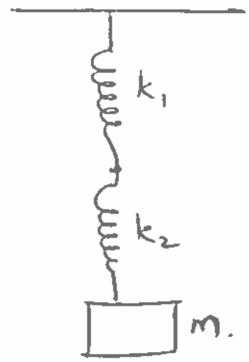
PHY 2053 Sec 72D7

Homework Assignment 8

Solutions

1)



a) Both springs are under the same tension T

$T = k_1 \Delta L_1$ $\qquad$ $T = k_2 \Delta L_2$

$\Delta L_1 = \dfrac{T}{k_1}$ $\qquad$ $\Delta L_2 = \dfrac{T}{k_2}$.

b) Total ~~exp~~ extension $\Delta L = \Delta L_1 + \Delta L_2$

$$= \frac{T}{k_1} + \frac{T}{k_2}$$

$\boxed{\text{If } \quad T = k_{eff}\,\Delta L \quad \text{find} \quad k_{eff}}$

$$T = k_{eff}\left[\frac{T}{k_1} + \frac{T}{k_2}\right] = k_{eff}\left[\frac{1}{k_1} + \frac{1}{k_2}\right] T$$

(or) $\quad k_{eff}\left[\dfrac{1}{k_1} + \dfrac{1}{k_2}\right] = 1$

$$\Rightarrow \quad \boxed{k_{eff} = \frac{k_1 k_2}{k_1 + k_2}}$$

c)

$$\boxed{\omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{k_1 k_2}{m(k_1+k_2)}}}$$

2)

a)

$$I_A = \frac{P}{4\pi d^2} \Rightarrow P = I_A \, 4\pi d^2$$

$$I_B = \frac{P}{4\pi (d+L)^2} \Rightarrow I_B = \frac{I_A \, 4\pi d^2}{4\pi (d+L)^2}$$

(or)

$$I_B = I_A \frac{d^2}{(d+L)^2}$$

b) Intensity received by ~~A~~ B will decrease

To see this

$$I_{B_{new}} = I_{A_{new}} \frac{d_{new}^2}{(d_{new} + L_{new})^2}$$

$d_{new} = d/2$, $L_{new} = L$; $I_{A_{new}} = I_A$

So $$I_{B_{new}} = I_A \frac{(d/2)^2}{(d/2 + L)^2}$$

$$\frac{I_{B,new}}{I_{B,old}} = \frac{\cancel{I_A}\left(d/2\right)^2}{\left(d/2 + L\right)^2} \times \frac{(d+L)^2}{\cancel{I_A}\, d^2}$$

$$= \frac{\cancel{d^2}(d+L)^2}{2^2\left(d/2+L\right)^2 \cancel{d^2}} = \frac{(d+L)^2}{(d+2L)^2} < 1$$

So, $I_{B,new} < I_{B,old}$.

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3)

a) $$\lambda_n = \frac{2L}{n}$$

λ_n is fixed by fret position.

b) $$v = \sqrt{\frac{TL}{m}}$$

This Speed can be changed by tuning (changing T)

c) $$f_n = \frac{v}{\lambda_n} = \frac{n}{2L}\sqrt{\frac{TL}{m}} = \frac{n}{2}\sqrt{\frac{T}{mL}}$$

$$\boxed{f_n = \frac{n}{2}\sqrt{\frac{T}{mL}}}$$

f_n can be changed by tuning. Changing T is sufficient to get any required f_n.

d) $f = \frac{v}{\lambda}$

~~v~~ λ is fixed by fret position

v can be changed by tuning.

e) No, the frequency will not change. This is because the v in the expression $f = \frac{v}{\lambda}$ is the speed of ~~sound~~ vibrations on a string. not speed of sound in air.

So, the guitar should sound the same

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4)

a) $\lambda_n = \frac{2L}{n}$

b) $f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$ where v is the speed of sound in the air in the resonant tube of the wind instrument.

c) Yes! And yes! Because this time v is speed of sound in air.

5)

$$A_{max} = A_1 + A_2$$

$$A_{min} = |A_1 - A_2| = A_1 - A_2 \quad \text{since} \quad A_1 > A_2.$$

Also, $I \propto A^2$

So, $\sqrt{I_1} = c A_1$

$\sqrt{I_2} = c A_2$

$\sqrt{I_{max}} = c A_{max}$. for some constant c.

$\sqrt{I_{min}} = c A_{min}$

~~So To solve for A_1 and A_2~~

To find I_1 and I_2, first find A_1 and A_2

$$A_1 + A_2 = A_{max}$$

$$A_1 - A_2 = A_{min}$$

Adding the two equations $2A_1 = A_{max} + A_{min}$

(or)

$$A_1 = \frac{A_{max} + A_{min}}{2}$$

Plugging this back into one of the equations:

$$A_2 = \frac{A_{max} - A_{min}}{2}$$

$$A_1 = \frac{A_{max} + A_{min}}{2} \Rightarrow \sqrt{I_1} = \frac{\sqrt{I_{max}} + \sqrt{I_{min}}}{2}$$

or $$I_1 = \left[\frac{\sqrt{I_{max}} + \sqrt{I_{min}}}{2}\right]^2 = \frac{I_{max} + I_{min} + 2\sqrt{I_{max} I_{min}}}{4}$$

Illy $$I_2 = \frac{I_{max} + I_{min} - 2\sqrt{I_{max} I_{min}}}{4}$$

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6) Since the speaking attempt is the same, λ is the same for normal air and helium.

$$f_{air} = \frac{V_{air}}{\lambda_{air}} \qquad f_{he} = \frac{V_{he}}{\lambda_{he}}$$

$$\lambda_{air} = \lambda_{he}.$$

$$f_{he} = \frac{V_{he}}{\lambda_{air}} = \frac{V_{he}}{(V_{air}/f_{air})} = \left[\frac{V_{he}}{V_{air}}\right] f_{air}.$$

$$f_{he} = \frac{1000}{340} \times 98\ Hz = 288.24\ Hz.$$

$$\boxed{f_{he} = 288.24\ Hz}$$

Higher pitch.

This sound is produced from the helium filled larynx, ~~but~~ and is transmitted into regular air. ~~For such cases,~~ In the transmission, frequency stays the same, wavelength changes.

$$\boxed{f_{heard} = 288.24 \text{ Hz.}}$$

$$\lambda_{heard} = \frac{V_{air}}{f_{heard}} = \frac{340}{288.24} \text{ m} = 1.18 \text{ m.}$$

$$\boxed{\lambda_{heard} = 1.18 \text{ m.}}$$

7)

a)

$$V = \frac{\omega}{k} = \frac{12\pi \text{ rad/s}}{3\pi \text{ rad/m}} = 4 \text{ m/s}$$

Direction: Left since ω and k have the same sign.

Max speed of string $= \omega A = 12\pi \times 0.12$ m/s

$= 4.52$ m/s.

b) The wave y_2 is shifted to the left by a phase of $\pi/6$. This corresponds to a distance d such that $kd = \pi/6$ or $d = \frac{\pi}{6k} = \frac{\pi}{6 \times 3\pi}$ m $= 0.056$ m

~~g) This wave waves~~

c) To create a standing wave, we need to add the following y_3 to y_1

$$y_3 = (12.0\text{ cm})\cos\left[\left(-3\pi\text{ rad/m}\right)x + \left(12\pi\text{ rad/s}\right)t\right]$$

It travels to the right, has the same wavelength

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{3\pi}\text{ m} = 0.67\text{ m}.$$

$$y_1 + y_3 = 2\,(12.0\text{cm})\cos\left[\left(3\pi\text{ rad/m}\right)x\right]\cos\left[\left(12\pi\text{ rad/s}\right)t\right]$$

Let Distance between 2 nodes be 'd'.

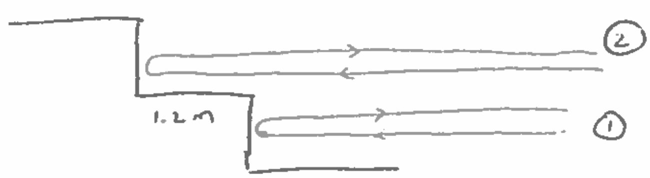
Let $\left[3\pi\text{ rad/m}\right]d = \pi$

$$d = \frac{\pi}{3\pi}\text{m} = 0.33\text{ m}.$$

or $d = \dfrac{\lambda}{2} = 0.33\text{ m}.$

8)

a)



Time between echo 1 and echo 2 reaching the stage $= \dfrac{2\times 1.2\text{m}}{340\text{ m/s}} = 0.00706\text{s}$

Corresponding frequency $= \frac{1}{T} = 142$ Hz

b) If width was smaller, time will reduce and frequency will increase.

9)

a) $\lambda_n = \frac{4L}{n}$ $n = 1, 3, 5.$

~~If~~ Let the speed of sound be 340 m/s

$f_n = \frac{v}{\lambda_n} = \frac{vn}{4L}$

If $n = 1$ & $f_n = 440$ Hz

$L = \frac{v}{4 \times 440 \text{ Hz}} = \frac{340 \text{ m/s}}{4 \times 440} = 0.19$ m

b) Other harmonics can ~~be~~ ~~also~~ also resonate in the tube

$f_3 = 440 \times 3$ Hz $= 1320$ Hz.

$f_5 = 440 \times 5$ Hz $= 2200$ Hz.

$f_7 = 440 \times 7$ Hz $= 3080$ Hz

Note that these will resonate in the tube even if 440 Hz isn't a fundamental frequency.

If $440 \text{ Hz} = f_i$ for some i in $1, 3, 5 \ldots$

1320 Hz will be $f_{(3 \times i)}$, and similarly for 2200 Hz & 3080 Hz

c) If 1320 Hz is fundametal.

$$L = \frac{340}{4 \times 1320} \text{ m} = 0.064 \text{ m}$$

If 2200 Hz is fundamental, $L = 0.039$ m

" 3080 Hz " " , $L = 0.0028$ m.

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11)

Part 1:

Russian sub to German sub:

$$f_o^G = f_s^R \left(\frac{V + V_G}{V - V_R} \right) = \cancel{1.00 \times 10^3 \text{ Hz}}$$

Part 2

German sub to Russian sub.

$$f_o^R = f_s^G \left(\frac{V + V_R}{V - V_G} \right)$$

Also $f_o^G = f_s^G$

So, $$f_o^R = f_s^R \left(\frac{V + V_G}{V - V_R} \right) \left(\frac{V + V_R}{V - V_G} \right)$$

Since the velocities are in ratios, we don't need to convert them to m/s.

$$f_o^R = 1.00 \times 10^3 \text{ Hz} \left(\frac{5470 + 50}{5470 - 60} \right) \frac{(5470 + 60)}{(5470 - 50)} = 1.04 \times 10^3 \text{ Hz}$$

10)

$$\beta = 80\,dB = 10\,dB \times \log_{10}\left(\frac{I}{I_0}\right)$$

$$\Rightarrow \log_{10}\frac{I}{I_0} = 8$$

$$I = I_0 \times 10^8 = 10^{-4}\ W/m^2.$$

a)

$$I_{new} = \frac{I}{3} = \frac{10^{-4}}{3}\ W/m^2.$$

$$\beta_{new} = 10\,dB \times \log_{10}\left(\frac{I_{new}}{I_0}\right)$$

$$= 10\,dB \times \log_{10}\left(\frac{10^8}{3}\right)$$

$$= 75\ dB.$$

Change in β = $\boxed{\Delta\beta = -5\,dB}$

b) At 5m $I_{5m} = 10^{-4}\ W/m^2$

At distance d $I_d = \frac{10^{-4}}{3}\ W/m^2.$

$$I_{5m} \times (5m)^2 = I_d \times d^2$$

~~[Using $I = \frac{P}{4\pi r^2}$]~~

So, $d = 5m\sqrt{\frac{I_{5m}}{I_d}} = 5m\sqrt{3}$ [Using $P = 4\pi I r^2$]

$$\boxed{d = 8.7\ m}$$

12)

a) That the sound produced by the string is close to the reference note [If the difference is large, you can hear the beats distinctly]

b) ~~If you~~ $f = \frac{v}{\lambda} = \frac{\sqrt{T/\mu}}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}$

i) If T increases, f increases.

ii) If beat frequency decreases, f is getting closer to the reference note (as $|f - f_{reference}|$ is decreasing)

These two suggest that f is less that the ~~refere~~ reference note, and is getting closer from below.

$\boxed{\text{String was tuned too low.}}$

c) $f_{ref} = 146.8\ Hz$

$f_{before\ tuning} = (146.8 - 1.5)\ Hz = 145.3\ Hz$

$f_{after\ tuning} = 146.8\ Hz$

$f = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}$

$$\Rightarrow \qquad T = \cancel{\xi}\,(\lambda^2 \mu)\, f^2$$

$$\text{Ans} = \frac{T_{\text{after tuning}} - T_{\text{before tuning}}}{T_{\text{before tuning}}} \times 100\%.$$

$$= \frac{f_{at}^2 - f_{bt}^2}{f_{bt}^2} \times 100\%.$$

$$= \frac{146.8^2 - 145.3^2}{145.3^2} \times 100\%.$$

$$= 2.08\%$$

The tension increased by 2.08%. to match the reference note.