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PHY2053 Health, Summer C 2016

Quiz 1

Date: Thursday, May 19, 2016

Problem 1: Mary runs on a straight road from point A to point B at 4 m/s, immediately turns around and heads back from point B to point A at a slower speed of 3 m/s. The entire run (A $\rightarrow$ B $\rightarrow$ A) takes 28 minutes.

(a) Make a guess for Mary's average **speed** during the entire run. [No penalty for wrong guesses for (a)]

(b) What is the distance between point A and point B?

(c) Now calculate her average **speed** during the entire run.

(d) What is the average **velocity** during the entire run?

Solution

(a) A common guess for the average speed is 3.5 m/s, since the two legs of the journey were traveled at 4 m/s and 3 m/s. But notice that lesser time will be spent from A to B, than from B to A. So, lesser time will be spent at 4 m/s than at 3 m/s. This suggests that the average speed will be closer to 3 m/s than 4 m/s. How much closer, we don't know. Again, in this part you just make a guess.

(b) Here we cannot use the guess we made for the average speed to find the distance. The total distance isn't 3.5 m/s $\times$ 24 min = 5880 m (which would give the distance between A and B as 5880/2 = 2940 m). And we cannot use the formula $\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$ either. That is true for uniformly accelerated motion (or if in the total run, equal times were spent at speeds v_i and v_f).

There are a few ways to approach this problem, we'll look at two mathematically equivalent ways here.

First method: Notice that for every 3 minutes run at 4 m/s, Mary has to run 4 minutes at 3 m/s to cover the same distance on the way back. This is because for the same distance, time and speed are inversely related (time = dist./speed). This means that, every 3 minutes out of 7 in the total time are spent running from A to B (the remaining 4 minutes are spent running from B to A). By this ratio,

$$t_{\mathrm{A}\rightarrow\mathrm{B}} = \frac{3}{7} \times 28 \text{ min} = 12 \text{ min}$$

$$t_{\mathrm{B}\rightarrow\mathrm{A}} = \frac{4}{7} \times 28 \text{ min} = 16 \text{ min}$$

Using these numbers it is easy to see that distance from A to B = 4 m/s $\times$ 12 min = **2880 m** (which is also 3 m/s $\times$ 16 min).

Second method: A more solid approach is to set up equations and solve them. Total time T_{tot} = 28×60 s = 1680 s. Let the distance between A and B be d. We have the following equations:

$$d = 4 \text{ m/s} \times t_{\mathrm{A}\rightarrow\mathrm{B}} \tag{1}$$

$$d = 3 \text{ m/s} \times t_{\mathrm{B}\rightarrow\mathrm{A}} \tag{2}$$

$$t_{\mathrm{A}\rightarrow\mathrm{B}} + t_{\mathrm{B}\rightarrow\mathrm{A}} = T_{\mathrm{tot}} \tag{3}$$

Solving these equations is farely straight forward. One approach is to combine the first two as

$$4 \text{ m/s} \times t_{\mathrm{A}\rightarrow\mathrm{B}} = 3 \text{ m/s} \times t_{\mathrm{B}\rightarrow\mathrm{A}}$$

and then use the third to eliminate $t_{\mathrm{B}\rightarrow\mathrm{A}}$. This gives

$$4 \text{ m/s} \times t_{\mathrm{A}\rightarrow\mathrm{B}} = 3 \text{ m/s} \times (T_{\mathrm{tot}} - t_{\mathrm{A}\rightarrow\mathrm{B}})$$

Solving this we'll get $t_{\mathrm{A}\rightarrow\mathrm{B}} = 720$ s (or 12 min). This will againt yield, $d =$ **2880** m if we plug it back into the first equation.

(c)

$$\text{Average speed} = \frac{\text{Total distance}}{\text{time}} = \frac{2880 \times 2}{1680} \text{ m/s} = 3.43 \text{ m/s} \ \text{(Closer to 3 m/s than to 4 m/s)}$$

(d) Since the total displacement is 0, average velocity is 0.

(Problem 2 on the next page)

Problem 2: You throw a rock from the edge of the top of a building at a speed of $20\sqrt{2}$ m/s at an angle of 45° above the horizontal. The rock falls on the ground below at a distance of 40 m from the building, i.e., 40 m is the *horizontal distance* between the launch position and final position of the rock. Use this information to find the height of the building. $\left[\ \sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}\ \right]$

Solution

We can look at this question as "When $\Delta x = 40$ m, what is Δy?"
Resolving the initial velocity, we find that

$$v_x = 20\sqrt{2}\cos 45^\circ = 20 \text{ m/s}$$

$$v_{y,i} = 20\sqrt{2}\sin 45^\circ = 20 \text{ m/s}$$

***x*-equation**

$$\Delta x = v_x \Delta t$$

***y*-equation**

$$\Delta y = v_{y,i}\Delta t - \frac{1}{2}g\left(\Delta t\right)^2$$

Plugging the values of Δx and v_x into the x-equation, we get $\Delta t = 2$ s.
Plugging this into the y-equation, we get $\Delta y = 20$ m.

And that should be the answer. So, why was the question deemed incorrect?

If the height of the building is 20 m, Δy would be -20 m, since the rock is supposed to land *below* where it started. In fact, for any acceptable height of the building, Δy should be negative. A positive number indicates that for the numbers provided in the question for speed and angle, by the time the rock travels 40 m horizontally, it won't get below the height from which it was launched. So, it can't have landed on the ground *below* at a horizontal distance of 40 m.

In fact, in 2 seconds, the rock would've just reached it's maximum height. It'll take another 2 seconds to get back to the level from which it was launched, and by this time it would've traveled 80 m horizontally. Beyond this time, it will be below the level, and so if the horizontal distance provided in the question is greater than 80 m (100 m, for example) instead of 40 m, the question would've been valid, and Δy would've been negative.