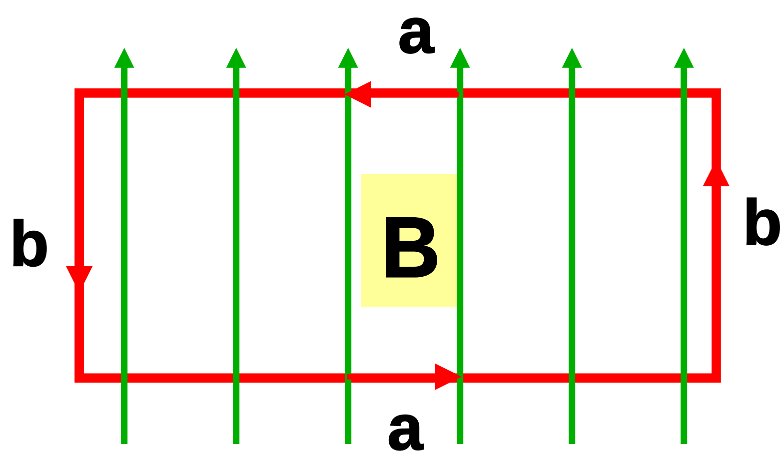
Torque on Current Loop

- ➔ Consider rectangular current loop
  - Forces in left, right branches = 0
  - Forces in top/bottom branches cancel
  - No net force! (true for any shape)
- ➔ But there is a net torque!
  - Bottom side up, top side down (RHR)
  - Rotates around horizontal axis

$$\tau = Fd = (iBa)b = iBab = iBA$$

- ➔ μ = NiA ⇒ "magnetic moment" (N turns)
  - True for any shape!!
  - Direction of μ given by RHR

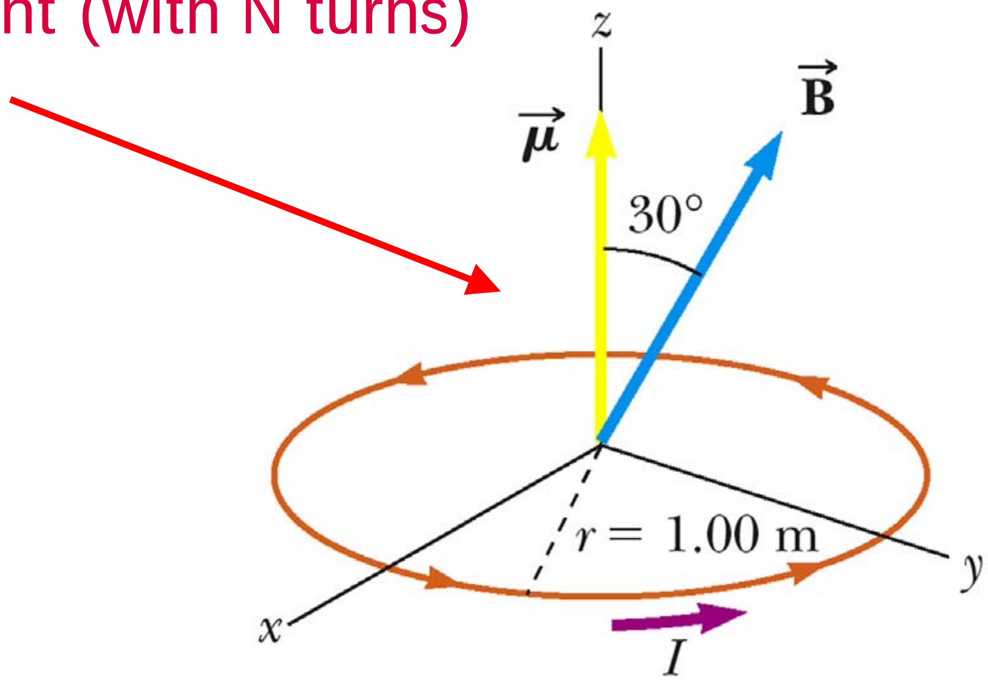


Plane normal is ⊥ B here

General Treatment of Magnetic Moment, Torque

➔ μ = NiA is magnetic moment (with N turns)

- Direction of μ given by RHR

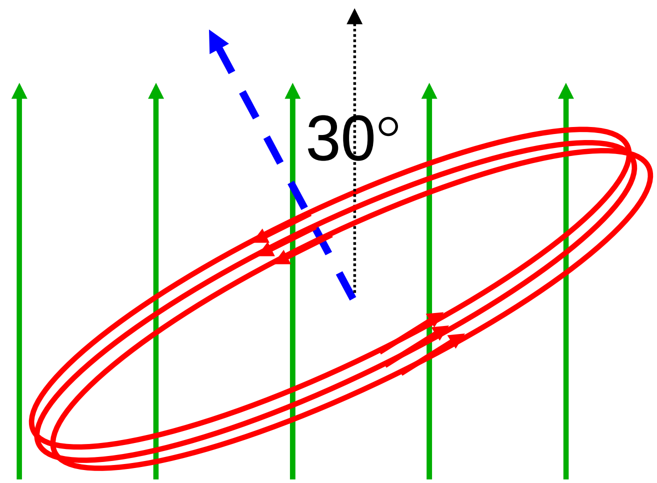


➔ Torque depends on angle θ between μ and B

$$\tau = \mu B \sin\theta$$

Torque Example

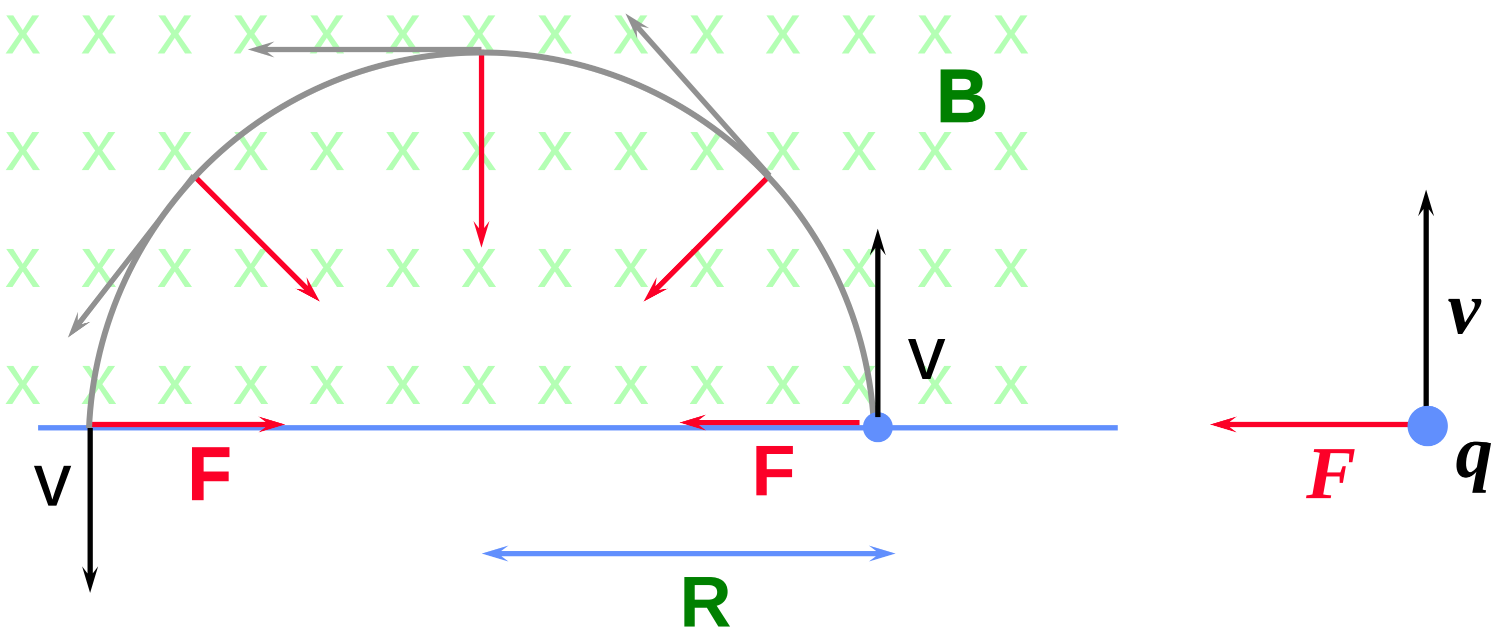
- A 3-turn circular loop of radius 3 cm carries 5A current in a B field of 2.5 T. Loop is tilted 30° to B field.



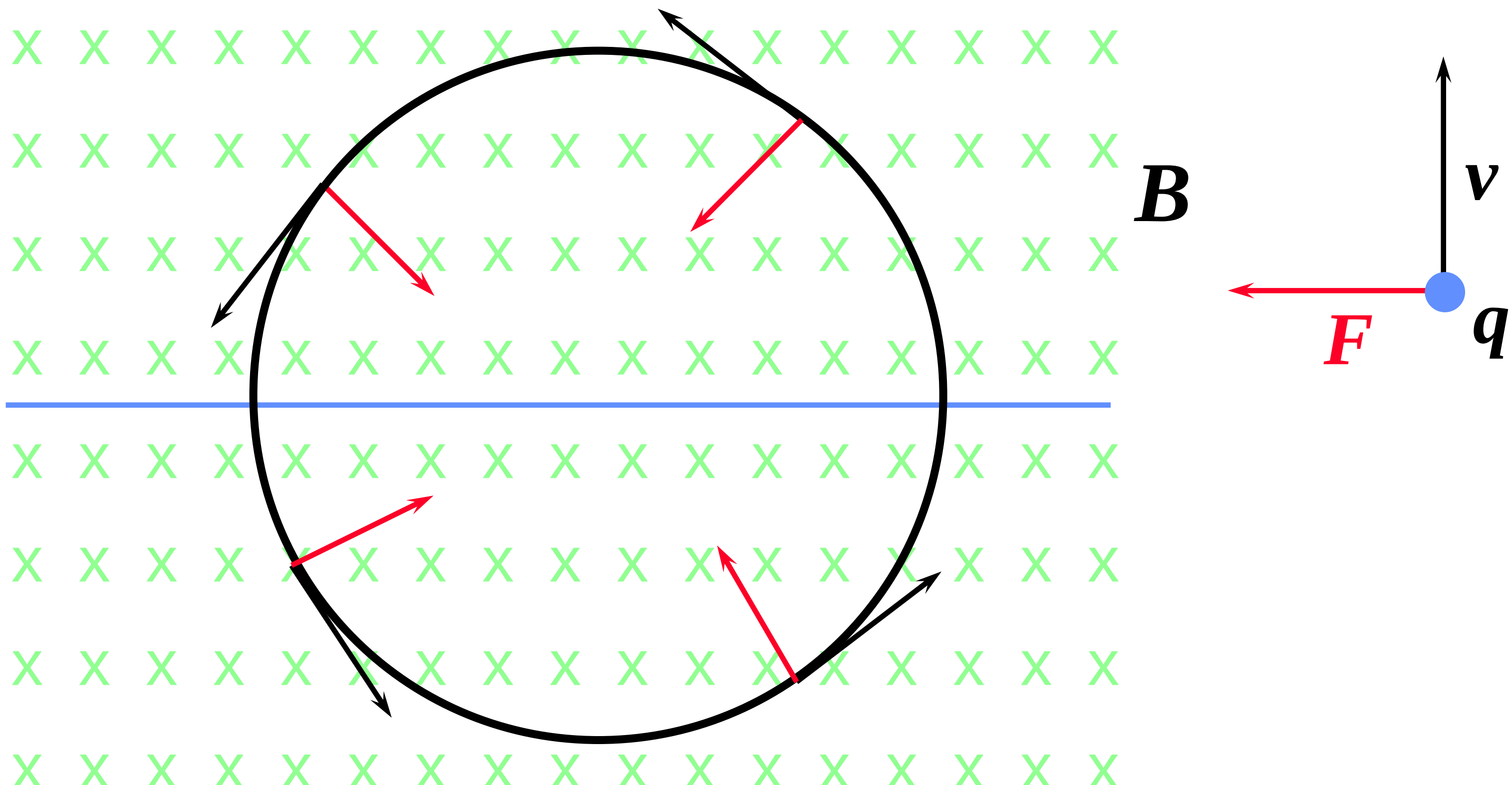
- $\mu = 3i\pi r^2 = 3\times 5\times 3.14\times (0.03)^2 = 0.0339\,\mathrm{A\cdot m^2}$
- $\tau = \mu B \sin 30 = 0.0339\times 2.5\times 0.5 = 0.042\,\mathrm{N\cdot m}$
- Rotation *always* in direction to align μ with B field

Trajectory in a Constant Magnetic Field

- ➔ A charge q enters B field with velocity v perpendicular to B. What path will q follow?
  - ◆ Force is always ⊥ velocity and ⊥ B
  - ◆ Path will be a circle. F is the centripetal force needed to keep the charge in its circular orbit. Let's calculate radius R



Circular Motion of Positive Particle



$$\frac{mv^2}{R} = qvB \Rightarrow \boxed{R = \frac{mv}{qB}}$$

Cosmic Ray Example

➔Protons with energy 1 MeV move ⊥ earth B field of 0.5 Gauss or B = 5 × 10^{-5} T. Find radius & frequency of orbit.

$$K = \tfrac{1}{2} m v^2 \Rightarrow \quad v = \sqrt{\frac{2K}{m}}$$

$$K = \left(10^6\right)\left(1.6\times10^{-19}\right) = 1.6\times10^{-13}\ \text{J}$$

$$m = 1.67\times10^{-27}\ \text{kg}$$

$$R = \frac{mv}{eB} = \frac{\sqrt{2mK}}{eB}$$

$$R = 2900\,\text{m}$$

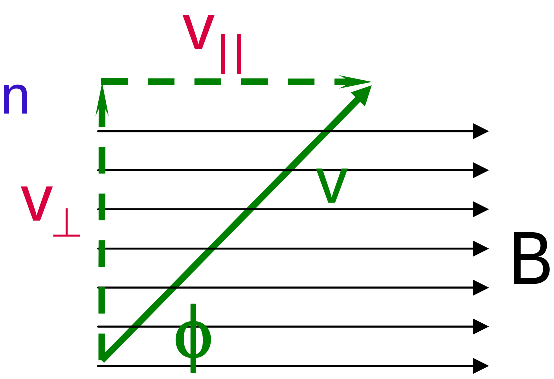
$$f = \frac{1}{T} = \frac{v}{2\pi R} = \frac{v}{2\pi\left(mv/eB\right)} = \frac{eB}{2\pi m}$$

$$f = 760\,\text{Hz}$$

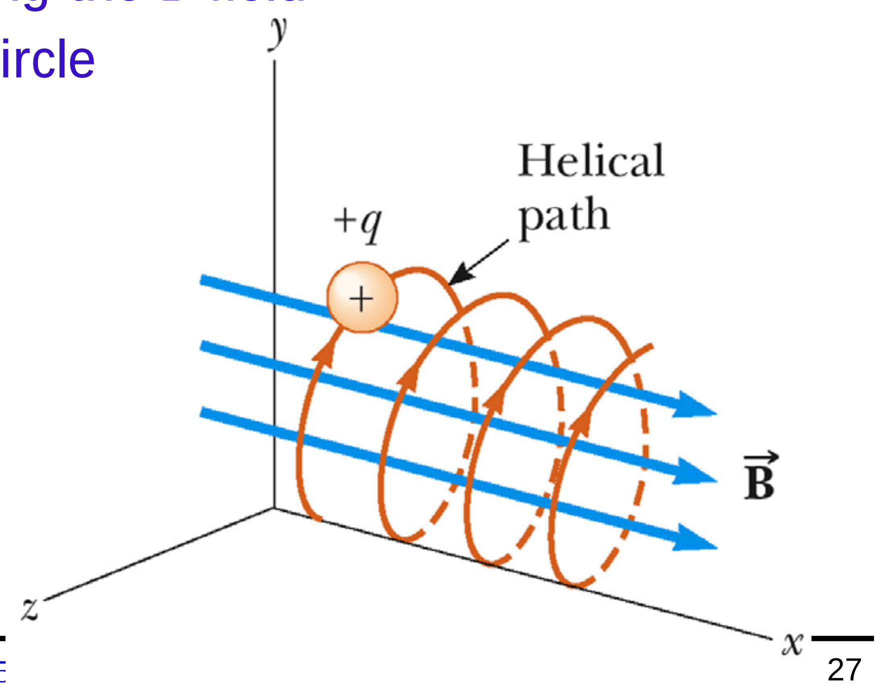
Frequency is independent of v!

Helical Motion in B Field

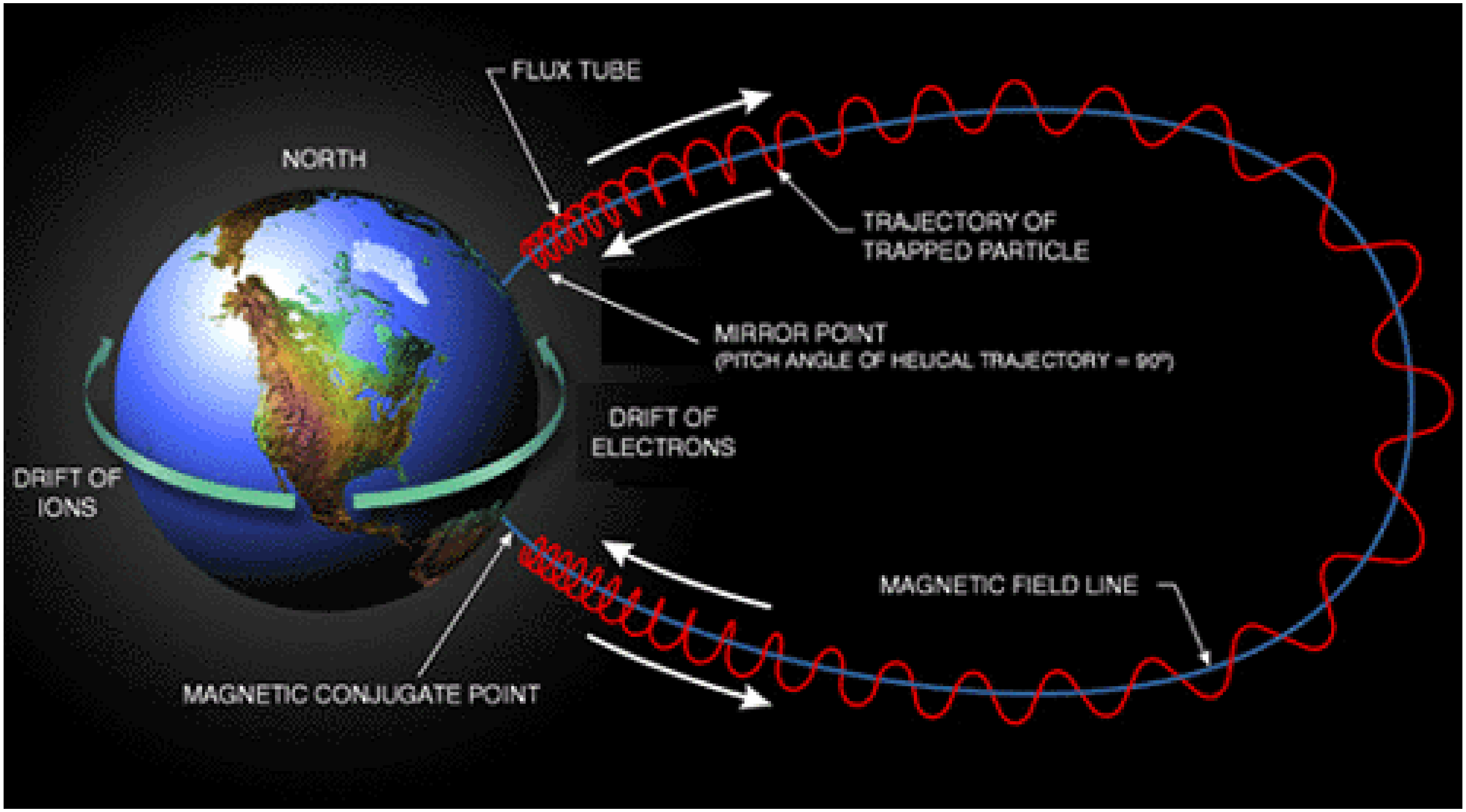
- Velocity of particle has 2 components
  - $\vec{v} = \vec{v}_{||} + \vec{v}_{\perp}$ (parallel to B and perp. to B)
  - Only $v_{\perp} = v \sin\phi$ contributes to circular motion
  - $v_{||} = v \cos\phi$ is unchanged
- So the particle moves in a helical path
  - $v_{||}$ is the constant velocity along the B field
  - $v_{\perp}$ is the velocity around the circle



$$R = \frac{mv_{\perp}}{qB}$$

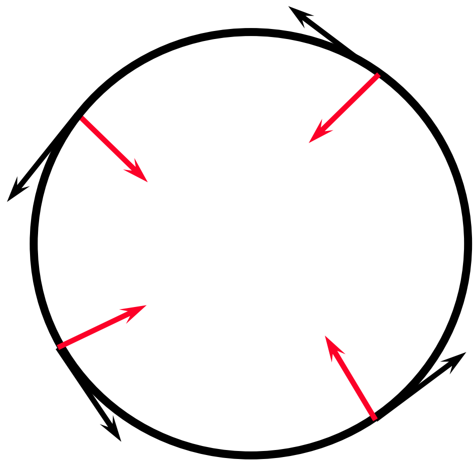


Helical Motion in Earth's B Field



Magnetic Field and Work

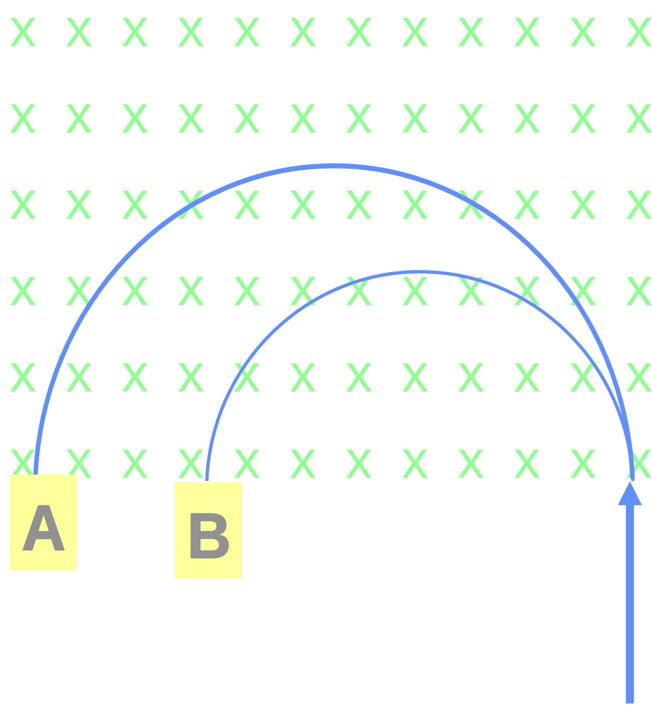
- ➔Magnetic force is *always* perpendicular to velocity
  - ◆Therefore B field does no work!
  - ◆Why? Because $\Delta K = \vec{F}\cdot\Delta\vec{x} = \vec{F}\cdot\left(\vec{v}\Delta t\right) = 0$
- ➔Consequences
  - ◆Kinetic energy does not change
  - ◆Speed does not change
  - ◆Only direction changes
  - ◆Particle moves in a circle (if $\vec{v}\perp\vec{B}$)



Magnetic Force

➔ Two particles of the same charge enter a magnetic field with the same speed. Which one has the bigger mass?

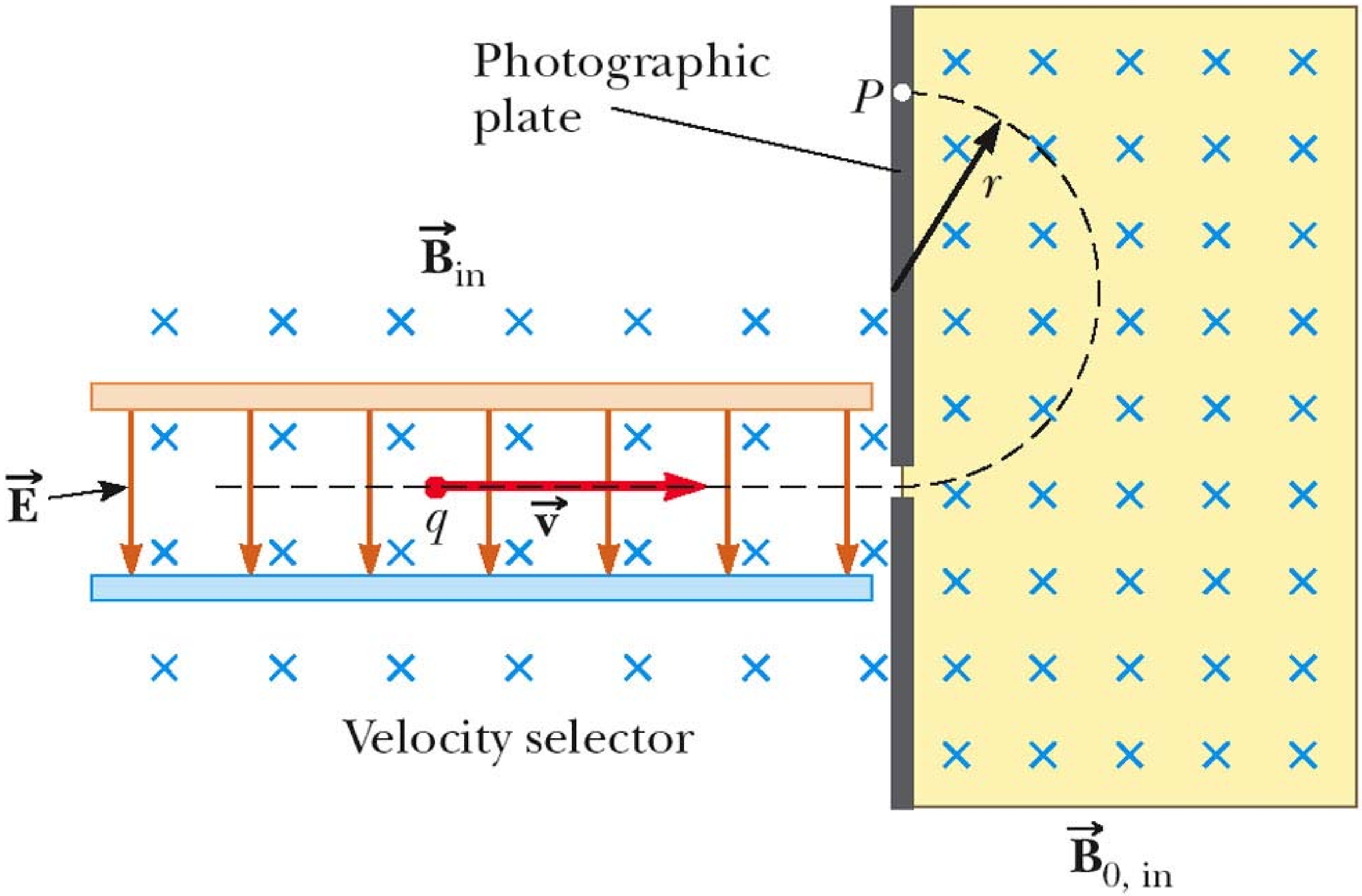
- A
- B
- Both masses are equal
- Cannot tell without more info



$$R = \frac{mv}{qB}$$

Bigger mass means bigger radius

Mass Spectrometer



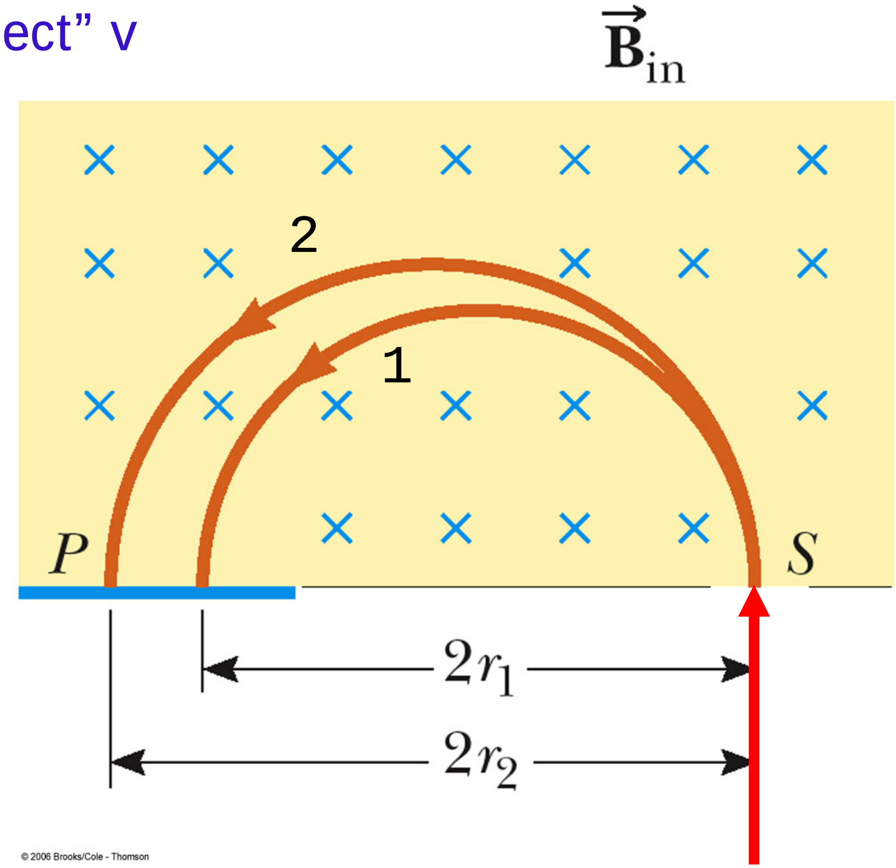
Mass Spectrometer Operation

- ➔Positive ions first enter a "velocity selector" where E ⊥ B and values are adjusted to allow only undeflected particles to enter mass spectrometer.
  - ◆Balance forces in selector ⇒ "select" v

$$qE = qvB$$
$$v = E / B$$

  - ◆Spectrometer: Determine mass from v and measured radius r

$$r_1 = \frac{m_1 v}{qB}$$
$$r_2 = \frac{m_2 v}{qB}$$



Mass Spectrometer Example

- ➔A beam of deuterons travels right at v = 5 x 10^5 m/s
  - ◆What value of B would make deuterons go undeflected through a region where E = 100,000 V/m pointing up vertically?

$$eE = evB$$

$$B = E/v = 10^5 / 5\times10^5 = \boxed{0.2\,\mathrm{T}}$$

  - ◆If the electric field is suddenly turned off, what is the radius and frequency of the circular orbit of the deuterons?

$$\frac{mv^2}{R} = evB \quad \Rightarrow R = \frac{mv}{eB} = \frac{\left(3.34\times10^{-27}\right)\left(5\times10^5\right)}{\left(1.6\times10^{-19}\right)(0.2)} = \boxed{5.2\times10^{-2}\ \mathrm{m}}$$

$$f = \frac{1}{T} = \frac{v}{2\pi R} = \frac{5\times10^5}{(6.28)\left(5.2\times10^{-2}\right)} = \boxed{1.5\times10^6\ \mathrm{Hz}}$$