Chapter 16

Electric Energy
and
Capacitance
Electric Potential Energy

- The electrostatic force is a conservative force.
- It is possible to define an electrical potential energy function with this force.
- Work done by a conservative force is equal to the negative of the change in potential energy.
Work and Potential Energy

- There is a uniform field between the two plates.
- As the charge moves from A to B, work is done on it.
- \[ W = Fd = q E_x (x_f - x_i) \]
- \[ \Delta PE = -W = -q E_x (x_f - x_i) \]
- only for a uniform field.
Potential Difference

- The potential difference between points A and B is defined as the change in the potential energy (final value minus initial value) of a charge q moved from A to B divided by the size of the charge.
  \[ \Delta V = V_B - V_A = \Delta PE / q \]
- Potential difference is *not* the same as potential energy.
Potential Difference, cont.

- Another way to relate the energy and the potential difference: \( \Delta PE = q \Delta V \)
- Both electric potential energy and potential difference are *scalar* quantities
- Units of potential difference
  - \( V = J/C \) : Volts=Jouls/Coulombs
- A special case occurs when there is a *uniform electric field*
  - \( \Delta V = V_B - V_A = -E_x \Delta x \)
  - Gives more information about units: \( N/C = V/m \)
Energy and Charge Movements

- A positive charge gains electrical potential energy when it is moved in a direction opposite the electric field.
- If a charge is released in the electric field, it experiences a force and accelerates, gaining kinetic energy.
  - As it gains kinetic energy, it loses an equal amount of electrical potential energy.
- A negative charge loses electrical potential energy when it moves in the direction opposite the electric field.
Energy and Charge Movements, cont

- When the electric field is directed downward, point B is at a lower potential than point A.
- A positive test charge that moves from A to B loses electric potential energy.
- It will gain the same amount of kinetic energy as it loses in potential energy.
Summary of Positive Charge Movements and Energy

- When a positive charge is placed in an electric field
  - It moves in the direction of the field
  - It moves from a point of higher potential to a point of lower potential
  - Its electrical potential energy decreases
  - Its kinetic energy increases
Summary of Negative Charge Movements and Energy

- When a negative charge is placed in an electric field
  - It moves opposite to the direction of the field
  - It moves from a point of lower potential to a point of higher potential
  - Its electrical potential energy increases
  - Its kinetic energy increases
  - Work has to be done on the charge for it to move from point A to point B
Electric Potential of a Point Charge

- The point of zero electric potential is taken to be at an infinite distance from the charge.
- The potential created by a point charge \( q \) at any distance \( r \) from the charge is
  \[
  V = k_e \frac{q}{r}
  \]
- A potential exists at some point in space whether or not there is a test charge at that point.
Electric Field and Electric Potential Depend on Distance

- The electric field is proportional to $1/r^2$
- The electric potential is proportional to $1/r$
Electric Potential of Multiple Point Charges

- Superposition principle applies
- The total electric potential at some point P due to several point charges is the *algebraic* sum of the electric potentials due to the individual charges
  - The algebraic sum is used because potentials are scalar quantities
Electrical Potential Energy of Two Charges

- $V_1$ is the electric potential due to $q_1$ at some point $P$
- The work required to bring $q_2$ from infinity to $P$ without acceleration is $q_2 V_1$
- This work is equal to the potential energy of the two particle system

$$PE = q_2 V_1 = k_e \frac{q_1 q_2}{r}$$
Notes About Electric Potential Energy of Two Charges

- If the charges have the *same* sign, PE is positive
  - Positive work must be done to force the two charges near one another
  - The like charges would repel
- If the charges have *opposite* signs, PE is negative
  - The force would be attractive
  - Work must be done to hold back the unlike charges from accelerating as they are brought close together
Problem Solving with Electric Potential (Point Charges)

- Draw a diagram of all charges
  - Note the point of interest
- Calculate the distance from each charge to the point of interest
- Use the basic equation $V = k_e q/r$
  - Include the sign
  - The potential is positive if the charge is positive and negative if the charge is negative
Problem Solving with Electric Potential, cont

- Use the superposition principle when you have multiple charges
  - Take the algebraic sum
- Remember that potential is a scalar quantity
  - So no components to worry about
Potentials and Charged Conductors

- Since $W = -q(V_B - V_A)$, no work is required to move a charge between two points that are at the same electric potential
  - $W = 0$ when $V_A = V_B$
- All points on the surface of a charged conductor in electrostatic equilibrium are at the same potential
- Therefore, the electric potential is a constant everywhere on the surface of a charged conductor in equilibrium
Conductors in Equilibrium

- The conductor has an excess of positive charge.
- All of the charge resides at the surface.
- \( E = 0 \) inside the conductor.
- The electric field just outside the conductor is perpendicular to the surface.
- The potential is a constant everywhere on the surface of the conductor.
- The potential everywhere inside the conductor is constant and equal to its value at the surface.
The Electron Volt

The electron volt (eV) is defined as the energy that an electron gains when accelerated through a potential difference of 1 V

- Electrons in normal atoms have energies of 10’s of eV
- Excited electrons have energies of 1000’s of eV
- High energy gamma rays have energies of millions of eV

1 eV = 1.6 x 10^{-19} J
An *equipotential surface* is a surface on which all points are at the same potential:

- No work is required to move a charge at a constant speed on an equipotential surface.
- The electric field at every point on an equipotential surface is perpendicular to the surface.
Equipotentials and Electric Fields Lines – Positive Charge

- The equipotentials for a point charge are a family of spheres centered on the point charge.
- The field lines are perpendicular to the electric potential at all points.
Equipotentials and Electric Fields Lines – Dipole

- Equipotential lines are shown in blue
- Electric field lines are shown in red
- The field lines are perpendicular to the equipotential lines at all points
Capacitance

- A capacitor is a device used in a variety of electric circuits.
- The *capacitance*, \( C \), of a capacitor is defined as the ratio of the magnitude of the charge on either conductor (plate) to the magnitude of the potential difference between the conductors (plates).
Capacitance, cont

- \[ C \equiv \frac{Q}{\Delta V} \]

- **Units:** Farad (F)
  - 1 F = 1 C / V
  - A Farad is very large
    - Often will see \( \mu F \) or pF
Parallel-Plate Capacitor

- The capacitance of a device depends on the geometric arrangement of the conductors.
- For a parallel-plate capacitor whose plates are separated by air:

\[ C = \varepsilon_0 \frac{A}{d} \]
Parallel-Plate Capacitor, Example

- The capacitor consists of two parallel plates
- Each have area A
- They are separated by a distance d
- The plates carry equal and opposite charges
- When connected to the battery, charge is pulled off one plate and transferred to the other plate
- The transfer stops when $\Delta V_{cap} = \Delta V_{battery}$
Electric Field in a Parallel-Plate Capacitor

- The electric field between the plates is uniform
  - Near the center
  - Nonuniform near the edges
- The field may be taken as constant throughout the region between the plates
Applications of Capacitors – Camera Flash

- The flash attachment on a camera uses a capacitor
  - A battery is used to charge the capacitor
  - The energy stored in the capacitor is released when the button is pushed to take a picture
  - The charge is delivered very quickly, illuminating the subject when more light is needed
Capacitors in Circuits

- A *circuit* is a collection of objects usually containing a source of electrical energy (such as a battery) connected to elements that convert electrical energy to other forms.

- A *circuit diagram* can be used to show the path of the real circuit.
Capacitors in Parallel

- When capacitors are first connected in the circuit, electrons are transferred from the left plates through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged.

- The flow of charges ceases when the voltage across the capacitors equals that of the battery.

- The capacitors reach their maximum charge when the flow of charge ceases.
Capacitors in Parallel

- The total charge is equal to the sum of the charges on the capacitors
  - $Q_{\text{total}} = Q_1 + Q_2$
- The potential difference across the capacitors is the same
  - And each is equal to the voltage of the battery
More About Capacitors in Parallel

- The capacitors can be replaced with one capacitor with a capacitance of $C_{eq}$
  - The equivalent capacitor must have exactly the same external effect on the circuit as the original capacitors
Capacitors in Parallel, final

- \( C_{eq} = C_1 + C_2 + \ldots \)
- The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors
Capacitors in Series

- When a battery is connected to the circuit, electrons are transferred from the left plate of C\textsubscript{1} to the right plate of C\textsubscript{2} through the battery.
- As this negative charge accumulates on the right plate of C\textsubscript{2}, an equivalent amount of negative charge is removed from the left plate of C\textsubscript{2}, leaving it with an excess positive charge.
- All of the right plates gain charges of \(-Q\) and all the left plates have charges of \(+Q\).
More About Capacitors in Series

- An equivalent capacitor can be found that performs the same function as the series combination.
- The potential differences add up to the battery voltage.
Capacitors in Series, cont

- $\Delta V = V_1 + V_2$
  \[
  \frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}
  \]
- The equivalent capacitance of a series combination is always less than any individual capacitor in the combination.
Problem-Solving Strategy

- Be careful with the choice of units
- Combine capacitors following the formulas
  - When two or more unequal capacitors are connected \textit{in series}, they carry the same charge, but the potential differences across them are not the same
    - The capacitances add as reciprocals and the equivalent capacitance is always less than the smallest individual capacitor
Combining capacitors

When two or more capacitors are connected *in parallel*, the potential differences across them are the same.

- The charge on each capacitor is proportional to its capacitance.
- The capacitors add directly to give the equivalent capacitance.
Repeat the process until there is only one single equivalent capacitor

A complicated circuit can often be reduced to one equivalent capacitor

- Replace capacitors in series or parallel with their equivalent
- Redraw the circuit and continue

To find the charge on, or the potential difference across, one of the capacitors, start with your final equivalent capacitor and work back through the circuit reductions
Problem-Solving Strategy,
Equation Summary

- Use the following equations when working through the circuit diagrams:
  - Capacitance equation: \( C = \frac{Q}{\Delta V} \)
  - Capacitors in parallel: \( C_{eq} = C_1 + C_2 + \ldots \)
  - Capacitors in parallel all have the same voltage differences as does the equivalent capacitance
  - Capacitors in series: \( \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots \)
  - Capacitors in series all have the same charge, \( Q \), as does their equivalent capacitance
Energy Stored in a Capacitor

- Energy stored = $\frac{1}{2} Q \Delta V$
- From the definition of capacitance, this can be rewritten in different forms

\[
\text{Energy} = \frac{1}{2} Q\Delta V = \frac{1}{2} C\Delta V^2 = \frac{Q^2}{2C}
\]
Applications

Defibrillators

- When fibrillation occurs, the heart produces a rapid, irregular pattern of beats
- A fast discharge of electrical energy through the heart can return the organ to its normal beat pattern

In general, capacitors act as energy reservoirs that can slowly charged and then discharged quickly to provide large amounts of energy in a short pulse
Capacitors with Dielectrics

- A *dielectric* is an insulating material that, when placed between the plates of a capacitor, increases the capacitance.
  - Dielectrics include rubber, plastic, or waxed paper.
- \[ C = \kappa C_o = \kappa \varepsilon_o (A/d) \]
  - The capacitance is multiplied by the factor \( \kappa \) when the dielectric completely fills the region between the plates.
Capacitors with Dielectrics

(a) $C_0$, $Q_0$, $\Delta V_0$

(b) $C$, $Q_0$, $\Delta V$
Dielectric Strength

- For any given plate separation, there is a maximum electric field that can be produced in the dielectric before it breaks down and begins to conduct.
- This maximum electric field is called the **dielectric strength**.
An Atomic Description of Dielectrics

- Polarization occurs when there is a separation between the “centers of gravity” of its negative charge and its positive charge.

- In a capacitor, the dielectric becomes polarized because it is in an electric field that exists between the plates.
The presence of the positive charge on the dielectric effectively reduces some of the negative charge on the metal. This allows more negative charge on the plates for a given applied voltage. The capacitance increases.