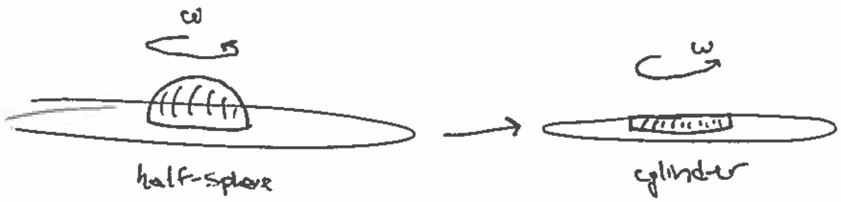
Homework Assignment 7

Due: Monday, July 18, 2016

Problem 1: A blob of clay ($\rho_{\text{clay}} = 2330$ kg/m^3) is to be turned into a plate on a pottery wheel. The clay starts out shaped like a half-sphere of radius $r_{\text{sphere}} = 12$ cm. The potter works for 3 minutes, spinning the pottery wheel at a constant angular speed of $\omega = 12.6$ rad/s, until the clay is shaped like a plate (really a cylinder, radius $r_{\text{cylinder}} = 24$ cm and height $h = 2$ cm). What average torque is required for the pottery wheel to maintain its constant speed during the shaping process?



$I_{sph} = \frac{1}{2}\left(\frac{2}{5}Mr^2\right)$ $\qquad I_{cyl} = \frac{1}{2}Mr^2$

We don't immediately know the mass. To find it, use the density and initial (or final) volume:

$$m = \rho V = \rho\left(\pi r_{cyl}^2 h\right)$$

$$= (2330)\left[\pi (0.24)^2 (0.02)\right]$$

$$= 8.43 \text{ kg}$$

The clay maintains a constant rotational speed, but the rotational inertia changes, so there must be a torque by

$$\tau = \frac{\Delta L}{\Delta t} = \frac{I_f\omega_f - I_i\omega_i}{\Delta t}$$

$$= \frac{(I_f - I_i)\,\omega}{\Delta t}$$

$$= \left(\frac{1}{2} m r_{cyl}^2 - \frac{1}{5} M r_{sp}^2\right)\frac{\omega}{\Delta t}$$

$$= \left(\frac{1}{2}(8.43)(0.24)^2 - \frac{1}{5}(8.43)(0.12)^2\right)\frac{12.6}{180}$$

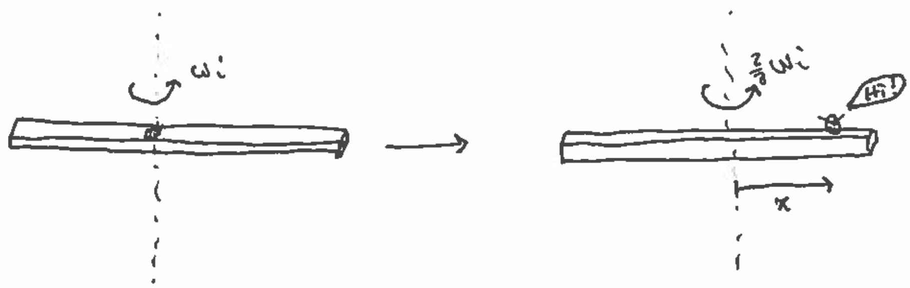
$$\boxed{\tau = 1.53\times 10^{-2} \text{ Nm}}$$

Problem 2: A 300 g rod of wood is 90 cm long and allowed to rotate about its center. A 55 g cockroach sits on the center of the rod. When the rod-cockroach system is brought to some initial angular speed, the cockroach decides to take a walk out along the rod, away from the center. How far must the cockroach walk for the angular speed of the system to be reduced to two-thirds of its initial value?

$M_{rod} = 0.3$ kg

$L_{rod} = 0.9$ m

$m_{cr} = 0.055$ kg



As there are no external torques, we may assume angular momentum is conserved. Treat the cockroach as a point-mass.

$$L_i = I_i \omega_i = I_{plank}\, \omega_i = \left[\frac{1}{12} M_{rod} (L_{rod})^2\right] \omega_i$$

$$L_f = I_f \omega_f = \left(I_{rod} + I_{cr}\right)\omega_f = \left[\frac{1}{12} M_{rod}(L_{rod})^2 + m_{cr} x^2\right]\omega_f .$$

Assuming $\omega_f = \frac{2}{3}\omega_i$, set $L_i = L_f$ and solve for x:

$$\frac{1}{12} M_{rod} (L_{rod})^2 \omega_i = \left[\frac{1}{12} M_{rod} (L_{rod}^2) + m_{cr} x^2\right]\left(\frac{2}{3}\omega_i\right)$$

$$\rightarrow \frac{1}{8} M_{rod} L_{rod}^2 = \frac{1}{12} M_{rod} L_{rod}^2 + m_{cr} x^2$$

$$\frac{1}{24} M_{rod} L_{rod}^2 = m_{cr} x^2$$

$$\rightarrow \boxed{x = \sqrt{\frac{M_{rod} L_{rod}^2}{24\, m_{cr}}} = 0.43 \text{ m}}$$

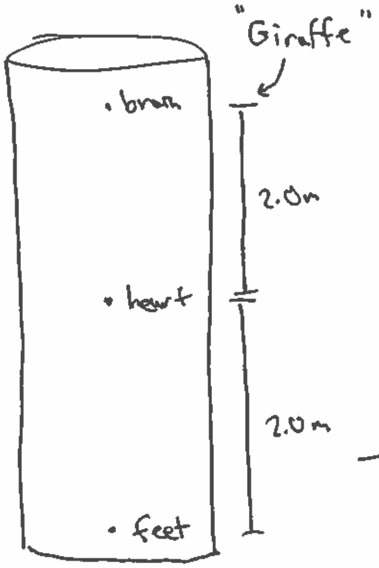
almost the end of the rod.

Problem 3: The average giraffe has its head 2.0 m above its heart and its heart 2.0 m above its feet. In this case, with the giraffe standing upright, the gauge pressure of the blood at its heart is about 33.3 kPa. The density of its blood is 1.00×10^3 kg/m^3.

(a) Find the blood pressure at the brain of the giraffe.

(b) Find the blood pressure at the feet of the giraffe.

(c) If the giraffe were to drink from a pond by lowering its head without splaying its legs, what would be the increase in the blood pressure to the brain? (Such an action would probably be lethal. Yikes!)



(a) We know the (gauge) blood pressure at the head is related to the (gauge) b.p. at the heart by

$$P_{head} = P_{heart} - \rho_{blood} g \, (2.0 \text{ m})$$

$$= 33.3 \text{ kPa} - (1000)(10)(2.0)$$

$$\boxed{P_{brain} = 13.3 \text{ kPa.}}$$

(b) Similarly,

$$P_{feet} = P_{heart} + \rho_{blood} \, g \, (2.0)$$

$$= 33.3 \text{ kP} + (1000)(10)(2.0)$$

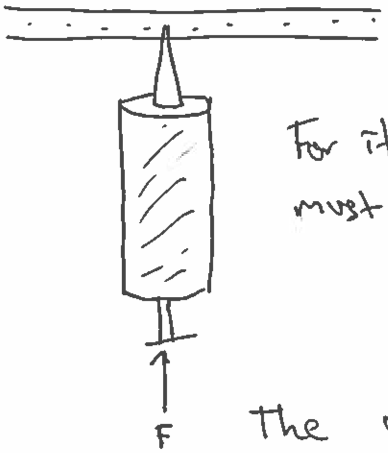
$$\boxed{P_{feet} = 53.3 \text{ kPa}}$$

(C) Assuming the blood pressure at the heart is constant, we find a pressure increase of

$$\Delta P = P_{feet} - P_{brain}$$

$$\boxed{\Delta P = 40 \text{ kPa}}$$

Problem 4: A nurse wishes to inject medicine into a patient. The piston of the syringe used for the injection is 5.00×10^{-5} m^2. If the patient has an average (gauge) blood pressure of 15 mm Hg, with what force must the nurse push on the piston for the medicine to flow into the patient?



With no force applied, the fluid in the syringe has a gauge pressure of 0.

For it to flow into the patient, the two gauge pressures must be equal:

$$P_{gauge,blood} = P_{gauge,syringe}.$$

The applied force increases the pressure in the fluid above the atmospheric pressure, so this amounts to the gauge pressure:

$$P_{gauge,syringe} = \frac{F}{A} = \frac{F}{5.0 \times 10^{-5} m^2}.$$ Thus we can simply solve for F:

$$F = P_{gauge,blood} \cdot A$$

$$= (15 mm\,Hg)(5.0 \times 10^{-5} m^2)$$

$$= (2000)(5.0 \times 10^{-5} m^2)$$

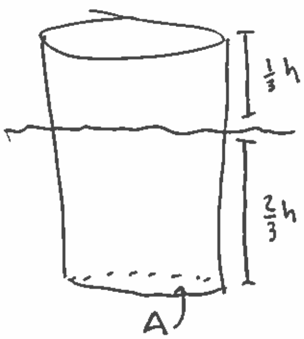
$$\boxed{F = 0.1\ N}$$

Problem 5: A right cylinder of height h and radius r floats partially submerged in a pool of incompressible liquid. The cylinder is made of a material with constant uniform density ρ_{cyl}.

(a) If the top of the cylinder is 1/3 h above the fluid's surface, find the density of the fluid in terms of the density of the cylinder.

(b) Imagine now that the top of the cylinder is held at a depth $d = 3.0$ m below the surface of the fluid. What is the net force exerted by the fluid on the cylinder? Write your answer in terms of the weight of the cylinder, W_{cylinder}.

(a)



As $F_{net} = 0 \longrightarrow F_B - mg = 0$. So $F_B = mg$.

From Archimedes' Princ: $F_B = \rho_f \, g \, V_{sub} = \rho_{fl} \, g \left(\frac{2}{3}h \cdot A\right)$

Also $mg = (\rho_{cyl} V_{cyl})g = \rho_{cyl}(h A) g$.

Thus

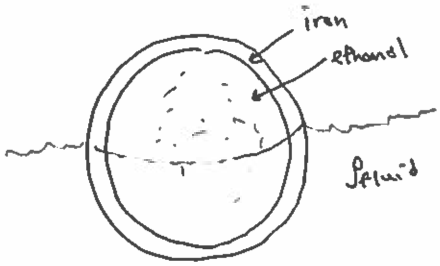
$$\rho_{fl} \, \cancel{g} \left(\frac{2}{3}\cancel{h}\cancel{A}\right) = \rho_{cyl}(\cancel{h}\cancel{A})\cancel{g}$$

$$\longrightarrow \rho_{fl}\left(\frac{2}{3}\right) = \rho_{cyl} \quad \text{or} \quad \boxed{\rho_{fl} = \frac{3}{2}\rho_{cyl}}$$

(b) The net force exerted by the fluid is the buoyant force, and as the cylinder is fully submerged:

$$F_B = \rho_{fl} \, g \, V_{cyl} = \frac{3}{2}\underbrace{\rho_{cyl} \, g \, V_{cyl}}_{W_{cyl}} = \boxed{\frac{3}{2}W_{cyl}.}$$

Problem 6: A hollow, metal shell of iron has inner radius $r_{in} = 12$ cm and outer radius $r_{out} = 14$ cm and is filled with ethanol. The shell floats in an unknown fluid so that exactly half of the shell is submerged. What is the density of the unknown fluid?



As the object floats at rest,

$$F_{net} = F_B - \text{Weight} = 0.$$

Here, the weight consists of the iron shell and ethanol,

$$\text{Weight} = W_I + W_e$$

$$W_I = \rho_{iron} V_{shell}\, g = \rho_{iron}\left(\frac{4}{3}\pi r_{out}^3 - \frac{4}{3}\pi r_{in}^3\right) g = 33.5 \text{ N}$$

$$W_e = \rho_{et} V_{et}\, g = \rho_{et}\, \frac{4}{3}\pi r_{in}^3\, g = 5.72 \text{ N}$$

$$\underline{\text{Weight} = 39.2 \text{ N}}$$

The buoyant force comes from the unknown fluid:

$$F_B = \rho_{Fl}\, g\, V_{sub} = \rho_{Fl}\, g \left(\frac{1}{2} V_{sphere}\right)$$

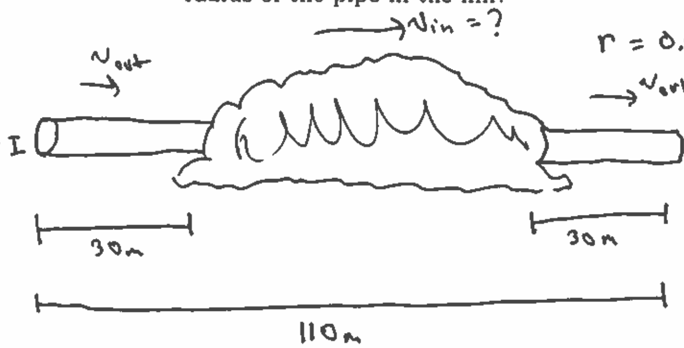
$$= \rho_{Fl}\, g \left(\frac{2}{3}\pi r_{out}^3\right).$$

For the forces to cancel we need

$$F_B = \text{Weight} \longrightarrow \rho_{Fl}\, g\left(\frac{2}{3}\pi r_{out}^3\right) = 39.2 \text{ N}$$

So $\boxed{\rho_{Fl} = 6820 \ \frac{kg}{m^3}}$

Problem 7: An old pipe system runs through a hill. On each side of the hill, the pipe has a radius of 2.50 cm, however the radius of the pipe within the hill is unknown. To determine the radius of the pipe inside the hill, engineers introduce a dye into the water stream and measure how long the dye takes to travel from one side of the hill to the other. The dye is introduced 30 m "upstream" of the hill and, after traveling for 88.9 s, reaches a point 30 m "downstream" of the hill. If the flow in the pipes outside of the hill is 2.50 m/s and the total distance traveled by the dye in passing through the hill is 110 m, what is the average radius of the pipe in the hill?



$r = 0.025\,m$; $\Delta t = 88.9\,s$; $v_{out} = 2.50\,m/s$

We first find the time required for the dye to pass through the mountain. The total time consists of the time spent outside plus the time spent in the mountain.

$$\Delta t = \text{time out} + \text{time inside}$$

As we know the speed and distance traveled on each side of the mountain, we can find t_{out}.

$$t_{out} = \frac{2 \times 30\,m}{2.50\,m/s} = 24\,s.$$

Thus $$\text{time}_{\text{inside}} = 88.9\,s - 24\,s = 64.9\,s.$$

We can also find the distance traveled inside the mountain:

$$110\,m - 60\,m = 50\,m \text{ inside}.$$

From this, we get the speed of the fluid flow

$$v_{in} = \frac{50\,m \text{ inside}}{64.9\,s} = 0.77\,m/s.$$

This speed is related to the outside speed by the continuity eqn.

$$v_{in} A_{in} = v_{out} A_{out} \longrightarrow \pi r_{in}^2 = \frac{v_{out}}{v_{in}} \pi r_{out}^2$$

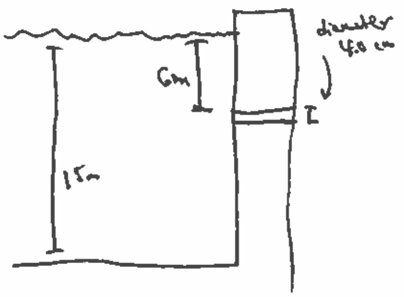
$$\longrightarrow r_{in} = \sqrt{\frac{v_{out}}{v_{in}}}\, r_{out}$$

$$\boxed{r_{in} = 0.045\,m \text{ or } 4.5\,cm}$$

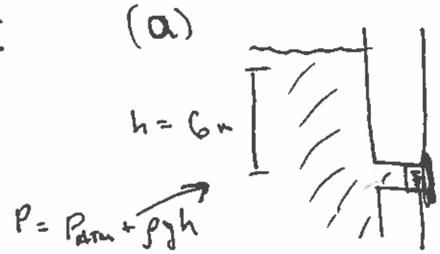
Problem 8: A large, fresh-water reservoir behind a dam has a depth of 15 m. A horizontal drainage pipe of 4.0 cm passes through the dam wall at a depth of 6.0 m, and is capped on the outside by a plug.

(a) Find the magnitude of the frictional force keeping the plug in place in the pipe.

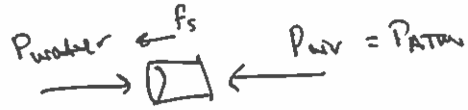
(b) After the plug is removed, find the volume of water the exits through the pipe after 2 hours.



(a)



On one side, the plug feels the push of the water. On the other it feels the push of the air. The static friction compensates for the difference in these pressures!



Looking at the forces (recall: $F = PA$)

$$F_{water} = F_{air} + f_s$$

or
$$f_s = F_{water} - F_{air}$$
$$= (P_{water} - P_{air})A$$
$$= (P_{ATM} + \rho g h - P_{ATM})A$$

So $f_s = \rho_{water}\, g\, (6\,m)\left[\pi \frac{.04^2}{4}\right] \rightarrow \boxed{f_s = 12.6\ N}$

(b) With the plug removed, the water flows out:

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

Assume
$h_1 = 6m$
$h_2 = 0$
$P_1 = P_2 = P_{ATM}$
$v_1 \approx 0$

$\rightarrow \rho g h = \frac{1}{2}\rho v_2^2$

or $v_2 = \sqrt{2gh}$

The volume flow is then given by

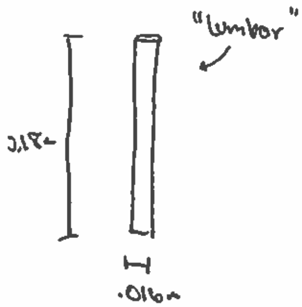
$$\frac{\Delta V}{\Delta t} = A v_2 = \pi \frac{(.04)^2}{4} v_2$$

Multiply this by time to get the volume:

$$V = \frac{\Delta V}{\Delta t}\Delta t = \pi \frac{0.04^2}{4} v_2 (7200\,s)$$

$$\boxed{V = 99.1\ m^3 \approx 100\ m^3}$$

Problem 9: The average human male lumbar vertabrae measure 18 cm in length and, on average, 1.6 cm in width. Assume that, when standing, the lumbar supports half the weight of a person and that the lumbar vertebrae have circular cross-section (not true, but this is a physics class, not anatomy!). By how much is the lumbar region of an 81 kg male compressed when standing, compared to when the man is lying down?



$W_{supported} = \frac{1}{2}\,\text{Weight} = 40.5\,\text{kg} \cdot g = 405\ \text{N}$

We are interested in ΔL:

$$\frac{F}{A} = Y\,\frac{\Delta L}{L} \rightarrow \Delta L = \frac{F}{A}\,\frac{L}{Y}$$

where F is the supported weight, L the resting length ("lying down length")

and A the area, $A = \pi\,(0.008\,\text{m})^2$

From Table 10.1, use Y for human vertabra (compression)

$$Y = 0.088 \times 10^{9}\ \text{Pa}$$

$$\rightarrow \boxed{\Delta L = \frac{405}{\pi\,(0.008)^2} \cdot \frac{0.18}{0.088 \times 10^{9}} = 4.1\ \text{mm}}$$

Problem 10: Jumping from large heights is dangerous for many reasons! Here we will investigate the dangers of landing on the ground with locked knees. A 65 kg "daredevil" teenager jumps from the roof of a two-story building (7 m in height) and lands on the ground with both feet. The teenager tries to cushion his fall by using his feet, but locks his knees in the process (meaning that the force of impact is sent into the femur of each leg). His descent is stopped suddenly, $\Delta t = 0.08$ s. Assume each femur has an area of 8.0 cm^2.

(a) Calculate the stress experienced by each femur in the teenager's legs.

(b) Using the information from Figure 10.4c in your book, will the teenager's femurs survive the fall and sudden stop?

We use impulse to find the force:

$$F\Delta t = \Delta p = m v_f - \underset{\text{starts at rest}}{0}$$

$$v_f = \sqrt{2gh} = 11.8 \text{ m/s}$$

So $$F = \frac{m(11.8)}{\Delta t} = 9588 \text{ N}$$ supported by both legs

So the stressing force is half of the total impulse force:

$$\underset{\text{per femur}}{\text{Stress}} = \frac{F}{A} = \frac{4794}{8\times10^{-4}} = 6\times10^{6} \text{ Pa}$$

$8.0 \text{ cm}^2 = 8\times10^{-4} \text{ m}^2$

(b) Looking at the table for compressive strain/stress (quadrant III) a compressive stress of $\sim 10^6$ Pa is well within the elastic regime, where the bone will remain unbroken. It's not until stresses of $\sim 18\times10^7$ Pa that we have problems. So even if he had landed on only one leg, his femur would still be okay. His knees, on the other hand... :(