Homework Assignment 6

Due: Thursday, July 14, 2016

Problem 1: A car of mass M is traveling on a road, with its wheels rolling wihout slipping. Each of its wheels has mass m and radius r. The distribution of mass on the wheels is such that the moment of inertia of each wheel is $c \times mr^2$, where c is some constant between 0 and 1. What fraction of the total kinetic energy of the car is the rotational kinetic energy of the wheels? Mass of the car M includes the mass of the wheels.

[Hint: Set the velocity of the car to be v and calculate the rotational and translational KE. Then calculate the required fraction. The final answer will not depend on v. Also, remember that a car has four wheels.]

Problem 2: Three uniform solid disks A, B, and C of masses 100 kg, 64 kg, and 49 kg, respectively and radii 10 m, 8 m, and 7 m, respectively are stacked on top of each other and stuck together so that their centers are directly on top of each other. What is the rotational inertia of this system for rotation about an axis passing through their centers? How much work is required to spin this system from rest to an angular velocity of 1 rad/s?

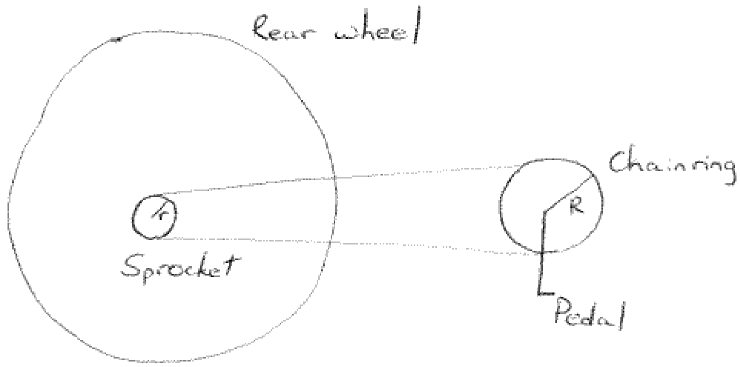
Problem 3: A wheel is composed of three sections: A uniform disk in the middle, and two smaller uniform disks attached to either side of the middle piece, so that their central axes coincide. The mass and radius of the middle piece are 3 kg and 50 cm respectively. The mass and radius of each of the smaller disks are 1 kg and 20 cm respectively. The wheel rolls without slipping from rest down a ramp. Using conservation of energy, find the speed of the wheel at the bottom of the ramp, if the height of the ramp is 10 m.

Problem 4: An object rolls without slipping along a roller coaster track. We are trying to understand how to control the speed of the object at different points in the track by varying it's mass, size and shape. Let M and R be the mass and radius (size) of the object. The shape of the object (or mass distribution) is characterized by the constant c, such that the rotational inertia of the object is $I = c \times MR^2$. If the object is released from rest on the track, find it's speed at a point on the track at height H below the starting point. On what variables (among mass, size, and shape) does this speed depend?

Problem 5: Bicycle gears

To keep a bicycle moving at a constant velocity will require no effort at, if not for the energy lost due to dissipative forces like rolling friction and air resistance, or the energy required to climb a hill. Assume that in a certain terrain under certain conditions, you need to pump energy into the bike at a rate of E per revolution-of-the-rear-wheel to keep the bike moving at a certain speed (This accounts for the energy dissipated from the system by dissipative forces).

You do this by applying a force or torque on the pedals. In addition, you can change the gears on the bike to make the ride easier/harder. The chain goes over a chainring on the pedal and a sprocket on the rear wheel. By changing gears you are essentially changing the radii of the chainring and the sprocket. Let the radii of the chainring and sprocket be R and r respectively.



(a) For one revolution of the rear wheel, how many revolutions do the pedals make? [Hint: The chain has a constant length. So, the distance the chain travels on the chainring equals the distance it travels on the sprocket.]

(b) Given that you need to perform a total work of E per revolution-of-the-rear-wheel, what is the torque you need to apply on the pedals to achieve this?

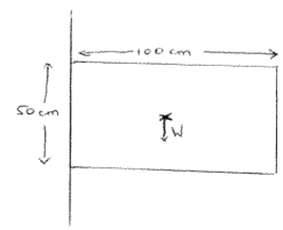
(c) As the ratio $\frac{R}{r}$ decreases, what happens to the torque required? What happens to the number of revolutions-of the-pedals per revolution-of-rear wheel? ['Higher' gears have higher $\frac{R}{r}$.]

Additional remarks: Humans have two limitations when riding bikes. One is the limitation on how much force (torque) we can exert. Another is, even if the torque required is small, there is only so fast we can move our legs in a pedaling motion. This should explain why, although lower gears help make rides 'easier' (lower torque), it is difficult to achieve speeds on lower gears which are easy to achieve on higher gears. Note that the speed of the bike is determined solely by how many revolutions the rear wheel makes per second.

So, there's no clear 'better' gear to ride a bike in. If a lot of power is required (windy/uphill), you can ease up by shifting to a lower gear. But you shouldn't stay there on easier conditions, if you want to travel at a good speed.

Problem 6: Equilibrium (20 points)

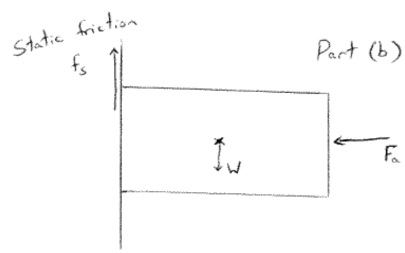
You're trying to hold a uniform box of mass 60 kg up against a wall. The wall is really rough, so instead of lifting it from below, you decide to push it horizontally against the wall, to let friction do the lifting. The coefficient of static friction between the wall and the box is 1.2. The dimensions of the box are as shown in the diagram. (100 cm and 50 cm, if the handwriting isn't clear)



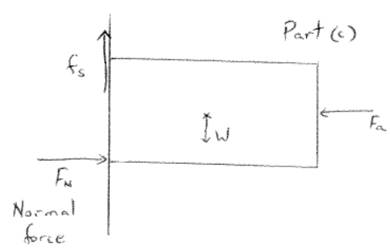
(a) What is the minimum force with which you need to push the box into the wall, so that the value of static friction can equal the weight of the box? If you apply more force that this, will the force of static friction increase?

(b) If you apply the force calculated in (a) right in the middle as shown in diagram, find the location of the effective normal force from the wall on the box. [You can do this by using the equation for rotational equilibrium.]

Does the effective normal force lie within the surface of contact between the wall and the box? What does your answer mean for our implicit assumption that the force calculated in (a) is sufficient to keep the box in equilibrium?



(c) Calculate the minimum force required to keep the box in equilibrium. This can be done by assuming that when we apply the minimum force required, the normal force from the wall passes through the lower edge of the box (why?). [Remember that static friction will not exceed the weight of the box, even when we apply greater force].

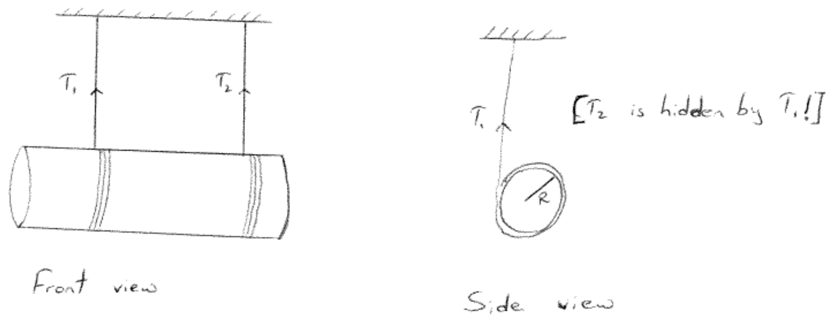


(d) Was it a good idea to push the box against the wall, instead of lifting it from below?

Additional remarks: Note that if the dimensions of the box had been different, the answer to (d) could've been different. There are many other factors that come into play in real life, and our intuition generally tends to work. Usually, a combination of lifting up and pushing in is the way to go. And a lot depends on where along the box we apply our force. In this problem we chose the middle. But it's a better idea to apply the force closer to the top edge, and a worse idea to apply it closer to the bottom edge. Can you see why that is the case?

Problem 7: $\sum F \neq 0; \sum \tau \neq 0$

A hollow cylinder of mass M and radius R is wound by two ropes and hung from the ceiling as shown below. Let the tension in rope 1 be T_1 and rope 2 be T_2. Let $T = T_1 + T_2$ be the total upward tension exerted by the ropes. Find the downward acceleration of the cylinder and the value of T in terms of M, R and g.

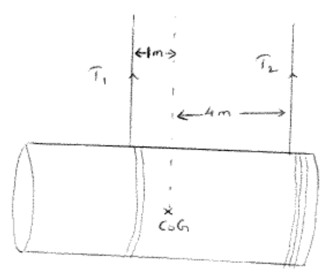


Problem 8: $\sum F \neq 0; \sum \tau = 0$

Introduction: In this continuation of problem 7, we are trying to calculate the values of T_1 and T_2, i.e., find out how the tension is shared by the two ropes. To do this, we have to use the condition that the cylinder isn't toppling, i.e., one rope doesn't unwind faster than the other. We're going to use the condition for non-toppling $\sum \tau = 0$, for rotation in the 'front view'. Contrast this with what we did in problem 7, where we considered rotations in the 'side-view'.

We had said in class that the choice of axis of rotation didn't matter for equilibrium problems. But this isn't an equilibrium problem, the cylinder is accelerating downwards, i.e., $\sum F \neq 0$. In such cases, $\sum \tau = 0$ *only if* the axis of rotation is chosen through the center of mass, the cylinder is actually toppling about other axes!

Question: In problem 7, using $\sum \tau = 0$ for rotation in the 'front-view' about an axis through the center of mass, find T_1 and T_2. The center of mass/center of gravity is marked on the diagram below. [Hint: All results from problem 7 are applicable here, in particular the expression for T, which is equal to $T_1 + T_2$.]



Additional remarks: Winding the two ropes in opposite directions will give a lesser value for the downward acceleration of the cylinder in problem 7. But to solve for the acceleration, you'll have to do the work involved in problems 7 and 8 - by writing down three equations (two from 7, and one from 8) for T_1, T_2 and a and solving them.

If the ropes are equidistant from the center of mass, the total tension will be split equally, regardless of which way they are wound. If the ropes are equidistant from the center of mass and wound in opposite directions, the acceleration will actually be 0! This is the physics/intuition behind the 'mixed grip' (one hand over, one hand under) in deadlifts. One of your hands tends to unwind in one direction, the other in the other direction, and the net result is a defence against the bar rolling down your hands.

Problem 9: $\sum F = 0; \sum \tau \neq 0$

A uniform circular disk of mass 10 kg and radius 10 m is free to rotate about a fixed axle perpendicular to it, passing through the center. It is acted upon by two forces in the clockwise direction. One has a magnitude of 10 N and acts at a distance of 1 m from the center, and the other has a magnitude of 20 N, and acts at a distance of 5 m from the center. The points of application of the forces lie in the same direction from the center (meaning a line segment can be drawn from the center to the edge of the circle, containing both the points of application of the forces)

(a) If the disk starts from rest, what is angular velocity after 1 minute?

(b) What is the magnitude of force on the disk by the axle about which the disk rotates?