Chapter 20: Electromagnetic Induction
Topics

Electromagnetic Induction

- Magnetic flux
- Induced emf
  - Faraday’s Law
  - Lenz’s Law
  - Motional emf
- Magnetic energy
- Inductance
- RL circuits
- Generators and transformers
Reading Quiz 1

Magnetic flux through a wire loop depends on:

1) thickness of the wire
2) resistivity of the wire
3) geometrical layout of the wire
4) material that the wire is made of
5) none of the above

\[ \Phi_B = BA \cos \theta \]

Flux depends only on geometrical properties
An induced emf produced in a motionless circuit is due to

1) a static (steady) magnetic field
2) a changing magnetic field
3) a strong magnetic field
4) the Earth’s magnetic field
5) a zero magnetic field

Faraday’s law
Reading Quiz 3

Motional emf relates to an induced emf in a conductor which is:

◆ 1) long
◆ 2) sad
◆ 3) stationary
◆ 4) insulated
◆ 5) moving

Potential difference proportional to velocity
Faraday’s law says that

1. an emf is induced in a loop when it moves through an electric field
2. the induced emf produces a current whose magnetic field opposes the original change
3. the induced emf is proportional to the rate of change of magnetic flux
Reading Quiz 5

A generator is a device that:

- a) transforms mechanical into electrical energy
- b) transforms electrical into mechanical energy
- c) transforms low voltage to high voltage
Magnetic Flux

- Define magnetic flux $\Phi_B$

$$\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA \cos \theta$$

* $\theta$ is angle between $\mathbf{B}$ and the normal to the plane

* Flux units are T-m$^2$ = “webers”

- When $\mathbf{B}$ field is not constant or area is not flat

  - Integrate over area

  $$\Phi_B = \int_A \mathbf{B} \cdot d\mathbf{A}$$

  - Won’t cover this case here
\[ \Phi_B = \mathbf{B} \cdot \mathbf{A} = BA \cos \theta \]

\[ \Phi_B = 0 \quad \Phi_B = \frac{1}{\sqrt{2}} BA \quad \Phi_B = BA \]
This effect can be quantified by Faraday’s Law
Faraday’s Law of Induction

\[ \mathcal{E} = -N \frac{\Delta \Phi_B}{\Delta t} \]

- The faster the change, the larger the induced emf
- Flux change caused by changes in either B, area, orientation
- Minus sign explained next slide
Faraday’s Law of Induction

\[ \mathcal{E} = -N \frac{\Delta \Phi_B}{\Delta t} \]

- minus sign from Lenz’s Law:
- induced current produces a magnetic field which opposes the original change in flux.
Comment on Lenz’s Law

Why does the induced current oppose the change in flux?

Consider the alternative

- If the induced current reinforced the change
- ... then the change would get bigger
- ... which would then induce a larger current
- ... and then the change would get even bigger
- ... and so on . . .

- A clear violation of conservation of energy!!
Direction of Induced Current

Bar magnet moves through coil
- Current induced in coil

Reverse pole
- Induced current changes sign

Coil moves past fixed bar magnet
- Current induced in coil as in (A)

Bar magnet stationary inside coil
- No current induced in coil
ConcepTest: Lenz’s Law

If a North pole moves towards the loop from above the page, in what direction is the induced current?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Must counter flux change in downward direction with upward B field
ConcepTest: Induced Currents

A wire loop is being pulled through a uniform magnetic field. What is the direction of the induced current?

(a) clockwise
(b) counter-clockwise
(c) no induced current

No change in flux, no induced current
ConcepTest: Induced Currents

In the cases above, what is the direction of the induced current?

1. Counterclockwise
2. None
ConcepTest: Lenz’s Law

If a coil is shrinking in a B field pointing into the page, in what direction is the induced current?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Downward flux is decreasing, so need to create downward B field.
Induced currents

A circular loop in the plane of the paper lies in a 3.0 T magnetic field pointing into the paper. The loop’s diameter changes from 100 cm to 60 cm in 0.5 s

- What is the magnitude of the average induced emf?
- What is the direction of the induced current?
- If the coil resistance is 0.05Ω, what is the average induced current?

\[
\left| V \right| = \frac{\Delta \Phi_B}{\Delta t} = 3.0 \times \left| \frac{\pi \left(0.3^2 - 0.5^2\right)}{0.5} \right| = 3.016 \text{ Volts}
\]

- Direction = clockwise (Lenz’s law)

- Current = 3.016 / 0.05 = 60.3 A
ConcepTest: Induced Currents

A wire loop is pulled away from a current-carrying wire. What is the direction of the induced current in the loop?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Downward flux through loop decreases, so need to create downward field.
ConcepTest: Induced Currents

A wire loop is moved in the direction of the current. What is the direction of the induced current in the loop?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Flux does not change when moved along wire
ConcepTest: Lenz’s Law

If the B field pointing out of the page suddenly drops to zero, in what direction is the induced current?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Flux into loop is increasing, so need to create field out of loop

If a coil is rotated as shown, in a B field pointing to the left, in what direction is the induced current?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Upward flux through loop decreases, so need to create upward field
ConcepTest: Induced Currents

Wire #1 (length L) forms a one-turn loop, and a bar magnet is dropped through. Wire #2 (length 2L) forms a two-turn loop, and the same magnet is dropped through. Compare the magnitude of the induced currents in these two cases.

- (a) $I_1 = 2 I_2$
- (b) $I_2 = 2 I_1$
- (c) $I_1 = I_2 \neq 0$
- (d) $I_1 = I_2 = 0$
- (e) Depends on the strength of the magnetic field

Voltage doubles, but R also doubles, leaving current the same
Motional EMF

Consider a conducting rod moving on metal rails in a uniform magnetic field:

\[ \mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = \frac{d(BLx)}{dt} = BL \frac{dx}{dt} \]

Current will flow counter-clockwise in this “circuit”. Why?
Force and Motional EMF

➢ Pull conducting rod out of B field

➢ Current is clockwise. Why?

\[ i = \frac{\mathcal{E}}{R} = \frac{BLv}{R} \]

➢ Current within B field causes force

\[ F = iLB = \frac{B^2L^2v}{R} \]

◆ Force opposes pull (RHR)
◆ Also follows from Lenz’s law

➢ We must pull with this force to maintain constant velocity
Power and Motional EMF

- Force required to pull loop: \( F = iLB = \frac{B^2 L^2 v}{R} \)

- Power required to pull loop: \( P = Fv = \frac{B^2 L^2 v^2}{R} \)

- Energy dissipation through resistance
  \[ P = i^2 R = \left( \frac{BLv}{R} \right)^2 R = \frac{B^2 L^2 v^2}{R} \]

- Same as pulling power! So power is dissipated as heat
  - Kinetic energy is constant, so energy has to go somewhere
  - Rod heats up as you pull it
Example

Pull a 30cm x 30cm conducting loop of aluminum through a 2T B field at 30cm/sec. Assume it is 1cm thick.

- Circumference = 120cm = 1.2m, cross sectional area = $10^{-4}$ m$^2$
- $R = \frac{\rho L}{A} = 2.75 \times 10^{-8} \times 1.2 / 10^{-4} = 3.3 \times 10^{-4}$ Ω

**EMF**

$$\mathcal{E} = BLv = 2 \times 0.3 \times 0.3 = 0.18 \text{ V}$$

**Current**

$$i = \frac{\mathcal{E}}{R} = \frac{0.18}{3.3 \times 10^{-4}} = 545 \text{ A}$$

**Force**

$$F = iLB = 545 \times 0.3 \times 2 = 327 \text{ N}$$

**Power**

$$P = i^2R = 98 \text{ W}$$

About 0.33°C per sec (from specific heat, density)
Electric Generators

➤ Rotate a loop of wire in a uniform magnetic field:
  ◆ changing θ ⇒ changing flux ⇒ induced emf
  ◆ \( \Phi_B = B A \cos \theta = B A \cos(\omega t) \)
Electric Generators

- Flux is changing in a sinusoidal manner
  - Leads to an alternating emf (AC generator)

\[
|E| = N \frac{d\Phi_B}{dt} = NBA \frac{d\cos(\omega t)}{dt} = NBA\omega\sin(\omega t)
\]

- This is how electricity is generated
- Water or steam (mechanical power) turns the blades of a turbine which rotates a loop
- Mechanical power converted to electrical power
A generator has a coil of wire rotating in a magnetic field. If the B field stays constant and the area of the coil remains constant, but the rotation rate increases, how is the maximum output voltage of the generator affected?

- (a) Increases
- (b) Decreases
- (c) Stays the same
- (d) Varies sinusoidally

\[ |\mathcal{E}| = NBA\omega \sin(\omega t) \]
Induction in Stationary Circuit

➡ Switch closed (or opened)
   ➢ Current induced in coil B (direction shown is for closing)

➡ Steady state current in coil A
   ➢ No current induced in coil B
Self - Inductance

Consider a single isolated coil:

- Current (red) starts to flow clockwise due to the battery
- But the buildup of current leads to changing flux in loop
- Induced emf (green) opposes the change

This is a self-induced emf (also called “back” emf)

\[ \mathcal{E} = -N \frac{d\Phi}{dt} = -L \frac{di}{dt} \]

L is the self-inductance
units = “Henry (H)”
Inductance of Solenoid

Total flux (length $l$)

$$B = \mu_0 in$$

$$N \Phi_B = (nl)(BA) = \mu_0 n^2 Al i$$

$$\mathcal{E} = -N \frac{d\Phi_B}{dt} = -\mu_0 n^2 Al \frac{di}{dt} = -L \frac{di}{dt}$$

$$L = \mu_0 n^2 Al$$

To make large inductance:
- Lots of windings
- Big area
- Long
LR Circuits

- Inductance and resistor in series with battery of EMF \( V \)

- Start with no initial current in circuit
  - Close switch at \( t = 0 \)
  - Current is initially 0 (initial increase causes voltage drop across inductor)

- Find \( i(t) \)
  - Resistor: \( \Delta V_R = Ri \)
  - Inductor: \( \Delta V_L = L \frac{di}{dt} \)
  - Use back emf of inductor

\[
i(t) = \frac{V - \Delta V_L}{R} = \frac{V - L \frac{di}{dt}}{R}
\]
Analysis of LR Circuit

General solution

\[ i = \left( \frac{V}{R} \right) \left( 1 - e^{-tR/L} \right) \]

Rise from 0 with time constant \( \tau = L / R \)

Final current (maximum)
Switch off battery: Find $i(t)$ if current starts at $i_0$.
Exponential Behavior

 tua L/R is the “characteristic time” of any RL circuit
  ♦ Only \( t / \tau \) is meaningful, just like for RC circuit

 tua
  ♦ Current falls to \( 1/e = 37\% \) of maximum value
  ♦ Current rises to 63\% of maximum value

 tua
  ♦ Current falls to \( 1/e^2 = 13.5\% \) of maximum value
  ♦ Current rises to 86.5\% of maximum value

 tua
  ♦ Current falls to \( 1/e^3 = 5\% \) of maximum value
  ♦ Current rises to 95\% of maximum value

 tua
  ♦ Current falls to \( 1/e^5 = 0.7\% \) of maximum value
  ♦ Current rises to 99.3\% of maximum value
ConcepTest: Generators and Motors

A current begins to flow in a wire loop placed in a magnetic field as shown. What does the loop do?

- (a) moves to the right
- (b) moves up
- (c) rotates around horizontal axis
- **(d) rotates around vertical axis**
- (e) moves out of the page

This is how a motor works!!
Electric Motors

- Current is supplied from an external source of emf (battery or power supply)
- Forces act to rotate the wire loop
- A motor is essentially a generator operated in reverse!
Motor

- Forces act to rotate the loop towards the vertical.

- When loop is vertical, current switches sign and the forces reverse, in order to keep the loop in rotation.

- This is why alternating current is necessary for a motor to operate.
Motors

Generators

Electrical $\Rightarrow$ mechanical energy          Mechanical $\Rightarrow$ electrical energy
Energy of an Inductor

- Energy is stored in inductor when current flows through it

\[ U_L = \frac{1}{2} Li^2 \]

- Energy is stored in the \( B \) field

- Similar to energy in charged capacitor

\[ U_C = \frac{q^2}{2C} = \frac{1}{2} CV^2 \]

- Energy is stored in the \( E \) field (between plates)
Energy of Solenoid

Recall B field of solenoid (current $i$, $n$ turns per meter)

\[ B = \mu_0 ni \quad \Phi_B = BA = \mu_0 niA \]

Inductance of solenoid

\[ L = \frac{N \Phi_B}{i} = \left( \frac{ni}{i} \right) \left( \mu_0 niA \right) = \mu_0 n^2 A l \]

Energy of solenoid can be written in terms of B field and volume ($V = lA$)

\[ U_L = \frac{1}{2} Li^2 = \frac{1}{2} \left( \mu_0 n^2 lA \right) \left( \frac{B}{\mu_0 n} \right)^2 = \frac{B^2}{2 \mu_0} Al \]

- Energy density of B field
- Volume
Energy of Solenoid (2)

- Typical MRI magnet
  - \( B = 0.5 - 1.5 \, \text{T} \)
  - \( R = 0.25 \, \text{m}, \, l = 2.5 \, \text{m} \)
  \[
  V = Al = \pi R^2 l \approx 0.5 \, \text{m}^3
  \]

- Assume \( B = 1 \, \text{T} \) typically

\[
U_B = \frac{B^2}{2 \mu_0} V = \frac{1^2}{2 \times 4\pi \times 10^{-7}} (0.5) \approx 200 \, \text{kJ}
\]

- This is a lot of energy
- Needs to be held in by strong materials
Gigajoule Magnet at CERN

CMS experiment at CERN
- p-p collisions at world’s highest energy in 2007
- Hope to discover new particles, find the origin of mass and new fundamental forces
Compact Muon Solenoid

- electromagnetic calorimeter
- solenoid
- muon chambers
- hadronic calorimeter
- inner tracker
- magnet yoke
- Human
CMS Experiment Magnet

Large central solenoid magnet to study particle production

- $B = 4T$, $R = 3.15\, m$, $l = 12.5\, m$
- $U_B = 2.6 \times 10^9\, J = 2.6\, \text{gigajoules}$

$$U_B = \frac{B^2}{2\mu_0} Al = \frac{4^2}{2 \times 4\pi \times 10^{-7}} \left(\pi \times 3.15^2\right)(12.5)$$

CMS Articles and Pictures

- **Home page**
  - [http://cmsinfo.cern.ch/Welcome.html/](http://cmsinfo.cern.ch/Welcome.html/)

- **Pictures of detector**

- **Interesting article on solenoid, with pictures**

- **Other documents & pictures about CMS**