{we can solve for } \theta:$$

$$\boxed{\theta = \arcsin\left(\frac{a}{g}\right) = \arcsin\left[\frac{2.5}{10}\right) = 14.5^\circ}$$

**Problem 2:** A 5000 kg spaceship floats at rest in deep space, far away from any other object. The ship's computer suddenly malfunctions, causing three of the ship's thrusters to randomly fire. The thrusters push with the following forces:

$$\vec{F}_1 = (5500\,\hat{x} + 1200\,\hat{y})\,\text{N},$$
$$\vec{F}_2 = (-3300\,\hat{x} - 2600\,\hat{y})\,\text{N},$$
$$\vec{F}_3 = (-2000\,\hat{x} + 4000\,\hat{y})\,\text{N}.$$

It takes the astronaut on board 90 s to fix the computer malfunction. What is the magnitude of the ship's displacement when the computer is fixed?

The strategy here is to calculate the net force acting on the spaceship. From $F_{net}$ and the mass we can find the acceleration and then use kinematics.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (5500 - 3300 - 2000)\hat{x} + (1200 - 2600 + 4000)\hat{y} \text{ N}$$

$$= 200\,\hat{x} + 2600\,\hat{y} \text{ N}$$

$$\longrightarrow F_{net} = \sqrt{200^2 + 2600^2} \text{ N} = 2608 \text{ N}$$

From Newton's 2nd law,

$$F_{net} = ma$$

$$\longrightarrow a = \frac{F_{net}}{m} = \frac{2608}{5000} = 0.52 \text{ m/s}^2.$$

This acceleration points in the direction of $\vec{F}_{net}$. We know $\Delta t = 90$ s and $v_i = 0$, so

$$\Delta x = v_i \Delta t + \frac{1}{2} a (\Delta t)^2 = \frac{1}{2} a (\Delta t)^2$$

2106 m

$$\longrightarrow \boxed{\Delta x = \frac{1}{2}(0.52)(90)^2 = \cancel{[illegible]} \text{ m}}$$

Quite far for a little mistake :(

**Problem 3:** Brad, an MMA fighter, has failed to qualify for his next fight by being slightly over the cut-off weight during his weigh-in. He steps on the scale (on the surface of the Earth) and finds that his weight is 3 N greater than the Middleweight cutoff of 840 N. He decides to solve this problem by fighting on Mars instead of Earth (naturally) where the acceleration due to gravity is roughly $g_{\text{mars}} = 4.0\ \text{m/s}^2$.

(a) What is Brad's mass? How does it change on the surface of Mars?

(b) Does Brad qualify for a Middleweight fight on Mars at his current mass? Assume there is no minimum weight cutoff. How much *more* mass could Brad "bulk up" and still qualify for his weight class on Mars?

(c) Will the current rules for defining weight classes be valid in the distant future, when humans have colonized multiple planets? What physical quantity would be better used to categorize fighters, and why?

(a) His weight is $W = 843\ \text{N}$, which is related to his mass and $g_{\text{Earth}}$:

$$W = mg$$

$$\rightarrow \boxed{m = \frac{W}{g} = 84.3\ \text{kg}.}$$

This does not depend on the planet he is on!

b) Well, his "weight" on Mars is

$$W_{\text{mars}} = m\, g_{\text{mars}} = (84.3\ \text{kg})(4.0\ \text{m/s}^2) = 337\ \text{N} < \text{cutoff}.$$

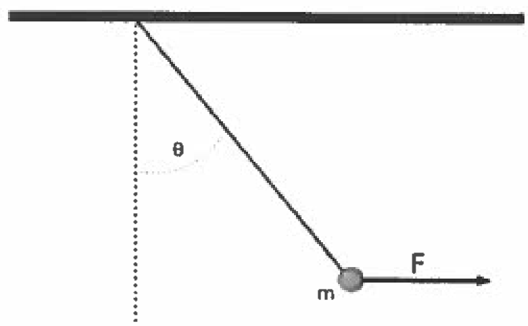
Yes, he qualifies. The max mass allowed to still qualify is

$$m_{\text{max}} = \frac{840\ \text{N}}{4} = 210\ \text{kg}$$

So he could add 125.7 kg of mass and still fight in his weightclass.

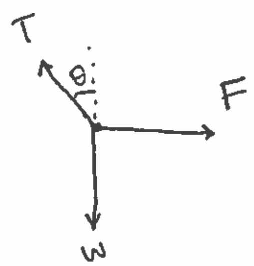
(c) We see that it would be more prudent to use mass as a limiting factor over weight, as it doesn't depend on the gravitational acceleration of the planet.

**Problem 4:** A ball of mass $m = 3.0$ kg hangs from a massless string. A force $F = 45$ N pushes on the ball horizontally, such that the ball hangs motionless as in the figure below:




(a) Find the tension in the string.

(b) What is the angle $\theta$?




$$F_{net_x} = F - T_x = 0 \qquad F = 45 \text{ N}$$

$$F_{net_y} = T_y - W = 0 \qquad W = 30 \text{ N}$$

From the net force equations, we see

$$T \sin\theta = F$$

$$T \cos\theta = W.$$

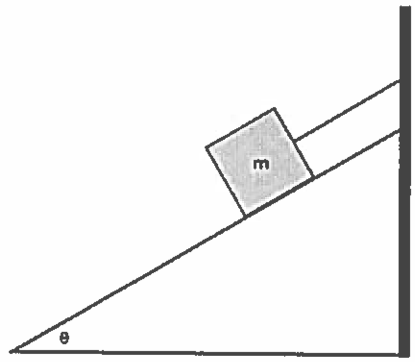
By dividing these equations, we get

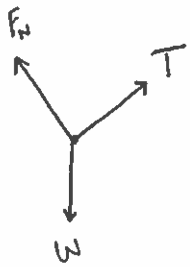
$$\begin{array}{rcl} T\sin\theta & = & F \\ \div\ T\cos\theta & = & W \\ \hline \tan\theta & = & F/W \end{array} \longrightarrow \boxed{\theta = \arctan\left(\frac{F}{W}\right) = 56.3^\circ} \text{ (b)}$$

We can use $\theta$ now above in either equation to find T. OR we notice that the x- and y-components of the tension are F and W, resp. Thus

$$\boxed{T = \sqrt{T_x^2 + T_y^2} = \sqrt{F^2 + W^2} = 54.1 \text{ N}} \text{ (a)}$$

**Problem 5:** A block, weighing 600 N, sits tied to a wall on a frictionless incline. The rope tying the block to the wall can support a maximum tension $T_{\max} = 400$ N before it snaps. What is the maximum allowed angle of inclination, $\theta$, before the string breaks? Assume that the tension is parallel to the incline.




$$F_{net_x} = T - W_x = T - mg\sin\theta = 0$$

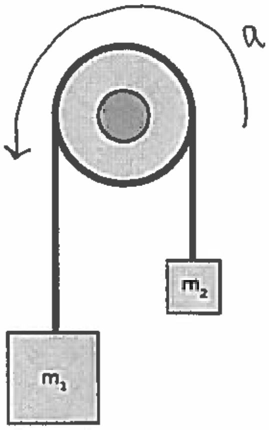
$$F_{net_y} = F_N - W_y = F_N - W\cos\theta = 0$$

We see that by evaluating $F_{net_x} = 0$ for $T_{max}$, this will give us an expression for $\theta$:

$$T_{max} - W\sin\theta = 0$$

$$\rightarrow \boxed{\theta = \arcsin\left(\frac{T_{max}}{W}\right) = 41.8^\circ}$$

**Problem 6:** The Atwood machine (pictured below) is a useful tool for comparing the masses of two objects; two objects hang from a massless string around a massless, frictionless pulley, and are released from rest. When $m_1 = m_2$, the system remains at rest. When the masses are unbalanced, the system accelerates. Assume for this problem that $m_1 = 12$ kg and $m_2 = 5$ kg.

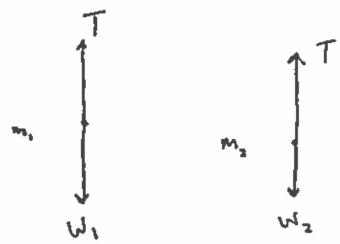



(a) Draw a free-body diagram for each mass, and write the net force equations for each.

(b) Find the acceleration of the system. What direction does each mass accelerate?

(c) What is the tension in the string connecting the two masses?

(a)




$F_{net_1} = T - W_1$

$F_{net_2} = T - W_2$

ignore x-direction.

(b) From above, we see that

$F_{net_1} = -m_1 a \,,\quad F_{net_2} = m_2 a$

$\rightarrow \; T - W_1 = -m_1 a \,,\quad T - W_2 = m_2 a$ ; we may eliminate T and find a:

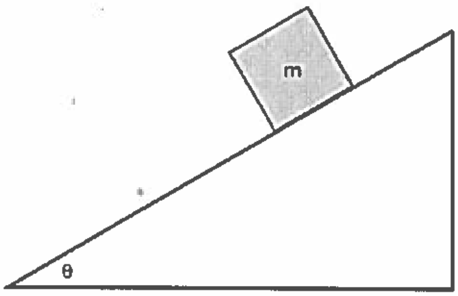
$\rightarrow \; W_1 - W_2 = (m_1 + m_2) a$ so $\boxed{a = \frac{W_1 - W_2}{m_1 + m_2} = 4.12 \text{ m/s}^2}$

Mass 1 accelerates down; mass 2 accelerates up.

(c) Now that we know a, we may solve for T:

$$\boxed{T = W_1 - m_1 a = 70.6 \text{ N}}$$

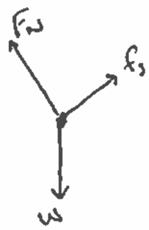
**Problem 7:** A cereal box of mass $m$ rests on an incline, $\theta = 30°$. The coefficients of static and kinetic friction between the cereal box and incline are $\mu_s = 0.55$ and $\mu_k = 0.3$, respectively.




(a) What amount of cereal (i.e. what mass $m$ of the box) will cause the box to slip down the incline? Will it ever slip on its own? Explain your answer.

(b) The box has now been pushed and is sliding down the ramp. If the box has a mass $m = 50$ kg (lots of cereal!), what is its acceleration down the incline?

a)




$F_N - mg\cos\theta = 0$

$f_s - mg\sin\theta = 0$

In general, we would think that, as the mass increases, $mg\sin\theta$ will increase against $f_s$ until we saturate $f_{s,max}$:

$\rightarrow f_{s,max} = mg\sin\theta$ Then we could increase m further and the block would begin to slide.

However, $f_{s,max} = \mu_s F_N = \mu_s mg\cos\theta$, so we see that both masses cancel out:

$\mu_s g\cos\theta = g\sin\theta$ [no m-dependence]

We see that the condition to slip depends only on $\mu_s$ and $\theta$, not m.

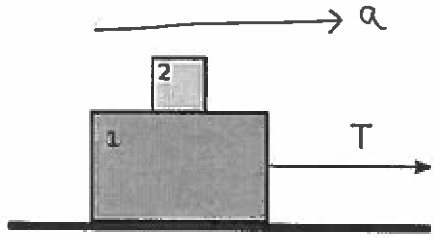
b) $\boxed{\text{Here, } f_s \rightarrow f_k}$ Here $\mu_s = 0.55$ and $\tan(30°) = 0.57$, which satisfies $\mu_s < \tan\theta$, so it slips!

Then $f_k - mg\sin\theta = -ma$ and $f_k = \mu_k F_N = \mu_k mg\cos\theta$.

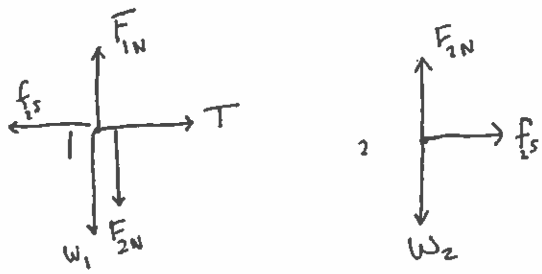
Thus $ma = mg\sin\theta - \mu_k mg\cos\theta$

$$\boxed{a = g(\sin\theta - \mu_k\cos\theta) = 2.4 \text{ m/s}^2 \text{ down incline}}$$

**Problem 8:** Block 1 rests on a frictionless table. Block 2 sits atop Block 1, and the coefficient of static friction between Blocks 1 and 2 is $\mu_s = 0.7$. A rope, tied to Block 1, pulls horizontally with a tension $T$. What maximum tension is allowed in the rope, such that the blocks move without Block 2 slipping? The masses of Blocks 1 and 2 are $m_1 = 25$ kg and $m_2 = 6$ kg, respectively. HINT: start by drawing a free-body diagram for each block and consider the third-law-pair between Blocks 1 and 2.




the FBD,




For block two to be on the verge of slipping, we see that

$$f_{21} = f_{2s,max} = \mu_s F_{2N}$$

So

$$f_{2s} = \mu_s W_2 .$$

$$F_{1x} = T - f_{2s} = m_1 a$$

$$F_{1y} = F_{1N} - W_1 - F_{2N} = 0$$

$$F_{2x} = f_{2s} = m_2 a$$

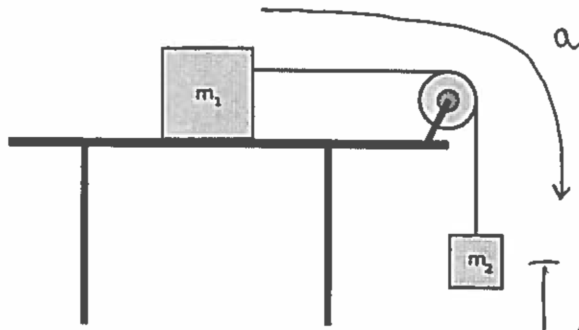
$$F_{2y} = F_{2N} - W_2 = 0$$

We use $F_{2x}$ to solve for $a$ and plug into $F_{1x}$:

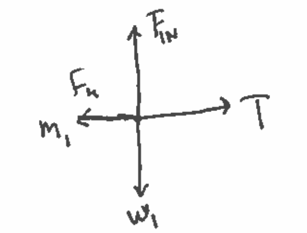
$$T - f_{2s} = \frac{m_1}{m_2} f_{2s}$$

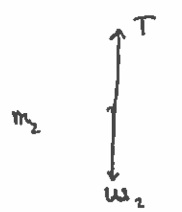
$$\rightarrow \boxed{T = \mu_s W_2 \left(1 + \frac{m_1}{m_2}\right) = 217 \text{ N}}$$

**Problem 9:** Two masses, $m_1 = 7$ kg and $m_2 = 13$ kg, are tied together by a massless rope run over a massless, frictionless pulley. Mass $m_1$ sits on a rough, horizontal table surface (not frictionless!). Both masses are released from rest. If mass $m_2$ falls a distance of 1.5 m in 0.7 s, what is the coefficient of friction between $m_1$ and the table? Is it static or kinetic?




$\Delta y = -1.5\text{m}$ $\quad \Delta t = 0.7\text{s}$




We may use this info to solve for a:

$$\Delta y = -\frac{1}{2} a (\Delta t)^2$$

$$\rightarrow a = \frac{2(1.5)}{(0.7)^2} = 6.12 \text{ m/s}^2$$

$F_{1x} = T - f_k = m_1 a$

$F_{1y} = F_{1N} - W_1 = 0$

$F_{2y} = T - W_2 = -m_2 a$

Solve for T and use in $F_{1x}$:

$$T = W_2 - m_2 a$$

$$(W_2 - m_2 a) - f_k = m_1 a$$

So we solve for $f_k$:

$$f_k = W_2 - (m_1 + m_2) a$$

and use $f_k = \mu_k F_{1N} = \mu_k W_1$

$$f_k = \mu_k W_1 = W_2 - (m_1 + m_2) a$$

$$\rightarrow \boxed{\mu_k = \frac{W_2 - (m_1+m_2)a}{W_1} = 0.11}$$

As there is relative motion between $m_1$ and the table it is kinetic friction.

**Problem 10:** Francine steps onto a scale in her bathroom and reads a weight of 168 N. She then enters a 1500 kg elevator (with her scale) and measures her weight to be 220 N as the elevator begins to move. What is the tension in the cable supporting the elevator? Ignore the mass of the scale.

Her true weight is $W_{F,true} = 168N$, while the normal force she experiences in the elevator is $W_{app} = F_N = 220N$. As $W_{app} > W_{true}$, she is accelerating upwards:

$$F_N - W_{true} = m_f a$$

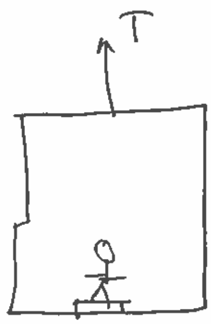
$$\rightarrow a = \frac{F_N - W_{true}}{m_f} = \frac{220-168}{16.8} = 3.1 m/s^2$$

Her mass is

$$m_f = \frac{W_{true}}{g} = 16.8 \text{ kg}.$$

Okay, Francine is 4 years old...

Now, consider the elevator:




$$T - W_{e,true} - W_{F,true} = (m_e + m_f) a$$

So

$$T = W_{e,true} + W_{f,true} + (m_e + m_f) a$$

$$= 15000 N + 168 N + (1500 + 16.8)(3.1 m/s^2)$$

$$\boxed{T = 19,870 N}$$