Homework Assignment 5

Due: Tuesday, July 5, 2016

Problem 1: Assume that collisions on a pool table between the cue ball and object ball are elastic and that the cue ball and object balls have the same mass (these are fairly realistic assumptions). You want to strike an object ball with the cue ball so that it gets into a pocket. But, you're not sure if this will deflect the cue ball into a different pocket, causing a scratch - which you don't want. In this problem we'll calculate the deflection of the cue ball. Choosing the x-direction to be along the shot, we can set the problem up as follows:

On a pool table, a cue ball traveling along the $+x$-direction hits an object ball at rest. After the collision, the object ball moves at an angle θ above the x-axis. The collision is elastic and the cue ball and object ball have the same mass.

(a) Find the direction in which the cue ball travels after collision. Answer in terms of θ.

(b) Find the angle between the trajectories of the object ball and the cue ball after collision.

This problem is trig-intensive and challenging. So, you may use the additional information that if the speed of the cue ball before collision is v, the speed of the object ball after collision will be $v\cos(\theta)$. This simplifies the problem. You may find the following trigonometric identities useful: $\sin^2(\theta) + \cos^2(\theta) = 1$, $\sin(90° - \theta) = \cos(\theta)$, $\cos(90° - \theta) = \sin(\theta)$

Problem 2: Two objects A and B of masses 5 kg and 2 kg, respectively, are traveling towards each other with velocities $4\hat{x}$ m/s and $-3\hat{x}$ m/s, respectively. The collision is totally inelastic, i.e., after collision they stick and move together.

(a) What is the final velocity of the combined object after collision?

(b) What are the total kinetic energies before and after collision? What is the loss in kinetic energy during the collision? What percentage of kinetic energy was lost during the collision?

(c) What is the velocity of the center of mass of A and B before collision?

Now assume that you're observing the process from a car that is traveling with the velocity you calculated in (c). This is known as the center of momentum frame of the system.

(d) In the new frame of reference, what are the velocities of A and B before collision?

(e) In the new frame of reference, what is the final velocity of the combined object after collision?

(f) In the new frame of reference, what are the total kinetic energies before and after collision? What is the loss in kinetic energy during the collision? What percentage of kinetic energy was lost during the collision?

Problem 3: You are dragging a box of mass 3 kg along a rough horizontal surface by applying a force of 50 N at an angle of 60° above the horizontal. We're looking at a segment of this effort, during which the object travels 10 m. The initial and final velocities of the object for this segment are respectively 3 m/s and 5 m/s.

(a) Calculate the work done by you during this segment.

(b) Calculate the total work done on the object during this segment.

Problem 4: You throw a bouncy ball vertically down towards the ground at a speed of 10 m/s from a height of 2 m. It bounces vertically upwards off the floor.

(a) What is the maximum height reached by the ball (from ground level), if the collision with the floor is elastic?

(b) What is the maximum height reached by the ball (from ground level), if exactly one-eighth of the kinetic energy of the ball just before collision is lost during the collision with the floor.

Problem 5: We have a rod of length 1 m composed of two parts of length 60 cm and 40 cm respectively, made from two different materials. The 60 cm part has a mass of 500 g and 40 cm part has a mass of 400 g. The rod is pivoted at its center (50 cm mark), so that it has two possible vertical orientations: one with the 40 cm part above and the 60 cm part below, and the other with the 60 cm part above and the 40 cm part below. Calculate the difference in the gravitational potential energy for the two orientations. Which orientation has the lesser energy? The rod will tend to/prefer to be in this configuration.

Problem 6: The gravitational field strength on the surface of some planet of radius 10000 km is 20 N/kg. At what altitude above the surface will the gravitational field strength drop down to 19 N/kg?

Problem 7: We have a planet of mass M and radius R with no atmosphere (so we can ignore air resistance).

(a) What is the minimum speed with which an object must be launched from the planet's surface, so that it escapes the gravitational attraction of the planet? Answer in terms of G, M, and R.

(b) If an object is launched vertically upwards from the planet's surface with half this minimum speed, what is the maximum distance (measured from the center of the planet) that this object reaches? Answer in terms of R only.

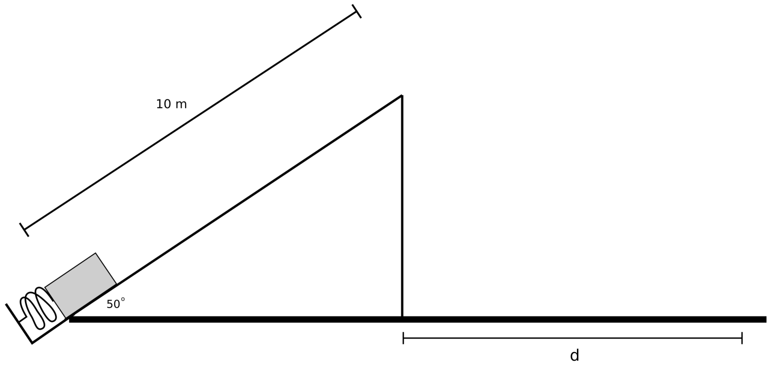
Problem 8: You stretch a rubber band between your fingers and aim it at a fellow classmate sitting in front of you. If the rubber band is stretched 7 cm beyond its equilibrium length and has an effective spring constant $k = 1700$ N/m:

(a) What is the stored energy in the rubber band?

(b) How fast does the rubber band fly out of your hand when you release it? The rubber band has a mass $m = 0.05$ kg; assume it leaves your hand unstretched.

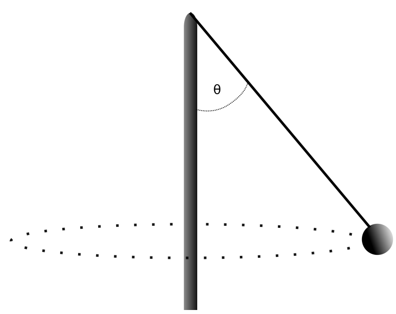
Problem 9: Ryan the adrenaline junkie, mass $m = 75$ kg, is about to do something very foolish. He is strapped to a 15 m bungee cable and is about to jump from the top of a 21-meter-tall bridge (over the freeway). If the bungee cord has a spring constant of 1200 N/m, does he stop before reaching the road beneath him? (He hasn't done the math himself.) Assume that he falls from rest off the bridge, and ignore his own height.

Problem 10: A spring, compressed by $\Delta x = 50$ cm with $k = 4300$ N/m, holds a block of mass $m = 5$ kg in place at the base of a 10 m ramp, inclined at an angle of 50° above the horizontal. The ramp is covered in smooth sandpaper, giving it a coefficient of kinetic friction $\mu_k = 0.2$ when the block slides on it. When the spring is released, the block shoots up the ramp, flying into the air and eventually landing on the ground (at the same height from which it was initially released). What distance d does the block travel from the base of the ramp to where it hits the ground?



Problem 11: A 500 kg rocket accelerates upward at a rate of 14 m/s^2 off the launchpad, where it was sitting at rest. What is the average power output of the rocket's engine if it reaches a vertical height of 2 km when the engines burn out. Assume the engine provides constant thrust.

Problem 12: A 0.5 kg ball is on the end of a massless rope that is 1.2 m in length and attached to a vertical pole. The ball, rope, and pole rotate about the pole's central axis at a constant rate; the ball makes one full rotation in $T = 1.5$ s. What is the angle θ that the rope makes with the pole?



Problem 13: A record sits at rest on a record player. Assume the record has a diameter of 12 inches.

(a) The record speeds up at a constant rate to the play speed (33 rpm) in a short time of 1.2 s. How many rotations does it undergo in this time?

(b) What is the angular acceleration of the record in part (a)?

(c) What is the linear speed of a speck of dust sitting half-way between the edge and the center of the record while the record is playing?

(d) The power to the record player is cut while the record is still spinning (33 rpm) and the needle is still in contact. If the record spins 14 complete times before coming to rest, what is the average angular acceleration created by the record needle on the record?

Problem 14: A curve in the highway has a radius of curvature $R = 900$ m. Assume the curve is unbanked and that cars traveling along this curve have constant speed.

(a) On a sunny day, the coefficient of static friction between the road and normal tires is $\mu_s = 0.6$. What is the fastest speed a car can travel around the curve before skidding off the road?

(b) During the winter, the ice and snow on the road make this curve dangerous to travel at the speed limit. If the fastest possible speed to take the curve is reduced to 30 km/h, what is the new coefficient of friction between the tires and the snowy road?

Problem 15: The International Space Station (ISS) maintains a stable circular orbit about the Earth at an average altitude of 412 km. Use the following values for the Earth's mass and radius, respectively: $M_E = 5.97 \times 10^{24}$ kg, $R_E = 6.37 \times 10^6$ m.

(a) How long does it take the ISS to complete one full orbit?

(b) If the ISS were to reduce its altitude to 350 km while maintaining a circular orbit, would its orbital velocity increase or decrease? By what percentage would its speed change?