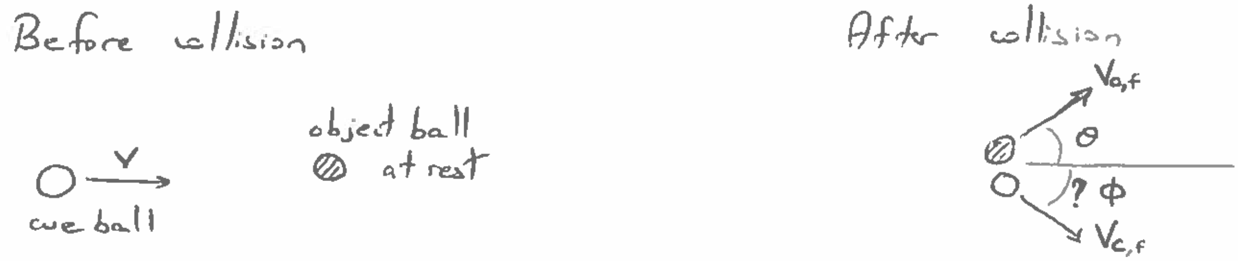
PHY 2053 Sec 72D7

Homework Assignment 5

Solutions

1.) There are multiple ways of solving this problem using vector algebra or trignometry or 'guess and verify.' Here we'll see a trignometric solution. I won't use the additional information provided in the problem.



If we assume that the initial speed of the cue ball v is known (it isn't) along with θ, the remaining unknowns are the two final speeds and the angle of cue ball's trajectory after collision. To solve for these three unknowns, we need three equations. Those will come from

1) Conservation of kinetic energy (elastic collision)
2) Conservation of momentum in the x-direction
3) " " " " " y-direction

Since neither the masses nor the initial velocity is given, the expectation should be that the final answer for ϕ will be independent of these.

Conservation of KE : $\frac{1}{2}Mv^2 + \phi = \frac{1}{2}MV_{c,f}^2 + \frac{1}{2}MV_{o,f}^2$

Conservation of momentum

In x-direction $\quad Mv + \phi = MV_{o,f}\cos\theta + MV_{c,f}\cos\phi$

In y-direction $\quad \phi + \phi = MV_{o,f}\sin\theta - MV_{c,f}\sin\phi$

We've used the fact that their masses are the same here. Dividing both sides by common factors we get

$$v^2 = V_{c,f}^2 + V_{o,f}^2 \qquad - \text{(1)}$$

$$v = V_{o,f}\cos\theta + V_{c,f}\cos\phi \qquad - \text{(2)}$$

$$0 = V_{o,f}\sin\theta - V_{c,f}\sin\phi \qquad - \text{(3)}$$

You can solve these in a few ways

1) Notice that conservation of momentum implies that $\vec{V}_{c,f}$, $\vec{V}_{o,f}$ and $\vec{v}$ can form a ~~vectors~~ triangle and (1) implies that that triangle is right-angled.

2) Use the additional info given in the problem along with equations (2) and (3) [no need for (1)]

Or as we show here:

To eliminate the v^2 in (1), square equation (2)

② $\Rightarrow \quad v^2 = V_{o,f}^2\cos^2\theta + V_{c,f}^2\cos^2\phi + 2V_{o,f}V_{c,f}\cos\theta\cos\phi$ — ②'

And since the other two ~~terms~~ equations now have only velocity^2 terms, square ③ as well

③ $\Rightarrow \quad 0 = V_{o,f}^2\sin^2\theta + V_{c,f}^2\sin^2\phi - 2V_{o,f}V_{c,f}\sin\theta\sin\phi$ — ③'

We know that $\sin^2\theta + \cos^2\theta = 1$

We can exploit this by adding equations ②' and ③'

②' + ③' $\Rightarrow \quad v^2 = V_{o,f}^2(\cos^2\theta + \sin^2\theta) + V_{c,f}^2(\cos^2\phi + \sin^2\phi)$

$$+ 2V_{o,f}V_{c,f}[\cos\theta\cos\phi - \sin\theta\sin\phi]$$

$$\Rightarrow \quad v^2 = V_{o,f}^2 + V_{c,f}^2 + 2V_{o,f}V_{c,f}[\cos\theta\cos\phi - \sin\theta\sin\phi]$$ — ④

Combining ① and ④, we get

$$2V_{o,f}V_{c,f}[\cos\theta\cos\phi - \sin\theta\sin\phi] = 0$$

(or)

$$\boxed{\cos\theta\cos\phi - \sin\theta\sin\phi = 0}$$

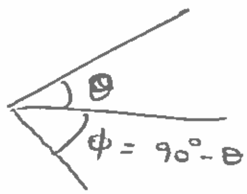
This is the final condition on ϕ given only θ

A solution is if $\sin\phi = \cos\theta$ and $\cos\phi = \sin\theta$, which happens when $\phi = 90° - \theta$. This can also be seen using the identity $\cos A\cos B - \sin A\sin B = \cos(A+B)$

So, $\cos(\theta + \phi) = 0$

$$\Rightarrow \theta + \phi = 90^\circ$$

If $\phi = 90^\circ - \theta$ for part (a), then the angle between



the trajectories will be 90° from each other.

This result is independent of the original speed of the cue ball, and the angle θ at which you want to hit the cue ball.

This result is fairly accurate in real with ~~very~~ slight modifications coming from the spin ~~of~~ on the shots. It is useful to ~~avoid~~ avoid ~~scratching~~ 'scratches' in pool.

2.) Let the velocities as measured from the ground be V and the velocities as measured from the car be U

$$m_A = 5 \text{ kg} \qquad m_B = 2 \text{ kg}$$

$$\vec{V}_{A,i} = 4\hat{x} \text{ m/s} \qquad \vec{V}_{B,i} = -3\hat{x} \text{ m/s}$$

a) $$\vec{V}_{AB,f} = ?$$

Using conservation of momentum

$$m_A \vec{V}_{A,i} + m_B \vec{V}_{B,i} = (m_A + m_B)\vec{V}_{AB,f}$$

Solving this we get

$$\vec{V}_{AB,f} = \frac{5\times 4 - 2\times 3}{7} = 2\hat{x} \text{ m/s}$$

b) ~~E~~ Kinetic energies K

$$K_{A,i} = \frac{1}{2} m_A V_{A,i}^2 = 40 \text{ kg m}^2/\text{s}^2$$

$$K_{B,i} = \frac{1}{2} m_B V_{B,i}^2 = 9 \text{ kg m}^2/\text{s}^2$$

$$K_{tot,i} = 49 \text{ kg m}^2/\text{s}^2$$

$$K_{tot,f} = \frac{1}{2}(m_A + m_B) V_{AB,f}^2 = 14 \text{ kg m}^2/\text{s}^2$$

Loss in kinetic energy $= 35 \text{ kg m}^2/\text{s}^2$ $\quad [49 - 14]$

$$\% \text{ loss} = \frac{35}{49} \times 100\% = 71.4\%$$

c) Velocity of center of mass before collision $= \dfrac{m_A \vec{V}_{A,i} + m_B \vec{V}_{B,i}}{m_A + m_B}$

$= 2\,\hat{x}$ m/s [Same as final velocity]

d) As mentioned earlier let the velocities as measured from the car be υ. And kinetic energies as measured from the car be $\mathcal{K}$ (script K)

For some object x $\upsilon_x = V_x$ − velocity of the car

So, $\vec{\upsilon}_{A,i} = \vec{V}_{A,i} - 2\,\hat{x}$ m/s $= \underline{\underline{2\,\hat{x}\text{ m/s}}}$

$\vec{\upsilon}_{B,i} = \vec{V}_{B,i} - 2\,\hat{x}$ m/s $= \underline{\underline{-5\,\hat{x}\text{ m/s}}}$

e) Using momentum conservation again, or using the result from (a) we get

$$\vec{\upsilon}_{AB,f} = \frac{m_A \vec{\upsilon}_{A,i} + m_B \vec{\upsilon}_{B,f}}{m_A + m_B} = \underline{\underline{0\text{ m/s}}}$$

f) $\mathcal{K}_{A,i} = \frac{1}{2} m_A \upsilon_{A,i}^2 = 10$ kg m²/s²

$\mathcal{K}_{B,i} = \frac{1}{2} m_B \upsilon_{B,i}^2 = 25$ kg m²/s²

$\mathcal{K}_{tot,i} = 10 + 25 = 35$ kg m²/s² (not 49 kg m²/s²)

$\mathcal{K}_{tot,f} = \frac{1}{2}(m_A + m_B)\,\upsilon_{AB,f}^2 = \phi$ (not 14 kg m²/s²)

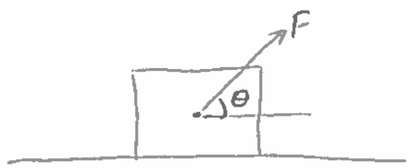
Loss of KE = 35 kg m²/s². % loss = 100%

The values of kinetic energies may be different in different frames, but the change in kinetic energy will be the same.

Also centre of momentum frame for two objects is the frame in which they have the least combined energy

---

3)



$m = 3\,kg$
$F = 50\,N$
$\theta = 60^\circ$
$\Delta x = 10\,m$

a) $W = F_{\parallel} \Delta x = F \cos\theta \, \Delta x$

$= 250\ J$

b) Total work $= \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$= \frac{1}{2} \times 3\,kg\,(25 - 9)\ m^2/s^2$

$= 24\ J$

This is because friction does $-250 + 24 = -226\ J$ of work on the box.

---

4) a) Since the collision conserves energy (elastic), we can use conservation of energy between the initial (throw) and final (max height) states.

$$\frac{1}{2} m v_i^2 + mgh_i = \cancel{\frac{1}{2} m v_f^2} + mgh_f$$

$$mg\,h_f = \frac{1}{2} m v_i^2 + mgh_i$$

$$\Rightarrow \quad h_f = \frac{v_i^2}{2g} + h_i$$

$$= 5\,m + 2\,m = 7\,m$$

b) In part (b), only 7/8 of the kinetic energy 'just before collision' survives the collision.

So, we need to multiply the kinetic energy just before collision by 7/8 ~~before~~ to find the max height. But KE just before collision includes the ~~poten~~ initial PE mgh_i which is converted to KE as the ball falls.

So, $$\frac{7}{8}\left[\frac{1}{2} m v_i^2 + mgh_i\right] = \cancel{\frac{1}{2} m v_f^2} + mgh_f$$

$$\Rightarrow h_f = \frac{7}{8}(7\,m) = 6.1\,m$$

5) The key here is the point that in the expression for gravitational P.E. mgh, h is the ~~height~~ altitude of the center of mass of an object.

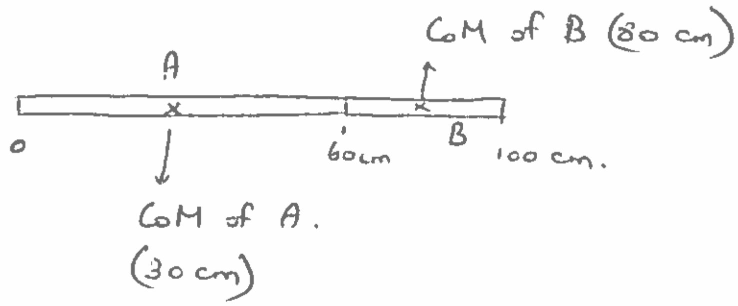
Call the 60 cm part 'A' and the 40 cm part B.

Configuration 1 : A below B.

Configuration 2 : A above B.

The center of mass of A alone is at it's midpoint.

Similarly the center of mass of B alone is at it's midpoint.



Where is the CoM of the entire stick? It is at

~~30cm~~

$$x_{\text{CoM of stick}} = \frac{(30\,\text{cm} \times 500\,\text{g}) + (80\,\text{cm} \times 400\,\text{g})}{(500\,\text{g} + 400\,\text{g})}$$

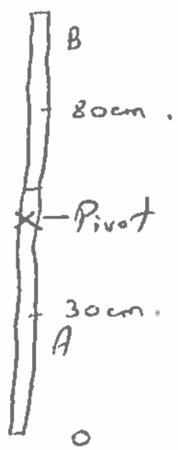
$$= \frac{15000 + 32000}{900}\ \text{cm}$$

$$= \frac{470}{9}\ \text{cm} \simeq 52.22\ \text{cm}.$$

Calculating the energies of the two configurations

Different points can be chosen as the 0 altitude point.
We'll pick the bottom of the stick as 0 altitude.

Configuration 1:



$$G.PE_1 = GPE_1^A + GPE_1^B$$

$$= m_A g h_A + m_B g h_B$$

$$= \left[(0.5 \times 10 \times 0.3) + (0.4 \times 10 \times 0.8)\right] J$$

$$\cancel{(m_A \times g \times h_A) + (m_B \times g \times h_B)}$$

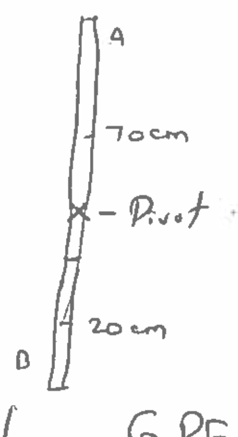
$$= 4.7 \text{ J}$$

Alternatively $GPE_1 = m_{stic}\, g\, h_{stick}$

$$= (0.5 + 0.4) \times 10 \times \frac{4.70}{9} \text{ J}$$

$$= 4.7 \text{ J}$$

Configuration 2:



$$G.PE_2 = G.PE_2^A + G.PE_2^B$$

$$= m_A g h_A + m_B g h_B$$

$$= \left[(0.5 \times 10 \times 0.7) + (0.4 \times 10 \times 0.2)\right] J$$

$$= 4.3 \text{ J}$$

Alternatively $G.PE_2 = (0.5 + 0.4) \times 10 \times \left(1 - \frac{4.7}{9}\right) \text{ J} = 4.3 \text{ J}$

Difference in energy is 0.4 J. Configuration 2 has lesser energy and is preferred. This makes sense as the denser segment (B) is at the bottom in configuration 2.

6) Gravitational field strength at surface $g_s = \frac{GM}{R^2}$

~~at~~ at altitude h $g_h = \frac{GM}{(R+h)^2}$.

Given $\frac{GM}{R^2} = 20\ N/kg$ — ①

$\frac{GM}{(R+h)^2} = 19\ N/kg$ — ②

find h

from ② $GM = [19\ N/kg](R+h)^2$

Plugging this into ① for GM.

$$[19\ N/kg]\frac{(R+h)^2}{R^2} = 20\ N/kg$$

$$\left[\frac{R+h}{R}\right]^2 = \frac{20}{19}$$

$$\frac{R+h}{R} = \sqrt{\frac{20}{19}} = 1.026$$

$$\Rightarrow\quad 1 + \frac{h}{R} = 1.026$$

(or) $$\frac{h}{R} = 0.026$$

$$h = 0.026R = 260\ km.$$

$$\boxed{h = 260\ km}$$

7)

a) Find escape velocity

If $V = V_{esc}$, $KE_{far\ away} = \emptyset$

So, $$\frac{1}{2} m V_{esc}^2 - \frac{GMm}{R} = \cancel{PE_{far\ away}} + \cancel{KE_{far\ away}}$$

$$\frac{1}{2} \cancel{m} V_{esc}^2 = \frac{GM\cancel{m}}{R}$$

$$V_{esc} = \sqrt{\frac{2GM}{R}}$$

b) Now $V = \frac{1}{2} V_{esc} = \sqrt{\frac{GM}{2R}}$

Maximum distance reached by object from center $= R_{max}$.

$$\frac{1}{2} m V^2 - \frac{GMm}{R} = PE_{R_{max}} + \cancel{KE_{R_{max}}}$$

$$\frac{1}{2} \cancel{m} \underbrace{\left(\frac{\cancel{GM}}{2R}\right)}_{\text{Since } V = \sqrt{\frac{GM}{2R}}} - \frac{\cancel{GMm}}{R} = -\frac{\cancel{GMm}}{R_{max}}$$

Find R_{max} if $\frac{1}{4R} - \frac{1}{R} = \frac{-1}{R_{max}}$.

$$\frac{-3}{4R} = -\frac{1}{R_{max}} \quad \text{or} \quad \boxed{R_{max} = \frac{4R}{3}}$$

8)

$k = 1700\ N/m \qquad m = 0.05\ kg.$

$\Delta x = 7\ cm = 0.07\ m$

a) Potential energy $= \frac{1}{2}k(\Delta x)^2$

$$= \frac{1}{2} \times 1700 \times (0.07)^2\ J$$

$$= 4.17\ J$$

b) All of this potential energy is converted into ~~$~~ kinetic when you release it

$$\frac{1}{2}mv^2 = 4.17\ J$$

$$v^2 = \frac{4.17 \times 2}{0.05}\ m^2/s^2 = 166.8\ m^2/s^2$$

$$\boxed{v = 12.9\ m/s}$$

---

9) There're ~~$~~ a few approaches to the problem.

1) Calculate the time at which he reaches the ground or the velocity when he reaches the ground. If these are real values, then he doesn't stop before reaching the ground. If they're imaginary, he doesn't reach the ground.

2) Actually ~~calc~~ calculate the stopping height and see if it is above or below the ground.

Demonstration of ~~m~~ approach (1)

Let's calculate the velocity at the bottom of the bridge:

At the top

$$V_t = \emptyset$$

$$KE_t = \emptyset$$

$$\text{Gravitational } PE_t = mg\,(21\text{ m})$$

$$\text{Elastic } PE_t = \emptyset$$

At the bottom.

$$KE_b = \frac{1}{2} m V_b^2$$

$$\text{Gravitational } PE_b = \emptyset$$

$$\text{Elastic } PE_b = \frac{1}{2} k\,(21\text{m} - 15\text{m})^2$$

From conservation of energy.

$$75\text{ kg} \times 10\ \text{m}/\text{s}^2 \times 21\text{ m} = \left[\frac{1}{2} \times 75\text{ kg} \times V_b^2\right] + \frac{1}{2} \times 1200\ \frac{\text{N}}{\text{m}} \times 36\text{ m}^2$$

(or)

$$\frac{75}{2} V_b^2 = 15750 - 21600 = -5850\ \text{m}^2/\text{s}^2$$

(or)

$$V_b = \sqrt{\frac{-11700}{75}}\ \text{m/s}$$ an imaginary value

So, ~~the~~ Ryan will never get to the bottom of the bridge.

Method 2

Let's say Ryan stops at a distance x above the ground. If x is positive he survives, if not he crashes.

At the top

$$V_t = 0$$

$$KE_t = 0$$

$$G.\ PE_t = mg\,(21\,m)$$

$$E.\ PE_t = 0$$

At height x:

$$V_x = 0 \qquad \text{(since he stops)}$$

$$KE_x = 0$$

$$G.\ PE_x = mg\,x$$

$$E.\ PE_x = \frac{1}{2}\,k\,(21m - x - 15m)^2$$

$$= \frac{1}{2}\,k\,(6m - x)^2$$

From conservation of energy

$$75 \times 10 \times 21 = (75 \times 10 \times x) + \frac{1}{2} \times 1200 \times (6 - x)^2$$

(or)

$$15750 = 750x + 21600 - 7200x + 600x^2$$

(or)

$$600x^2 - 6450x + 5850 = 0$$

The solutions are $\boxed{x = 1\,m \text{ or } 39/4\,m}$

The $39/4$ m doesn't work because ~~our~~ our expression for ~~PE~~ E.PE_x is only valid 15 m or ~~m~~more below the bridge (Bungee cord only pulls up when stretched unlike springs).

So, $x = 1$ m is the valid answer. Ryan stop 1m above the ground.

10)

Energy at bottom of ramp $= \frac{1}{2} \times 4300 \times (.5)^2$ J

$= 537.5$ J

Energy at the top $= mgh + \frac{1}{2} m v^2$

$= [5 \times 10 \times 10 \sin 50°]$ J $+ \frac{5}{2}$ kg v^2

$= 383.02$ J $+ 2.5$ kg v^2

Work done by friction $= -|f_k| \times 10$ m

$= -\mu_k F_N \times 10$ m

$= -\mu_k (mg \cos 50°) \times 10$ m

$= -0.2 \times 5 \times 10 \times \cos(50°) \times 10$ J

$= -64.28$ J

From work energy theorem:

537.5 J $- 64.28$ J $= 383.02$ J $+ 2.5$ kg v^2.

(or)

$$v^2 = 35.36 \ m^2/s^2$$

or

$$v = 5.95 \ m/s.$$

Now the question becomes:

If a block is thrown at a speed ~~angle~~ of 5.95 m/s, at an angle 50° above the ground, from a height of ($10 \sin 50°$) m, what is the horizontal distance traveled before it falls to the ground.

$$\Delta y = -(10 \sin 50°) \ m$$

$$v_{i,y} = (5.95 \sin 50°) \ m/s$$

$$\Delta x = ?$$

$$v_{i,x} = (5.95 \cos 50°) \ m/s$$

$$\Delta y = v_{i,y} \Delta t - \frac{1}{2} g \Delta t^2.$$

Solving this we get $\Delta t = 1.77 \ s$

Now $\Delta x = v_{i,x} \Delta t = 6.77 \ m$

(or)

$$\boxed{d = 6.77 m}$$

11)

~~Thrust prom~~

The forces acting on the rocket are the engine's thrust F and it's own weight W.

$$F - W = ma$$

$$F = W + ma = m(g+a) = 500\,(10+14)\ \text{N}$$

$$= 12000\ \text{N}$$

Time to reach 2 km:

$$2000\ \text{m} = \cancel{V_i t} + \frac{1}{2} a \Delta t^2$$

$$\Delta t = \sqrt{\frac{4000}{14}}\ \text{s} = 16.90\ \text{s}$$

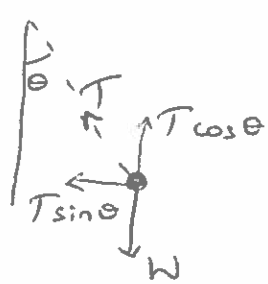
$$\text{Power} = \frac{\text{Work done}}{\text{Time}} = \frac{12000\ \text{N} \times 2000\ \text{m}}{16.90\ \text{s}}$$

$$= 1420\ \text{kW}$$

---

12)

Forces on the ball: Tension T, weight W.



$L = 1.2\ \text{m}$

r (radius of circle) $= L\sin\theta$

In the y-direction $T\cos\theta = W = mg$ (or) $T = \dfrac{mg}{\cos\theta}$

In the radial direction

$$T\sin\theta = \text{Centripetal force} = m\omega^2 r$$

(or)

$$\frac{\cancel{m}g}{\cos\theta}\cancel{\sin\theta} = m\omega^2 r = \cancel{m}\omega^2 L\cancel{\sin\theta}$$

(or)

$$\cos\theta = \frac{g}{\omega^2 L}$$

$$\omega = \frac{2\pi}{T}$$

s

So, $$\cos\theta = \frac{gT^2}{4\pi^2 L} = 0.47$$

$$\boxed{\theta = 61.64^\circ}$$

---

13)

$d = 12$ inches

$r = 6$ inches

a) $\omega_i = 0$

$$\omega_f = \frac{33\ \text{rot}}{60\ \text{s}} = \frac{33\times 2\pi}{60}\ \text{rad/s}.$$

$\Delta t = 1.2\ \text{s}$

$\Delta\theta = ?$

$$\Delta\theta = \frac{1}{2}(\omega_f+\omega_i)\,\Delta t = \frac{1}{2}\times\frac{33}{60}\frac{\text{rot}}{\text{s}}\times 1.2\ \text{s} = 0.33\ \text{rot}$$

b) $\alpha = \dfrac{\omega_f - \omega_i}{\Delta t} = \dfrac{33}{60 \times 1.2} \text{ rot}/s^2$

$= \dfrac{33 \times 2\pi}{60 \times 1.2} \text{ rad}/s^2 = 2.88 \text{ rad}/s^2.$

c) v at distance $\frac{r}{2}$ $(= 3 \text{ in})$

$= \omega_f \times 3 \text{ in} = \dfrac{33 \times 2\pi}{60} \times 3 \text{ in}/s$

$= 10.37 \text{ in}/s = 0.26 \text{ m}/s.$

d)

Now $\omega_i = \dfrac{33 \times 2\pi}{60} \text{ rad}/s.$

$\omega_f = 0.$

$\Delta\theta = 14 \times 2\pi \text{ rad}.$

$\alpha = ?$

$\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$

$\cancel{0 - 66\pi}$

$\cancel{0} - (1.1\pi)^2 \text{ rad}^2/s^2 = 2\alpha \, 28\pi \text{ rad}$

$\alpha = \dfrac{-1.21\pi}{2 \times 28} \text{ rad}/s^2 = -0.068 \text{ rad}/s^2.$

14)

Let v be the fastest safe speed

$$\text{Centripetal force at this speed} = f_s^{max}$$

$$\frac{\cancel{m}v^2}{R} = \mu_s \cancel{m} g$$

$$v^2 = \mu_s g R$$

a) $\mu_s = 0.6$ find v

$$v^2 = 0.6 \times 10 \times 900 \ m^2/s^2$$

$$= 5400 \ m^2/s^2$$

$$v = 73.48 \ m/s$$

b) $v = 30 \ km/hr = \frac{30 \times 10\cancel{0}\cancel{0}}{36\cancel{0}\cancel{0}} \ m/s = \frac{100}{12} \ m/s.$

Find μ_s.

$$\mu_s = \frac{v^2}{gR} = 0.0077$$

15)

Gravitational force provides centripetal force

So, $\frac{GM_E\cancel{m}}{R^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{R}}$

$$\boxed{v^2 = \frac{GM_E}{R}} \quad - \ (1)$$

Also, period of revolution $T = \frac{2\pi R}{v}$ or $v = \frac{2\pi R}{T}$

So, $\left[\frac{2\pi R}{T}\right]^2 = \frac{G M_E}{R}$ Using ①

(or)

$$\boxed{T^2 = \frac{4\pi^2}{G M_E} R^3} \quad — ②$$

a) $R = R_E + 412 \text{ km} = (6370 + 412) \text{ km}$

$= 6782 \text{ km.}$

$= 6.782 \times 10^6 \text{ m}$

Using eqn ②

$$T^2 = \frac{4\pi^2 \times (6.782 \times 10^6)^3}{6.674 \times 10^{-11} \times 5.97 \times 10^{24} \text{ kg}} \text{ s}^2$$

$$\simeq 3.09 \times 10^7 \text{ s}^2$$

$$\boxed{T \simeq 5560 \text{ s} \simeq 93 \text{ min}}$$

b) Using ①

$$v = \sqrt{\frac{G M_E}{R}}$$

If R decreases, v increases

$$\frac{V_{300}}{V_{412}} = \sqrt{\frac{R_{412}}{R_{350}}} = \sqrt{\frac{(6370 + 412) \text{ km}}{(6370 + 350) \text{ km}}} = 1.0046$$

That's a 0.46% increase in orbital speed.