The continuity equation
The continuity equation for an incompressible fluid equates the volume flow rates past two different points,

\[ A_1 v_1 = A_2 v_2 \]

Bernoulli’s Equation
Using energy ideas, the pressure of the fluid in a constriction cannot be the same as the pressure before or after the constriction. For horizontal flow the speed is higher where the pressure is lower. This is called the Bernoulli effect.

“The Bernoulli effect can seem counterintuitive at first; isn’t rapidly moving fluid at high pressure? For instance, if you were hit with the fast-moving water out of a firehose, you would be knocked over easily. The force that knocks you over is indeed due to fluid pressure; you would justifiably conclude that the pressure was high. However, the pressure is not high until you slow down the water by getting in its way. The rapidly moving water in the jet is, in fact, approximately at atmospheric pressure (zero gauge pressure), but when you stop the water, its pressure increases dramatically.” (p. 341)

For a more general situation where the pipe is not horizontal, we can use energy considerations to derive Bernoulli’s equation. (The derivation is given on page 342. I will only quote the result.)

\[ P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2 \]

or

\[ P + \rho g y + \frac{1}{2} \rho v^2 = \text{constant} \]

Hopefully, this reminds you of

\[ W_{nc} + m g y_1 + \frac{1}{2} m v_1^2 = m g y_2 + \frac{1}{2} m v_2^2 \]

Bernoulli’s equation relates the pressure, flow speed, and height at two points in an ideal fluid.
Problem 56. Suppose air, with a density of 1.29 kg/m$^3$ is flowing into a Venturi meter. The narrow section of the pipe at point A has a diameter that is 1/3 of the diameter of the larger section of the pipe at point B. The U-shaped tube is filled with water and the difference in height between the two sections of pipe is 1.75 cm. How fast is the air moving at point B?

Strategy Use the continuity equation to relate the speeds at points A and B. Then, use Bernoulli’s equation to find the speed of the air at point B.

Solution Relate the air speeds at points A and B.

$$A_Bv_B = A_Av_A, \text{ so } v_B = \frac{A_B}{A_A}v_A = \frac{d_B^2}{(d_B/3)^2}v_B = 9v_B.$$ 

Find the speed of the air at point B. Note that $y_A = y_B$.

$$P_B + \frac{1}{2}\rho v_B^2 + \rho g y_B = P_A + \frac{1}{2}\rho v_A^2 + \rho g y_A$$

$$P_B + \frac{1}{2}\rho v_B^2 = P_A + \frac{1}{2}\rho v_A^2$$

$$\frac{1}{2}\rho v_B^2 = P_A - P_B + \frac{1}{2}\rho (9v_B)^2$$

$$\frac{1}{2}\rho v_B^2 = P_A - P_B + \frac{81}{2}\rho v_B^2$$

$$40\rho v_B^2 = P_B - P_A$$

The pressure difference causes the height difference in the manometer

$$P_B - P_A = \rho_w gh$$

Substituting

$$40\rho v_B^2 = \rho_w gh$$

$$v_B = \sqrt{\frac{\rho_w gh}{40\rho}}$$

$$= \sqrt{\frac{(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1.75 \times 10^{-2} \text{ cm})}{40(1.29 \text{ kg/m}^3)}}$$

$$= 1.82 \text{ m/s}$$

Problem 52. In a tornado or hurricane, a roof may tear away from the house because of a difference in pressure between the air inside and the air outside. Suppose that air is blowing across the top of a 2000 ft$^2$ roof at 150 mph. What is the magnitude of the force on the roof?
Lesson 13: Fluid dynamics, Hooke's law, Simple harmonic motion (Sections 9.8-10.6)

Strategy  Use Bernoulli’s equation to find the pressure difference at the roof.

Solution  Let the region above the roof be labeled 1. The region inside the house near the roof is labeled 2. Assume the air under the roof is still.

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \]
\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \rho g y_2 \]

Now \( y_1 \) is almost equal to \( y_2 \) and we can assume that the difference in height has negligible effect on the pressure.

\[ P_1 + \frac{1}{2} \rho v_1^2 = P_2 \]
\[ P_2 - P_1 = \frac{1}{2} \rho v_1^2 \]

Which side is at the higher pressure, the inside or outside? The magnitude of the force on the roof is

\[ F = \Delta P A \]
\[ = \frac{1}{2} \rho_{air} v_1^2 A \]
\[ = \frac{1}{2} \left( 1.20 \text{ kg/m}^3 \right) \left( \frac{150 \text{ mi}}{h} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \left( \frac{1609 \text{ m}}{1 \text{ mi}} \right)^2 \left( \frac{2000 \text{ ft}^2}{1 \text{ mi}^2} \right) \left( \frac{1 \text{ m}}{3.281 \text{ ft}} \right)^2 \]
\[ = 5.0 \times 10^5 \text{ N} \]

which is equal to 56 tons!

Viscosity
Bernoulli’s equation ignores viscosity. When real fluids flow, the different layers of fluid drag against each other. A pressure difference is needed to maintain the flow. This is similar to needing a constant force to overcome kinetic friction. Fluid layers further away from the wall flow faster than those close to the wall.
Poiseuille’s Law

The volume flow rate of viscous fluid through a horizontal cylindrical pipe depends on

- Pressure gradient $\frac{\Delta P}{L}$
- Viscosity. The higher the viscosity, the lower the flow rate.
- Radius of the pipe.

The French physician Poiseuille \( (pwahzoy) \) formulated his law after studying blood flow

$$\frac{\Delta V}{\Delta t} = \frac{\pi \Delta P / L}{8 \eta} r^4$$

Viscosity is $\eta$ (Greek letter eta), measured in Pa·s. Other units are poise \( (pwaz) \) (P) and cP. Conversions are 1 P = 0.1 Pa·s and 1 cP = 0.001 Pa·s.

**Problem.** Water flows through a pipe of radius 8.50 mm. The viscosity of water is 1.005 cP. If the flow speed at the center is 0.200 m/s and the flow is laminar, find the pressure drop due to viscosity along a 3.00 m section of pipe.

**Strategy** Use Poiseuille’s Law to find the pressure drop.

**Solution** Poiseuille’s law is

$$\frac{\Delta V}{\Delta t} = \frac{\pi \Delta P / L}{8 \eta} r^4$$

Solving for $\Delta P$

$$\Delta P = \frac{\Delta V \cdot 8 \eta L}{\Delta t \pi r^4}$$

The volume flow rate is related to the area of the tube and the speed of the flow (see the continuity equation)

$$\frac{\Delta V}{\Delta t} = Av = \pi r^2 v = \pi (8.50 \times 10^{-3} \text{ m})^2 (0.200 \text{ m/s}) = 4.54 \times 10^{-5} \text{ m}^3 / \text{s}$$

Viscosity is not in the correct units. $\eta = 1.005 \times 10^{-3} \text{ Pa·s}$. The pressure difference is
Lesson 13: Fluid dynamics, Hooke's law, Simple harmonic motion (Sections 9.8-10.6)

$$\Delta P = \frac{\Delta V}{\Delta t} \frac{8 \eta L}{\pi r^4} = \left(4.54 \times 10^{-3} \text{ m}^3/\text{s} \right) \frac{8 \left(1.005 \times 10^{-3} \text{ Pa} \cdot \text{s} \right)(3 \text{ m})}{\pi \left(8.50 \times 10^{-3} \text{ m}^3 \right)} = 67 \text{ Pa}$$

Magnus effect. Tennis: top spin, slice, side spin; Baseball: curveballs; Golf: slice, hook
http://www.youtube.com/watch?v=Fk2xU8pEIlI&index=2&list=PLB0A49B27192A26D5
https://www.youtube.com/watch?v=2OSrvzNW9FE

Viscous Drag
An object moving through a fluid experiences drag. Clearly the drag depends on the viscosity of the fluid, the speed of the object, and its size.

Viscous drag is very complicated, but there is a well understood example. For a sphere of radius $r$ traveling at appropriate speed $v$ (so there is no turbulence), Stoke’s Law holds

$$F_D = 6\pi \eta rv$$

When the viscous drag is equal to a falling object’s weight, the object reaches terminal velocity. This is how parachutes work.

Surface Tension
The surface of a liquid has special properties not associated with the interior of the liquid. The surface acts like a stretched membrane under tension.

The surface tension ($\gamma$) of a liquid is the force per unit length with which the surface pulls on its edge.

Soaps break up the surface tension so that water can reach into small places and clean them.

“The high surface tension of water is a hindrance in the lungs. The exchange of oxygen and carbon dioxide between inspired air and the blood takes place in tiny sacs called alveoli, 0.05 to 0.15 mm in radius, at the end of the bronchial tubes. If the mucus coating the alveoli had the same surface tension as other body fluids, the pressure difference between the inside and outside of the alveoli would not be great enough for them to expand and fill with air. The alveoli secrete a surfactant that decreases the surface tension in their mucous coating so they can inflate during inhalation.” (p. 350)
Lesson 13: Fluid dynamics, Hooke's law, Simple harmonic motion (Sections 9.8-10.6)

Bubbles
A gas bubble inside a fluid is in equilibrium between the surface tension trying to shrink the bubble and the pressure inside trying to expand it. The pressure inside the bubble must be greater than the fluid pressure outside. It can be shown that the pressure difference, $\Delta P$, is

$$\Delta P = P_{in} - P_{out} = \frac{2\gamma}{r}$$

As the bubbles rise to the surface from the bottom, they expand. $P_{in}$ reduces with the expansion and $r$ increases as well. The difference, $P_{in} - P_{out}$, becomes smaller.

Chapter 10 Elasticity and Oscillations

In this problem

we assume that the beam is perfectly rigid. The weight hanging from the end will not affect it. This assumption is not true.

In this chapter we study the physical properties of solids. We studied fluids in the last chapter.

Lesson 13, page 6
Some Vocabulary

- A deformation is a change in the size or shape of an object. The deformation may be too small to see if the object is strong and the deforming force is small.
- When the forces are removed from an elastic object, the object returns to its original shape and size.

A More General Hooke's Law

Suppose a force is applied to both ends of a wire.

How does the amount of elongation $\Delta L$ depend on the original length of the wire?

If $F$ causes $L$ to increase by $\Delta L$, the same force $F$ should cause $2L$ to increase by $2\Delta L$.

Since the amount elongation depends on the original length of the wire, we defined the strain as

$$\text{strain} = \frac{\Delta L}{L}$$

Does changing the cross-sectional area of the wire change the effect of $F$?

A thicker cable could be considered to be made of several thinner cables placed side by side. The force would be spread over a bigger area and it would be less effective.

We define the stress as

$$\text{stress} = \frac{F}{A}$$

Stress is measured in N/m$^2$ or Pa.

We rewrite Hooke’s law in terms of stress and strain. In general

$$\text{stress} \propto \text{strain}$$

In terms of the definitions

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$
The constant of proportionality is called the elastic modulus or Young’s modulus. If the same units as stress. \( Y \) is a property of the material used.

Hooke’s law holds up to a maximum stress called the proportional limit.

Beyond the Proportional Limit
If the stress exceeds the proportional limit, the strain is no longer proportional to the stress. The solid will return to its original shape when the stress is removed as long as the elastic limit (see below) is not exceeded.

Prince Rupert’s drop: [http://www.youtube.com/watch?v=xe-f4gokRBs](http://www.youtube.com/watch?v=xe-f4gokRBs)

Beyond the Hooke’s Law
If the stress exceeds the proportional limit but not the elastic limit, the strain is no longer proportional to the stress. The solid will return to its original shape when the stress is removed.

Some more vocabulary
- **Elastic limit** – When the stress is less than the elastic limit, removing the stress will return the solid to the original shape. If the elastic limit is surpassed, the solid remains permanently deformed.
- **Ultimate strength** – If the ultimate strength is surpassed, the solid fractures. The ultimate strength can be different for tensile and compressive stresses.
- **Ductile** – A ductile material continues to stretch beyond its ultimate strength without breaking and the stress decreases from the ultimate strength. When soft metals like gold, silver, copper, lead, et al are pulled, they become thinner and thinner until they break.
- **Brittle** – A brittle material has the ultimate strength and the breaking point close together. Bone is an example of a brittle material. “Babies have more flexible bones than adults because they have built up less of the calcium compound hydroxyapatite. As people age, their bones become more brittle as the collagen fibers lose flexibility and their bones also become weaker as calcium gets reabsorbed (a condition called osteoporosis).” (p. 368)
Other Deformations
There are other ways to deform a solid. Two additional ways are shear deformation and volume deformation.

Shear Deformation
The forces act parallel to the edge of the solid. Tensile and compressive forces act perpendicular to the edges.

\[
\text{shear stress} = \frac{F}{A}
\]

It looks like the previous definition but the picture to the right shows otherwise.

\[
\text{shear strain} = \frac{\Delta x}{L}
\]

Define the shear modulus \( S \) as

\[
\frac{F}{A} = S \frac{\Delta x}{L}
\]

The shear modulus is also measured in Pa.

Volume Deformation
As an example, consider a solid immersed in a fluid. The pressure exerted on all sides will change its volume.

The volume stress is created by the pressure.

\[
\text{volume stress} = \frac{F}{A} = P
\]

The strain will be the change in volume caused by the pressure

\[
\text{volume strain} = \frac{\Delta V}{V}
\]

The bulk modulus is defined in

\[
\Delta P = -B \frac{\Delta V}{V}
\]
Like the other moduli, $B$ is measured in Pa. $\Delta P$ refers to the additional pressure above an atmosphere. Why is there a minus sign?

Unlike the previous stresses and strains, volume stress can be applied to a fluid.

**Problem** A certain man’s biceps muscle has a maximum cross-sectional area of 12 cm$^2 = 1.2 \times 10^{-3}$ m$^2$. What is the stress in the muscle if it exerts a force of 300 N?

**Solution** From the definition of tensile stress, we have

$$\text{Stress} = \frac{F}{A} = \frac{300 \text{ N}}{1.2 \times 10^{-3} \text{ m}^2} = 2.5 \times 10^5 \text{ Pa}$$

**Problem** A wire 1.5 m long has a cross-sectional area of 2.4 mm$^2$. It is hung vertically and stretches 0.32 mm when a 10-kg block is attached to it. Find (a) the stress, (b) the strain, and (c) Young’s modulus for the wire.

**Solution** All measurements must be in SI units.

$$A = 2.4 \text{ mm}^2 \times \left( \frac{1 \text{ m}}{1000 \text{ mm}} \right)^2 = 2.4 \times 10^{-6} \text{ m}^2$$

$$\Delta L = 0.32 \text{ mm} \times \left( \frac{1 \text{ m}}{1000 \text{ mm}} \right) = 3.2 \times 10^{-4} \text{ m}$$

(a) Use the definition of stress

$$\text{Stress} = \frac{F}{A} = \frac{mg}{A} = \frac{(10 \text{ kg})(9.8 \text{ m/s}^2)}{2.4 \times 10^{-6} \text{ m}^2} = 4.08 \times 10^7 \text{ Pa}$$

(b) The definition of strain

$$\text{Strain} = \frac{\Delta L}{L} = \frac{3.2 \times 10^{-4} \text{ m}}{1.5 \text{ m}} = 2.13 \times 10^{-4}$$

(c) Young’s modulus

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$

$$\text{Stress} = Y \text{ Strain}$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{4.08 \times 10^7 \text{ N/m}^2}{2.13 \times 10^{-4}} = 1.92 \times 10^7 \text{ Pa}$$
Simple Harmonic Motion  
Vibration is repeated motion back and forth along the same path. Vibrations occur in the vicinity of a point of **stable equilibrium**.

An equilibrium points is stable is the net force on the object when it is displaced a small distance from equilibrium points back towards the equilibrium point. This is Hooke’s law!

\[ F_x = -kx \]

This type of force is called a restoring force. Simple harmonic motion (SHM) occurs whenever the restoring force is proportional to the displacement from equilibrium.

Simple harmonic motion can be used to approximate small vibrations.

**Energy Analysis**  
For a spring-mass system, we know that energy is conserved.

\[ E = K + U = \text{constant} \]

Recalling the definitions for \( K \) and \( U \)

\[ E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \]

We will call the maximum displacement of the body the amplitude \( A \). At the maximum displacement, the object stops.

\[ E_{\text{total}} = \frac{1}{2}m(0)^2 + \frac{1}{2}kA^2 = \frac{1}{2}kA^2 \]

The maximum speed occurs at equilibrium \((x = 0)\) and \( U = 0 \)
Lesson 13: Fluid dynamics, Hooke's law, Simple harmonic motion (Sections 9.8-10.6)

\[ E_{\text{total}} = \frac{1}{2} mv_m^2 + \frac{1}{2} k(0)^2 = \frac{1}{2} mv_m^2 \]

Equating the two forms of \( E_{\text{total}} \) gives

\[ v_m = \sqrt{\frac{k}{m}} \]

**Acceleration in SHM**

From Hooke’s law for springs

\[ F_x = -kx \]

\[ ma_x = -kx \]

\[ a_x = \frac{-kx}{m} \]

The acceleration (not a constant) is proportional to the displacement and in the opposite direction. The largest acceleration \( (a_m) \) occurs at the largest displacement \( (A) \)

\[ a_m = \frac{kA}{m} \]

**Period and Frequency**

One cycle means that the particle is at the same location and heading the same direction. The period \( (T) \) is the time to complete one cycle. The frequency \( (f) \) is the number of cycles per second. Just as before

\[ f = \frac{1}{T} \]

Not all vibrations are simple harmonic.