PHY 2053 Sec 72D7

Homework Assignment 6

Solutions

1)

Let the car be travelling at speed v

Translational $KE_{trans} = \frac{1}{2} M v^2$

Rotation $KE_{rot} = 4 \times \frac{1}{2} I \omega^2$

$$= 4 \times \frac{1}{2} (c m r^2) \frac{v^2}{r^2}$$

$$= 2 c m v^2$$

Total $KE_{tot} = \left[\frac{1}{2} M v^2 + 2 c m v^2\right]$

Required fraction $= \dfrac{KE_{rot}}{KE_{tot}} = \dfrac{2 c m v^2}{\frac{1}{2} M v^2 + 2 c m v^2} = \dfrac{2cm}{\frac{1}{2} M + 2cm}$

$$= \frac{1}{1 + \frac{M}{4cm}}$$

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3)

$$I = \left\{\left[\frac{1}{2} \times 3 \times (0.5)^2\right] + \left[2 \times \frac{1}{2} \times 1 \times (0.2)^2\right]\right\} \text{ kg m}^2$$

$$= 0.415 \text{ kg m}^2$$

Using energy conservation.

$$Mgh = KE^f_{trans} + KE^f_{rot} = \frac{1}{2} M v_f^2 + \frac{1}{2} I \omega_f^2$$

$M = 3 + (2 \times 1) = 5$ kg ; $\omega_f = \dfrac{v_f}{0.5 m}$ since the wheel rolls along the middle section

$$5 \times 10 \times 10 \text{ J} = \left[\frac{1}{2} \times 5 \times V_f^2 + \frac{1}{2} \times \frac{0.415}{(0.5)^2} V_f^2\right] \text{kg}$$

$$= 3.33 \text{ kg } V_f^2$$

$$V_f^2 = \frac{500}{3.33} \text{ m}^2/\text{s}^2$$

$$\boxed{V_f = 12.25 \text{ m/s}}$$

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2) Rotational inertia is additive

So, $I_{total} = I_A + I_B + I_c = \frac{1}{2}\left[100 \times 10^2 + 64 \times 8^2 + 49 \times 7^2\right] \text{kg m}^2$

$$= 8248.5 \text{ kg m}^2$$

$\omega_i = 0$

$\omega_f = 1 \text{ rad/s}$

$$\Delta KE = \frac{1}{2} I \omega_f^2 - \cancel{\frac{1}{2} I \omega_i^2}$$

$$= 4124.25 \text{ J}$$

$$\boxed{\text{Work done} = 4124.25 \text{ J}}$$

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4)

$KE_{trans} = \frac{1}{2} M v^2$, $KE_{rot} = \frac{1}{2} I \omega^2$

$$= \frac{1}{2} c M R^2 \frac{v^2}{R^2} = \frac{c}{2} M v^2$$

$$KE_{tot} = \frac{1}{2} M v^2 + \frac{c}{2} M v^2 = \left(\frac{c+1}{2}\right) M v^2$$

Using conservation of energy

$$\cancel{M}gH = \left(\frac{c+1}{2}\right)\cancel{M}v^2$$

$$v^2 = \frac{2gH}{c+1}$$

$$\boxed{v = \sqrt{\frac{2gH}{c+1}}}$$

v depends only on c (among c, M and R), i.e., only shape.

5)

a) Distance traveled by the chain on the sprocket equals that traveled on the chain ring

So, $$\Delta L_{chainring} = \Delta L_{sprocket}$$

$$R\,\Delta\theta_{chainring} = r\,\Delta\theta_{sprocket}$$

If $\Delta\theta_{sprocket} = 1$ rev

$$\boxed{\Delta\theta_{chainring} = \frac{r}{R}\text{ rev}}$$

b) ~~Energy~~ Work per revolution of back wheel = E

So, ~~energy~~ Work per $\frac{r}{R}$ rev of pedal = E

Using $W = \tau\Delta\theta$.

$$E = \tau\left[\frac{r}{R}\times 2\pi\right] \quad \text{(converting to radians)}$$

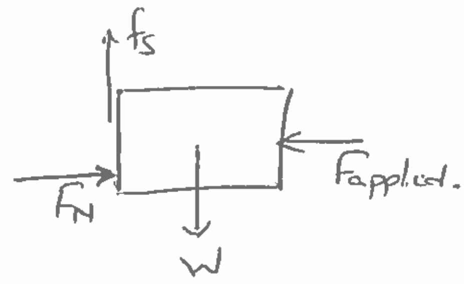
So, $\boxed{\tau = \frac{RE}{2\pi r}}$ or $\left(\frac{E}{2\pi}\right)\left(\frac{R}{r}\right)$

c) As $\frac{R}{r}$ increases, torque required increases.

Number of revolutions of pedals decreases.

6)

a) Friction acts upwards



$$\boxed{\begin{aligned} F_N &= F_{applied} \\ f_s &= W \end{aligned}}$$ for equilibrium

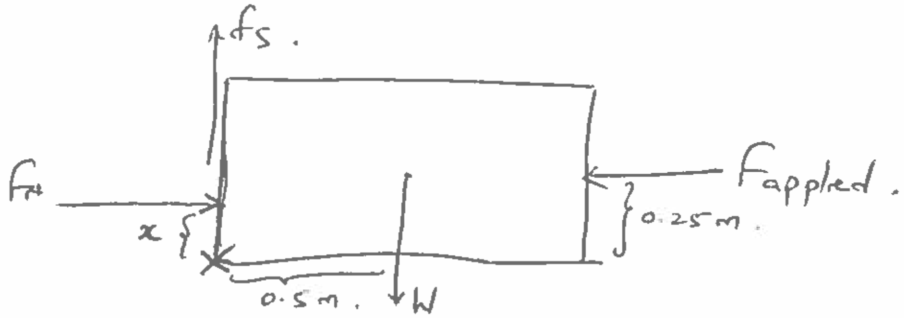
$$f_s \leq f_s^{max} = \mu_s F_N$$

So, $$F_N \geq \frac{f_s}{\mu_s} = \frac{60 \times 10\ N}{1.2} = 500\ N$$

So, $F_{applied} \geq 500\ N$

$F_{applied}$ must be ~~at~~ at least 500N. If we apply more force, f_s will remain 600 N.

b) We'll pick the pivot to be the bottom left corner.



To find: x

$$\tau_{f_s} = 0$$

$$\tau_W = -600\ N \times 0.5\ m = -300\ Nm$$

$$\tau_{F_{applied}} = 500\ N \times 0.25\ m = 125\ Nm$$

$\tau_{F_N} = -F_N\, x = -500\,\text{N}\, x$

Using $\Sigma \tau = 0$ for equilibrium.

$$0 - 300\ \text{Nm} + 125\ \text{Nm} - 500\,\text{N}\, x = 0.$$

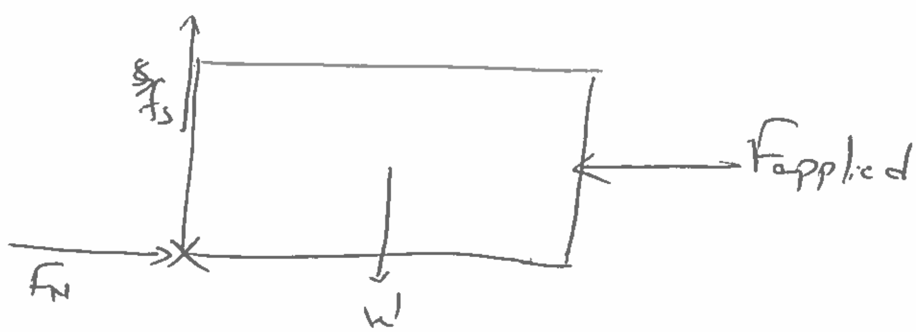
$$x = \frac{-175\ \text{m}}{500} = -0.35\ \text{m}.$$

$$\boxed{x = -0.35\ \text{m}}$$

This means that F_N acts below the chosen pivot, i.e., outside the surface of contact

This means that 500N isn't sufficient to keep the box pinned to the wall.

c)



With the same pivot $\tau_{f_s} = \tau_{F_N} = 0$.

$\tau_W = -600 \times 0.5\ \text{m} = -300\ \text{Nm}$

$\tau_{F_{applied}} = F_{applied} \times 0.25\ \text{m}$

Using $\Sigma \tau = 0$: $-300\ \text{Nm} + F_{applied} \times 0.25\text{m} = 0.$

$$F_{applied} = \frac{300}{0.25}\ \text{N} = 1200\ \text{N}.$$

So, ~~the~~ atleast 1200 N of force is needed to keep the box pinned to the wall.

d) If we'd tried to lift the box directly, only 600 N would be required. So, it seems like a bad idea. But the answer is maybe. Maybe the human body finds it easier to excert horizontal force than vertical.

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7)

~~Upw~~ Vertical forces are tensions T_1 and T_2, and weight W.

$$\Sigma F = T_1 + T_2 - W = -Ma$$

where a is the downward acceleration

If $T = T_1 + T_2$, then

$$\boxed{T - Mg = -Ma} \quad - \quad 1$$

Also,

$$|\tau_{T_1}| = T_1 R$$

$$|\tau_{T_2}| = T_2 R.$$

$$|\tau_{T_1} + \tau_{T_2}| = (T_1 + T_2) R = TR$$

$$\Sigma \tau = I\alpha$$

$$\Rightarrow \quad TR = (MR^2)\left(\frac{a}{R}\right) = MaR$$

$$\Rightarrow \quad \boxed{T = Ma} \quad - \ (2)$$

From (1) and (2)

$$Ma - Mg = -Ma$$

or $\quad 2Ma = Mg$

or $\quad \boxed{a = g/2}$

and $\quad \boxed{T = Mg/2}$

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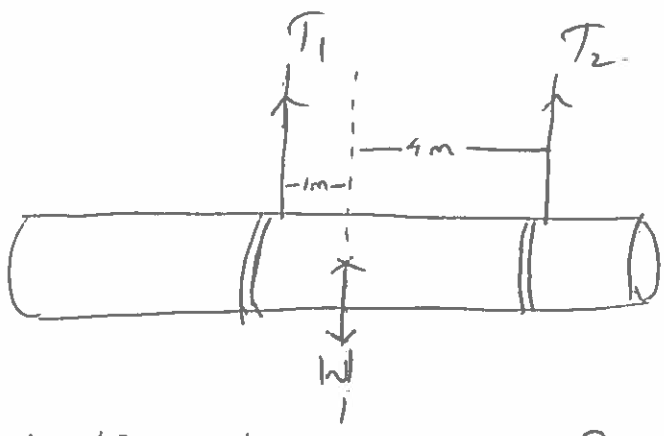
8)



About the drawn axis of rotation:

$$\tau_W = 0$$

$$\tau_{T_1} = -T_1 \times 1\,m$$

$$\tau_{T_2} = T_2 \times 4m$$

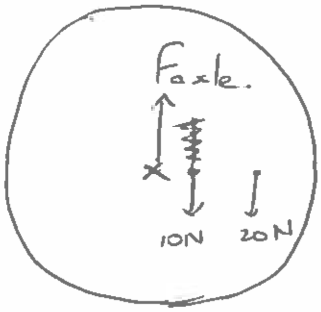
So, $\quad -T_1 \times 1m + T_2 \times 4m = 0$

$$T_1 = 4\,T_2$$

Also $\quad T_1 + T_2 = T = Mg/2$

So, $\quad 5T_2 = Mg/2 \quad$ or $\quad \boxed{T_2 = Mg/10 \ ; \ T_1 = 2Mg/5}$

9)



a) $I = \frac{1}{2} MR^2 = \frac{1}{2} \times 10 \times 10^2 \text{ kg m}^2 = 500 \text{ kg m}^2$

$\tau_{10N} = -10\text{ N} \times 1\text{ m} = -10 \text{ Nm}$

$\tau_{20N} = -20\text{ N} \times 5\text{ m} = -100 \text{ Nm}.$

$\tau_{tot} = -110 \text{ Nm}$

$\tau_{tot} = I\alpha$

$$\alpha = \frac{-110 \text{ Nm}}{500 \text{ kg m}^2} = \frac{-11}{50} \text{ rad/s}^2.$$

$$\omega_f = \cancel{\omega_i} + \alpha \Delta t = \frac{-11}{50} \times 60 \text{ rad/s}$$

$\boxed{\omega_f = 13.2 \text{ rad/s clockwise}}$

b) $\Sigma F = 0.$

$F_{axle} - 10\text{ N} - 20\text{ N} = 0$

$\boxed{F_{axle} = 30\text{ N}}$