Homework Assignment 4

Due: Monday, June 13, 2016

Problem 1: A lion of mass 150 kg and a grizzly bear of mass 300 kg are charging directly towards each other at 75 km/h and 40 km/h respectively. Find the magnitude of momentum of

(a) the lion.

(b) the bear.

(c) the lion-bear system.

Solution

Let the direction of the lion's motion be positive, and that of the bear be negative. The speeds of the lion and bear in m/s are respectively 20.83 m/s and 11.11 m/s.

(a) $p_{\text{lion}} = m_{\text{lion}} v_{\text{lion}} =$ **3124.5 kg m/s or 11250 kg km/hr**

(b) $p_{\text{bear}} = m_{\text{bear}} v_{\text{bear}} =$ **−3333.3 kg m/s or −12000 kg km/hr**. The minus sign will be dropped in the magnitude.

(c) $p_{\text{system}} = p_{\text{lion}} + p_{\text{bear}} =$ **−208.8 kg m/s or −750 kg km/hr**. Again, the minus sign will be dropped in the magnitude.

Problem 2: A tennis ball of mass 60 g hits a wall horizotally at a speed of 30 m/s and rebounds with the same speed. Just before and just after collision, the ball is traveling horizontally.

(a) What is the magnitude of momentum of the ball just before collision? What is the magnitude of momentum of the ball just after collision?

(b) What is the magnitude of the change in momentum of the ball?

(c) If the collision between the wall and the ball lasts for 0.1 s, what is the magnitude of average force during collision?

Solution

Let the direction in which the ball is traveling before collision be positive, and the direction after collision be negative. This choice won't affect any of the answers because we're only interested in the magnitudes.

(a) $p_i = .060 \times 30$ kg m/s = **1.8 kg m/s**. $p_i = .060 \times (-30)$ kg m/s = **−1.8 kg m/s**.

(b) $\Delta p = p_f - p_i =$ **−3.6 kg m/s**.

(c) Using impulse-momentum theorem $\Delta p = F_{\text{av}} \Delta t$, we get $F_{\text{av}} =$ **−36 N**.

The minus signs will be dropped in the magnitudes for all three parts.

Problem 3: A person of mass 70 kg is standing on an icy (frictionless) horizontal surface with a metal ball of mass 5 kg. The person throws the ball at a speed of 2 m/s, at an angle of 60° above the horizontal. Both speed and angle are as measured *with respect to the ground*.

(a) What is the recoil speed with which the person slides after throwing the ball?

(b) What is velocity of the ball as seen by the person? Give the magnitude and direction (angle above horizontal).

Solution

(a) In this problem, only the x-component of momentum is conserved in the collision (the throw). The person's velocity in the y-direction is 0 throughout. Because there are relative velocities invloved, we'll be carefull with indices in our solution (B for ball, P for person, and G for the ground). We'll do away with the index for initial and final; all velocities in the solution are after the throw. We're given that the velocity of the ball with respect to the ground is 2 m/s at an angle of 60° above the horizontal. Let's pick the x-axis along the horizontal component of the ball's velocity. Then,

$$v_{BG}^x = 2\cos 60^\circ \text{ m/s} = 1 \text{ m/s}$$
$$v_{BG}^y = 2\sin 60^\circ \text{ m/s} = 1.73 \text{ m/s}$$

$$v_{PG}^y = 0$$
$$v_{PG}^x = ?$$

From conservation of momentum in the x-direction we have

$$m_B v_{BG}^x + m_P v_{PG}^x = 0 \tag{1}$$

Solving this we get $v_{PG}^x = \mathbf{-0.07 \text{ m/s}}$, -0.071 m/s if we include an extra sig-fig. This is the recoil velocity of the person; the magnitude will hae the minus sign dropped.

(b) Now we use

$$\vec{v}_{BP} = \vec{v}_{BG} - \vec{v}_{PG} \tag{2}$$

to find

$$v_{BP}^x = 1 - (-0.07) = 1.07 \text{ m/s}$$
$$v_{BP}^y = 1.73 - 0 = 1.73 \text{ m/s}$$

This corresponds to a magnitude of **2.04 m/s** and an angle of **58.26°**. This is very close to the velocity with respect to the ground, because the mass of the person is much larger than the ball's, so the recoil is relly slow.

Problem 4: Same problem, slightly different math. A person of mass 70 kg is standing on an icy (frictionless) horizontal surface with a metal ball of mass 5 kg. The person throws the ball at a speed of 2 m/s, at an angle of 60° above the horizontal. Both speed and angle are as measured *with respect to the person.* What is the velocity of the ball with respect to the ground? Give the magnitude and direction (angle above horizontal).

Solution

We've almost completely solved problem 4, in solving problem 3. The only difference is in what's given and what's asked for. But before that, let's look at some incorrect approches, which will help better understand some concepts.

The recoil velocity of the person will not be the same as before. To see this notice that the ball's velocity with respect to the person isn't the same in the two problems, although close. We have to set up the equation for conservation of momentum again, if we want to find (and potentially use) the recoil velocity of the person.

Now to use the conservation of momentum, we should resolve the given velocity of the ball in the x- and y directions and plug directly into equation (1) in problem 3, right? **Not quite**. This is because, the velocity given to us is in the frame of the person, but that equation needs the velocity in the ground frame of reference. We've always understood this implicitly, but in writing the conservation of momentum equations, we look at all momenta (and velocities) from the same frame of reference. It won't make much sense to apply conservation to a problem involving collisions on a pool table, if all velocities are measured in the table's frame, except one which is measured from a moving car.

Now, onto the actual solution. We will solve the same two numbered equations seen before, except for different variables. Re-writing equation (1) above,

$$m_B v^x_{BG} + m_P v^x_{PG} = 0$$

In the x-direction, equation (2) gives

$$v^x_{BP} = v^x_{BG} - v^x_{PG}$$

Solving these (we know that $v^x_{BP} = 2\cos 60° = 1$ m/s), you'll find that $v^x_{BG} = 0.93$ m/s and $v^x_{PG} = -0.07$ m/s (-0.067 m/s if you include an extra sig-fig).

In the y-direction, equation (2) still gives $v^y_{BP} = v^y_{BG} = 1.73$ m/s. From this we can find the velocity of the ball with respect to the ground to be **1.98 m/s** at an angle of **61.68°**.

You'll get a similar answer if you assume incorrectly from problem 3 that $v^x_{PG} = -0.07$ m/s. That's because the recoil velocity isn't much, and so using momenta from different frames in momentum conservation doesn't affect the answer much. As the problem setter, I can increase the degree to which the incorrect method's answer will be off by making the mass of the ball closer to that of the person, thereby making the recoil significant. But although you can use $v^x_{PG} = -0.07$ m/s, $v^x_{PG} = -0.07 v^x_{BG}$ is a result that holds in both problem 3 and 4. In fact, with enough familiarity with the properties of linear equations and their solutions, you can solve the problem by saying that the ratio $v^x_{BG} : v^x_{BP}$ should be the same in both problem 3 and 4. So,

$$\frac{v^x_{BG}}{1} = \frac{1}{1.07} \text{ m/s}$$

leading to $v^x BG = 0.93$ m/s. Don't worry too much if this way of solving the problem is not clear.

Problem 5: Particles A, B and C of masses 5 g, 10 g and 15 g respectively are located at coordinates (5 m, 7 m), (3 m, 4 m), (-6 m, 2m) respectively [cartesian (x, y) coordinates]. Find the coordinates of the center of mass.

Solution

This problem is more or less just plug and chug.

$$x_{\text{CoM}} = \frac{m_\text{A} x_\text{A} + m_\text{B} x_\text{B} + m_\text{C} x_\text{C}}{m_\text{A} + m_\text{B} + m_\text{C}} = -1.17 \text{ m}$$

$$y_{\text{CoM}} = \frac{m_\text{A} y_\text{A} + m_\text{B} y_\text{B} + m_\text{C} y_\text{C}}{m_\text{A} + m_\text{B} + m_\text{C}} = 3.5 \text{ m}$$

The coordinates of the center of mass is **(-1.17 m, 3.5 m)**. Note that even in this form of expressing the position, the units are important.

Problem 6: A two object system is known to have its center of mass moving at 3 m/s in the +x-direction. The two components have masses M and $M/2$. The heavier component is moving at 4 m/s at an angle of 60° above the +x-axis. Find the velocity (magnitude and direction) of the lighter compenent.

Solution

The given velocities are to be plugged into the equation for the velocity of center of mass, to find the velocity of the lighter component. **Since velocity is a vector, this needs to be done separately to the x and y components of velocity. The same applies to the law of conservation of momentum as well, if the momentum is conserved in more than one dimension.**

Let the heavy object be object #1, and the lighter object be object #2. The equations for the x and y components of the velocity of the center of mass give

$$3 \text{ m/s} = \frac{M \times 4\cos 60^\circ \text{ m/s} + \frac{M}{2} v_{2,x}}{M + \frac{M}{2}}$$
$$0 = \frac{M \times 4\sin 60^\circ \text{ m/s} + \frac{M}{2} v_{2,y}}{M + \frac{M}{2}}$$

Solving these, we get $v_{2,x} = 5$ m/s and $v_{2,y} = -6.93$ m/s. This corresponds to a magnitude of **8.54 m/s** and a direction of **−54.18°** or **54.18° below the x-axis**. Note, that in solving for $v_{2,x}$ and $v_{2,y}$, M cancel out.

Problem 7: On a pool table, the cue ball, traveling at a speed of v, hits an object ball at rest head on (one dimensional collision). The cue ball and the object ball have the same mass. After collision, the cue ball keeps moving forward at a third of its original speed. What is the speed of the object ball after collision?

Solution

Conservation of momentum in one dimension:

$$\begin{aligned} v_{c,i} &= v \\ v_{o,i} &= 0 \\ v_{c,f} &= \frac{1}{3}v \\ v_{o,f} &= ? \end{aligned}$$

$$m_c v_{c,i} + m_o v_{o,i} = m_c v_{c,f} + m_o v_{o,f}$$

Since $m_c = m_o$, we can cancel out the masses. Solving this equation, we get $v_{o,f} = \mathbf{\frac{2}{3}v}$.

Problem 8: A car is traveling down a road at 50 m/s when Captain America latches onto it from the side of the road. If Captain America's mass is 100 kg, and the speed of the car immediately after he gets on it is 45 m/s, what is the mass of the car (without including Captain America)? Assume that Cap was at rest on the ground before latching on.

Solution

One dimensional momentum conservation again. This time we're solving for the mass of one of the objects.

$$\begin{aligned} m_{\text{car}} \times 50 + m_{\text{cap}} \times 0 &= (m_{\text{car}} + m_{\text{cap}}) \times 45 \\ \Rightarrow m_{\text{car}} \times (50 - 45) &= m_{\text{cap}} \times 45 = 4500 \text{ kg} \\ \Rightarrow m_{\text{car}} &= \frac{4500}{5} \text{ kg} = \mathbf{900\ kg} \end{aligned}$$

Problem 9: On a pool table, the cue ball traveling at speed of 2 m/s in the +x-direction hits an object ball at rest. The cue ball and the object ball have the same mass. After collision, the x-component of the cue ball's velocity is 0.5 m/s, and the y-component of the object ball's velocity is $\frac{\sqrt{3}}{2}$ m/s. Find the velocities of the cue ball and object ball (magnitude and direction). What is the angle between the velocity vectors of the cue ball and object ball after collision?

Solution

Two dimension conservation momentum, this time.

Conservation of momentum in the x-direction gives $v^x_{o,f} = 1.5$ m/s; and in the y- direction it gives $v^y_{c,f} = -\frac{\sqrt{3}}{2}$ m/s.

Converting to magnitudes and angles, we find that the cue ball travels at **1 m/s** at an angle of $-\mathbf{60}°$; and the object ball travels at $\sqrt{\mathbf{3}}$ **m/s** at an angle of $+\mathbf{30}°$. The angle between their final trajectories is $\mathbf{90}°$.

Problem 10: Two objects A and B collide. Just before collision the two objects are moving only in the x-direction (either +x or -x direction). Just after collision, object A keeps moving in the x-direction (either +x or -x direction). Can object B have a non-zero compenent of velocity along the y-direction just after collision? If yes, give an example. If no, explain why not.

Solution

Before collision, the total momentum in the y-direction is 0. So by conservation of momentum, after collision as well the total momentum in the y-direction should be 0. This means that if B has a non-zero component of velocity in the y-direction, then A's velocity will need to have some component in the y-direction as well, so that the momenta can can out (in the y-direction); a contradiction since A is given to be moving only in the x-direction even after collision. So, B cannot have a non-zero y-component to its velocity. To explain with an equation,

$$m_{\mathrm{A}} v^y_{\mathrm{A},i} + m_{\mathrm{B}} v^y_{\mathrm{B},i} = m_{\mathrm{A}} v^y_{\mathrm{A},f} + m_{\mathrm{B}} v^y_{\mathrm{B},f}$$

If $v^y_{\mathrm{A},i}$, $v^y_{\mathrm{B},i}$, and $v^y_{\mathrm{A},f}$, then so is $v^y_{\mathrm{B},f}$ (irrespective of what the masses of the objects are, and what the angle of contact is between the objects colliding).

This was the expected answer. And all this is true unless, as some of you have noted, in the problem the momentum of the 'A+B system' isn't conserved in the y-direction.