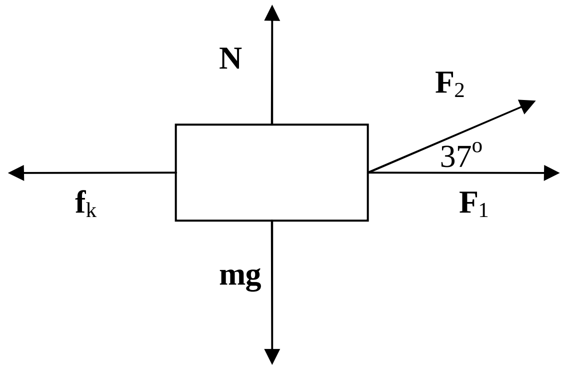
PHY2053 Summer 2012
Exam 2
Solutions

1. The free-body diagram for the block is



Using Newton's second law for the *x*-components

$$\sum F_x = ma_x$$
$$F_1 + F_2 \cos 37^\circ - f_k = 0$$
$$f_k = F_1 + F_2 \cos 37^\circ = (10\text{ N}) + (15\text{ N})\cos 37^\circ = 22\text{ N}$$

The work done by kinetic friction

$$W = f_k \Delta r \cos\theta = (22\text{ N})(6\text{ m})\cos 180^\circ = -130\text{ N}$$

2. Mechanical energy is conserved

$$U_1 + K_1 = U_2 + K_2$$
$$mgy_1 + \tfrac{1}{2}mv_1^{\,2} = mgy_2 + \tfrac{1}{2}mv_2^{\,2}$$
$$gy_1 + \tfrac{1}{2}v_1^{\,2} = gy_2 + \tfrac{1}{2}v_2^{\,2}$$
$$v_2 = \sqrt{v_1^{\,2} + 2g(y_1 - y_2)} = \sqrt{(3\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(60\text{ m} - 30\text{ m})} = 24\text{ m/s}$$

3. Mechanical energy is not conserved since the sled stops moving.

$$\begin{aligned} W_{nc} &= \Delta K + \Delta U \\ &= (K_f - K_i) + (U_f - U_i) \\ &= (0 - 0) + (0 - mgy_i) \\ &= -mgy_i = -(45\text{ kg} + 15\text{ kg})(9.8\text{ m/s}^2)(15\text{ m}) = -8800\text{ J} \end{aligned}$$

Friction does the dissipative work

$$W_{nc} = f_k \Delta r \cos\theta$$
$$f_k = \frac{W_{nc}}{\Delta r \cos 180^\circ} = \frac{-8800 \text{ J}}{(44 \text{ m})(-1)} = 200 \text{ N}$$

4. The force information gives the force constant for the spring

$$F = kx$$
$$k = \frac{F}{x} = \frac{80 \text{ N}}{0.20 \text{ m}} = 400 \text{ N/m}$$

Mechanical energy is conserved as the ball exits the gun

$$U_1 + K_1 = U_2 + K_2$$
$$\tfrac{1}{2}kx_1^2 + \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}kx_2^2 + \tfrac{1}{2}mv_2^2$$
$$\tfrac{1}{2}kx_1^2 + 0 = 0 + \tfrac{1}{2}mv_2^2$$
$$v_2 = x_1\sqrt{\frac{k}{m}} = (0.20 \text{ m})\sqrt{\frac{400 \text{ N/m}}{0.018 \text{ kg}}} = 30 \text{ m/s}$$

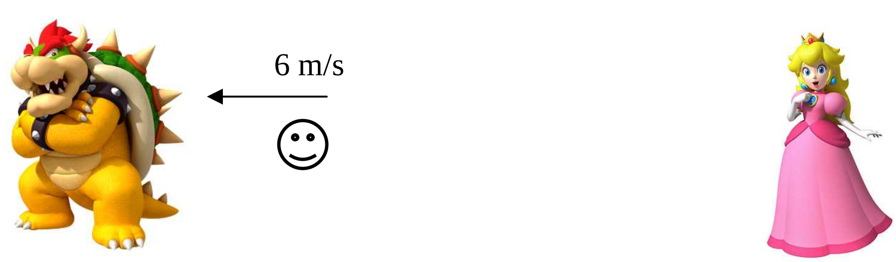
5. The work done by the engine increases the car's kinetic energy.

$$\begin{aligned} W &= \Delta K \\ &= K_f - K_i \\ &= \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \\ &= \tfrac{1}{2}(1000 \text{ kg})(40 \text{ m/s})^2 - \tfrac{1}{2}(1000 \text{ kg})(10 \text{ m/s})^2 = 7.5\times10^5 \text{ J} \end{aligned}$$

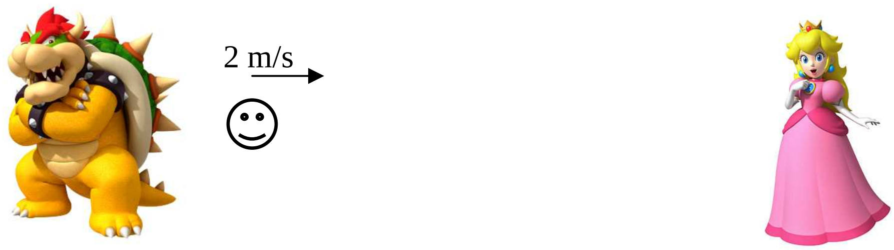
The power output by the engine rate of the work done by the engine

$$P = \frac{W}{t} = \frac{7.5\times10^5 \text{ J}}{10 \text{ s}} = 7.5\times10^4 \text{ W}$$

6. The force is found from the impulse-momentum theorem. Before the collision



After the collision



The impulse-momentum theorem is

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}_{av}\Delta t$$

Since this is a vector equation, we must take components. Since the motion is only along the *x*-axis, only the *x*-component is needed.

$$\Delta p_x = F_{av,x}\Delta t$$

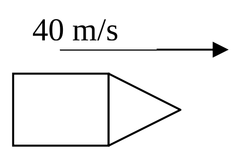
The change in momentum is

$$\Delta p_x = mv_{fx} - mv_{ix} = (1.5 \text{ kg})(2 \text{ m/s}) - (1.5 \text{ kg})(-6 \text{ m/s}) = 12 \text{ kg}\cdot\text{m/s}$$

The average force is

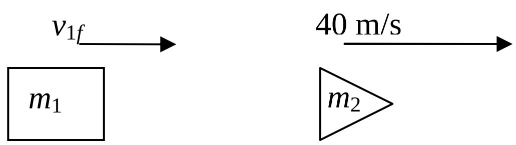
$$F_{av,x} = \frac{\Delta p_x}{\Delta t} = \frac{12 \text{ kg}\cdot\text{m/s}}{0.20 \text{ s}} = 60 \text{ N}$$

7. Linear momentum is conserved since the explosion is an internal force. Before the explosion



$$p_i = mv_i = (15 \text{ kg})(40 \text{ m/s}) = 600 \text{ kg}\cdot\text{m/s}$$

After the explosion

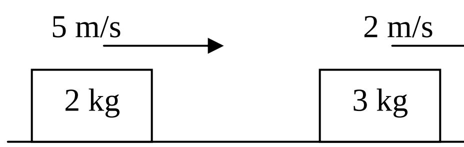


$$p_f = m_1 v_{1f} + m_2 v_{2f}$$

Since linear momentum is conserved,

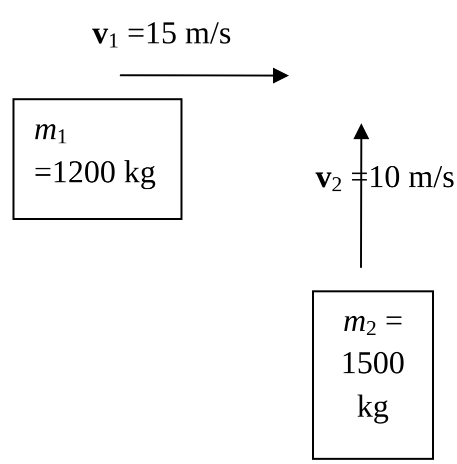
$$p_i = p_f$$
$$p_i = m_1 v_{1f} + m_2 v_{2f}$$
$$v_{1f} = \frac{p_i - m_2 v_{2f}}{m_1} = \frac{(600\ \text{kg}\cdot\text{m/s}) - (5\ \text{kg})(60\ \text{kg}\cdot\text{m/s})}{10\ \text{kg}} = 30\ \text{m/s}$$

8. Use the equations for a one dimensional elastic collision derived in lecture



$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}$$
$$= \left(\frac{2(2\ \text{kg})}{2\ \text{kg} + 3\ \text{kg}}\right)(5\ \text{m/s}) + \left(\frac{3\ \text{kg} - 2\ \text{kg}}{2\ \text{kg} + 3\ \text{kg}}\right)(2\ \text{m/s}) = 4.4\ \text{m/s}$$

9. Momentum is conserved in the collision. Before the collision



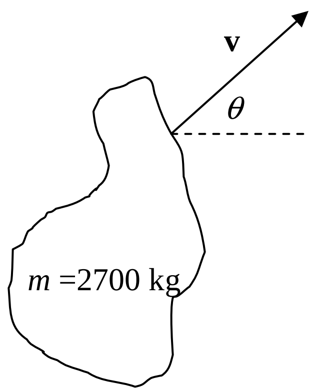
For the *x*-component

$$p_{ix} = m_1 v_1 = (1200 \text{ kg})(15 \text{ m/s}) = 1.8\times10^4 \text{ kg}\cdot\text{m/s}$$

For the *y*-component

$$p_{iy} = m_2 v_2 = (1500 \text{ kg})(10 \text{ m/s}) = 1.5\times10^4 \text{ kg}\cdot\text{m/s}$$

After the collision



For the *x*-component

$$p_{fx} = mv\cos\theta$$

And the *y*-component

$$p_{fy} = mv\sin\theta$$

Using the conservation of linear momentum

$$p_{ix} = p_{fx}$$
$$1.8\times10^4 \text{ kg}\cdot\text{m/s} = mv\cos\theta$$

And

$$p_{iy} = p_{fy}$$
$$1.5\times10^4 \text{ kg}\cdot\text{m/s} = mv\sin\theta$$

There are two equations

$$1.8\times10^{4}\ \text{kg}\cdot\text{m/s} = mv\cos\theta$$
$$1.5\times10^{4}\ \text{kg}\cdot\text{m/s} = mv\sin\theta$$

To solve for v directly, square the equations and add them together.

$$(mv\cos\theta)^2 + (mv\sin\theta)^2 = (1.8\times10^{4}\ \text{kg}\cdot\text{m/s})^2 + (1.5\times10^{4}\ \text{kg}\cdot\text{m/s})^2$$
$$(mv)^2(\cos^2\theta + \sin^2\theta) = 5.49\times10^{8}\ \text{kg}^2\cdot\text{m}^2/\text{s}^2$$
$$v = \sqrt{\frac{5.49\times10^{8}\ \text{kg}^2\cdot\text{m}^2/\text{s}^2}{m^2}} = \sqrt{\frac{5.49\times10^{8}\ \text{kg}^2\cdot\text{m}^2/\text{s}^2}{(2700\ \text{kg})^2}} = 8.7\ \text{m/s}$$

10. The object looks like

The definition of the center of mass is

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

The 1 subscript refers to the rod and the 2 subscript refers to the additional mass. Solving for m_2:

$$x_{cm} = \frac{m_1 + m_2}{m_1 + m_2}$$
$$x_{cm}(m_1 + m_2) = m_1x_1 + m_2x_2$$
$$m_1x_{cm} + m_2x_{cm} = m_1x_1 + m_2x_2$$
$$m_2(x_{cm} - x_2) = m_1(x_1 - x_{cm})$$
$$m_2 = m_1\frac{(x_1 - x_{cm})}{(x_{cm} - x_2)}$$

Measuring the locations from the left end of the rod, the location of the rod is x_1 = 1 m, the location of the added mass is x_2 = 0, and the location of the center of mass is x_{cm} = 0.75 m. So

$$m_2 = m_1\frac{(x_1 - x_{cm})}{(x_{cm} - x_2)} = (3\ \text{kg})\frac{(1\ \text{m} - 0.75\ \text{m})}{(0.75\ \text{m} - 0)} = 1.0\ \text{kg}$$

11. The object consists of two parts. The rotational inertia can be decomposed into

$$I = I_{disk} + I_{mass}$$

The rotational inertia of the disk is

$$I_{disk} = \tfrac{1}{2} m_{disk} R^2 = \tfrac{1}{2}(3\text{ kg})(0.6\text{ m})^2 = 0.54\text{ kg}\cdot\text{m}^2$$

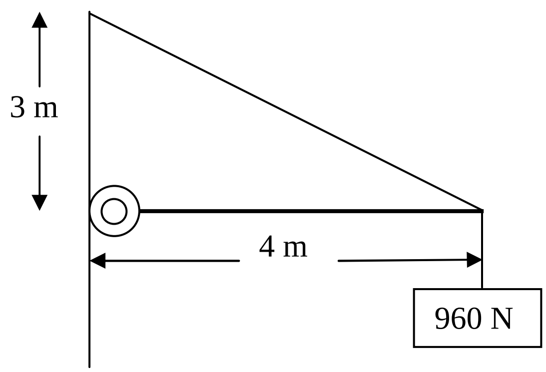
The rotational inertia of the extra mass is

$$I_{disk} = mr^2 = (1\text{ kg})(0.6\text{ m})^2 = 0.36\text{ kg}\cdot\text{m}^2$$

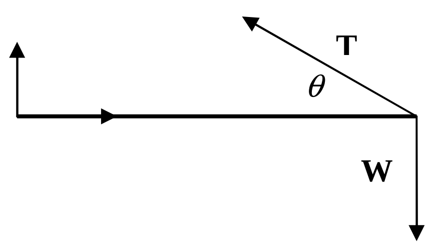
Finally,

$$I = I_{disk} + I_{mass} = 0.54\text{ kg}\cdot\text{m}^2 + 0.36\text{ kg}\cdot\text{m}^2 = 0.90\text{ kg}\cdot\text{m}^2$$

12.



The forces on the beam are



The condition for equilibrium is

$$\sum \tau = 0$$

Taking torques about the left end (the hinge)

$$\sum \tau = 0$$
$$\tau_W + \tau_T = 0$$

The torque due to the weight is clockwise. Its value is

$$\tau_W = -Wr_{\perp W} = -(960\text{ N})(4\text{ m}) = 3840\text{ N}\cdot\text{m}$$

To find the torque due to the tension, we need the angle θ

$$\theta = \tan^{-1}\left(\frac{3\text{ m}}{4\text{ m}}\right) = 37^\circ$$

The torque is counterclockwise,

$$\tau_T = +Tr_{\perp T} = T((4\text{ m})\sin 37^\circ) = (2.4\text{ m})T$$

The tension can be found

$$\begin{aligned} \tau_W + \tau_T &= 0 \\ -3840\text{ N}\cdot\text{m} + (2.4\text{ m})T &= 0 \\ T &= \frac{3840\text{ N}\cdot\text{m}}{2.4\text{ m}} = 1600\text{ N} \end{aligned}$$

13. The rotational inertia of the hoop is

$$I = MR^2 = (100\text{ kg})(2\text{ m})^2 = 400\text{ kg}\cdot\text{m}^2$$

Its angular acceleration

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(0-20\text{ rad/s})}{200\text{ s}} = -0.10\text{ rad/s}^2$$

We don't care about the direction of the acceleration. Drop the minus sign. The torque is

$$\begin{aligned} \sum\tau &= I\alpha \\ \tau &= I\alpha = (400\text{ kg}\cdot\text{m}^2)(0.10\text{ rad/s}^2) = 40\text{ N}\cdot\text{m} \end{aligned}$$

14. The fastest object reaches the bottom first. Use energy to find the fastest. Take position 1 at the top of the ramp and position 2 at the bottom of the ramp.

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + mgy &= (\tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2) + 0 \end{aligned}$$

The rotational inertia for the shapes can be summarized (like our text does) by

$$I = \beta m R^2$$

For the sphere $\beta = 2/5$, the cylinder $\beta = 1/2$, and the ring $\beta = 1$. Also use $\omega = v/R$ in the energy relation:

$$mgy = \tfrac{1}{2} mv^2 + \tfrac{1}{2}\beta m R^2 \left(\frac{v}{R}\right)^2$$
$$gy = \tfrac{1}{2} v^2 (1+\beta)$$
$$v = \sqrt{\frac{2gy}{(1+\beta)}}$$

The largest β will be the slowest. The order will be sphere, cylinder, and ring.

15. Angular momentum will be conserved.

$$L_i = L_f$$
$$I_i \omega_i = I_f \omega_f$$

The time for one rotation (T) is related to the angular speed (ω)

$$\omega T = 2\pi$$
$$\omega = \frac{2\pi}{T}$$

Substituting

$$I_i \omega_i = I_f \omega_f$$
$$I_i \frac{2\pi}{T_i} = I_f \frac{2\pi}{T_f}$$
$$T_f = T_i \frac{I_f}{I_i} = (1.8\ \text{s})\left(\frac{\tfrac{1}{2} I_i}{I_i}\right) = 0.90\ \text{s}$$

16. No.

17. At the depth of 2 m

$$P_2 = P_1 + \rho g d = (1.01\times10^5\ \text{Pa}) + (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(2\ \text{m}) = 1.21\times10^5\ \text{Pa}$$

Double that number and find the depth

$$P_2 = P_1 + \rho g d$$

$$d = \frac{P_2 - P_1}{\rho g} = \frac{2(1.21\times10^5\ \text{Pa}) - 1.01\times10^5\ \text{Pa}}{(1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)} = 14\ \text{m}$$

18. From the density and the mass the volume is found

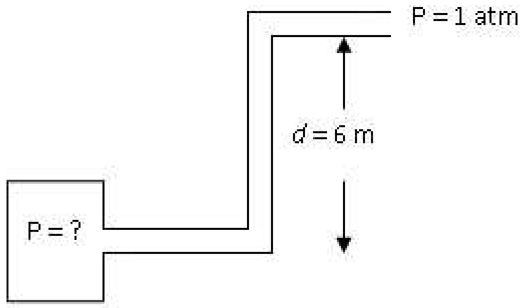
$$\rho = \frac{m}{V}$$

$$V = \frac{m}{\rho} = \frac{15\ \text{kg}}{3000\ \text{kg/m}^3} = 5.0\times10^{-3}\ \text{m}^3$$

Use Archimedes' principle to find the buoyant force

$$F_B = \rho_f g V = (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(5.0\times10^{-3}\ \text{m}^3) = 49\ \text{N}$$

19. Call the position at the bottom of the pipe 1 and the position at the top of the pipe 2. Applying Bernoulli's equation



$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^{\,2} = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^{\,2}$$

The pipe's diameter does not change so

$$A_1 = A_2$$

By the continuity equation

$$A_1 v_1 = A_2 v_2$$

$$v_1 = v_2$$

Since the end of the pipe is exposed to the atmosphere $P_2 = P_{atm}$. Heights are measured from the lowest point so $y_1 = 0$. Making these substitutions

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho {v_1}^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho {v_2}^2$$
$$P_1 + 0 + \tfrac{1}{2}\rho {v_1}^2 = P_{atm} + \rho g y_2 + \tfrac{1}{2}\rho {v_1}^2$$
$$P_1 = P_{atm} + \rho g y_2 = 1.01\times10^5 \text{ Pa} + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(6 \text{ m}) = 1.60\times10^5 \text{ Pa}$$

20. Poiseuille's law is

$$\frac{\Delta V}{\Delta t} = \frac{\pi}{8}\frac{\Delta P / L}{\eta} r^4$$
$$\Delta P / L = \left(\frac{\Delta V}{\Delta t}\right)\frac{8\eta}{\pi r^4} = (2.30\times10^{-2} \text{ m}^3\text{/s})\left(\frac{(8)(1.0\times10^{-3} \text{ Pa}\cdot\text{s})}{\pi(0.025 \text{ m})^4}\right) = 150 \text{ Pa/m}$$