PHY2053 Health, Summer C 2016

Quiz 3

Date: Wednesday, June 8, 2016

Problem 1: An projectile is thrown from the ground at a speed of 100 m/s at an angle above the horizontal. It reaches a maximum height of 375 m and lands on the ground at a distance of 600 m from the launch point. During it's motion through the air, the projectile experiences a constant air drag of 15 N against the motion in the *horizontal direction only.*

(a) During this process, *launch to landing*, what is the work done by gravity on the projectile?

(b) During this process, what is the work done by air drag on the projectile?

Optional exercise (do not attempt during the quiz): Find the mass of the projectile. There actually is enough information in the problem to calculate the mass.

Solution

(a) The force of gravity on the projectile is a constant. So, work done by gravity is just

$$W_g = -mg\Delta y$$

Since gravitational force is mg downwards. The key phrase is we need the work done by gravity from *launch to landing.* So $\Delta y = 0$, which means that work done by gravity is $\mathbf{0}$.

Gravity does negative work on the projectile's way up and and an equal in magnitude positive work on the way down. If you want to calcualte the work done on the way up, you need the mass of the projectile, which isn't given in the problem statement (but can be calculated as we'll see below).

(b) Here, $F_x = -15$ N and $\Delta x = +600$ m which gives $W = -15 \times 600 = -\mathbf{9000\ J}$. **Note that work is a scalar, is not pointing in the backward direction or anything**.

Optional exercise: With a bit of effort we can calculate the mass of the projectile.

First we'll use the maximum height to find the initial velocity in the y-direction (and with that the initial velocity in the x-direction also).

$$v_{f,y}^2 - v_{i,y}^2 = -2g\Delta y$$

We can write this because air drag acts only the x-direction, so in the y-direction the acceleration is still $-g$. Using $v_y = 0$ at the maximum height, we get $v_{i,y}^2 = 2g(\Delta y)_{\max} = 7500\ \mathrm{m^2/s^2}$ and $v_{i,y} \approx 86.60$ m/s.

We have the initial speed, and the y-component of the initial velocity, so we can either use trigonometry or Pythagorean theorem to find the x-component of the initial velocity. $v_{i,x}^2 + v_{i,y}^2 = v_i^2$ or

$$v_{i,x}^2 = v_i^2 - v_{i,y}^2 = 10000 - 7500 = 2500\ \mathrm{m^2/s^2}$$

This gives $v_{i,x} = 50$ m/s (the angle of launch is $60°$ above the horizontal).

Now, in the x-direction: We know that the motion isn't uniform, but uniformly accelerated since there is a constant air drag acting. We have the initial velocity and the total displacement. If we can extract acceleration a_x out of this, we can find the mass (as we know the total force in the x-direction as well). If you try solving for a_x using kinematics, you'll find that we are one piece of information short. We need time Δt for which we again need the y-direction.

There are a few ways of finding how long it'll take the projectile to get back to ground using y-direction kinematics. One is to use $\Delta y = v_{i,y}\Delta t - \frac{1}{2}g(\Delta t)^2$ to find $\Delta t \approx 17.32$ s. We've milked the y-direction enough and are in a position to find a_x. In the x-direction, the motion is uniformly accelerated and:

$$v_{i,x} = 50\ \mathrm{m/s}$$
$$\Delta t = 17.32\ \mathrm{s}$$
$$\Delta x = 600\ \mathrm{m}$$

Using $\Delta x = v_{i,x}\Delta t + \frac{1}{2}a_x(\Delta t)^2$, we get

$$a_x = 2 \times \frac{\Delta x - v_{i,x}\Delta t}{(\Delta t)^2} \approx -1.77\ \mathrm{m/s^2}$$

. Combining this with $F_x^{\mathrm{net}} = -15$ N, we get $\mathbf{m = 8.46\ kg}$.

(Problem 2 on the next page)

Problem 2: An object originally at rest at the origin ($x = y = 0$) explodes into three pieces, A, B, and C, of masses M, $2M$ and $3M$ respectively (the mass of the original object is $6M$). After the explosion, B is seen to be moving in the **negative** x-direction at a speed of 2 m/s and and C is seen to be moving in the **positive** y-direction at a speed of 1 m/s. There are no other forces acting on A, B, C, or the original object.

(a) Find the velocity of A after the explosion. Providing the x- and y-components of the velocity is sufficient, no need to calculate the magnitude and direction.

(b) Find the positions of A, B, and C 5 seconds after the explosion. Answer in terms of their x- and y-coordinates.

(c) Find the location of the center of mass of A, B, and C 5 seconds after the explosion. Again answer in terms of x- and y- coordinates.

Solution

(a) Since these are no external forces acting on the objects involved, we can conserve momentum. Total momentum before collision is 0. So total momentum after collision must be 0. The corresponding (two) equations are

$$m_{\mathrm{A}} v_{\mathrm{A},x} + m_{\mathrm{B}} v_{\mathrm{B},x} + m_{\mathrm{C}} v_{\mathrm{C},x} = 0 \quad [= (m_{\mathrm{A}} + m_{\mathrm{B}} + m_{\mathrm{C}}) \times v_i]$$
$$m_{\mathrm{A}} v_{\mathrm{A},y} + m_{\mathrm{B}} v_{\mathrm{B},y} + m_{\mathrm{C}} v_{\mathrm{C},y} = 0 \quad [= (m_{\mathrm{A}} + m_{\mathrm{B}} + m_{\mathrm{C}}) \times v_i]$$

The masses are $m_{\mathrm{A}} = M$, $m_{\mathrm{B}} = M$, and $m_{\mathrm{C}} = 3M$ and the velocities are $v_{\mathrm{B},x} = -2$ m/s, $v_{\mathrm{B},y} = 0$, $v_{\mathrm{C},x} = 0$, $v_{\mathrm{C},y} = +1$ m/s. Plugging these in the x-equation we get

$$M v_{\mathrm{A},x} + 2M \times (-2 \text{ m/s}) + 0 = 0$$

which gives $\mathbf{v_{A,x}} = \mathbf{+4\ m/s}$. Similarly from the y-equation we get $\mathbf{v_{A,y}} = \mathbf{-3\ m/s}$.

(b) After 5 seconds, the x and y coordinates of A, B and C:

$$x_{\mathrm{A}} = +4 \times 5 = \mathbf{20\ m}$$
$$y_{\mathrm{A}} = -3 \times 5 = \mathbf{-15\ m}$$

$$x_{\mathrm{B}} = -2 \times 5 = \mathbf{-10\ m}$$
$$y_{\mathrm{B}} = 0 \times 5 = \mathbf{0}$$

$$x_{\mathrm{C}} = 0 \times 5 = \mathbf{0}$$
$$y_{\mathrm{C}} = +1 \times 5 = \mathbf{5\ m}$$

(c) The position of the center of mass, from the formula will be

$$x_{\mathrm{com}} = \frac{m_{\mathrm{A}} x_{\mathrm{A}} + m_{\mathrm{B}} x_{\mathrm{B}} + m_{\mathrm{C}} x_{\mathrm{C}}}{m_{\mathrm{A}} + m_{\mathrm{B}} + m_{\mathrm{C}}} = \frac{M \times 20 + 2M \times (-10) + 3M \times 0}{6M} \text{ m} = \mathbf{0}$$
$$y_{\mathrm{com}} = \frac{m_{\mathrm{A}} y_{\mathrm{A}} + m_{\mathrm{B}} y_{\mathrm{B}} + m_{\mathrm{C}} y_{\mathrm{C}}}{m_{\mathrm{A}} + m_{\mathrm{B}} + m_{\mathrm{C}}} = \frac{M \times (-15) + 2M \times 0 + 3M \times 5}{6M} \text{ m} = \mathbf{0}$$

This shouldn't be surprising since in the absence of external forces (the explosion is due to internal forces), the centre of mass maintains it's state of rest or uniform motion. Remember that a system composed of multiple components also obey Newton's three laws of motion. If you look at the second law as $\vec{F}_{\mathrm{net}} = m\vec{a}$ (instead of as $\vec{F}_{\mathrm{net}}$ is the rate of change of total momentum), then $\vec{a}$ is the acceleration (or the rate of change of velocity) of the center of mass.

This is the reason you might feel a similarity between the expression for total momentum of a system and the expression for velocity of it's centre of mass.