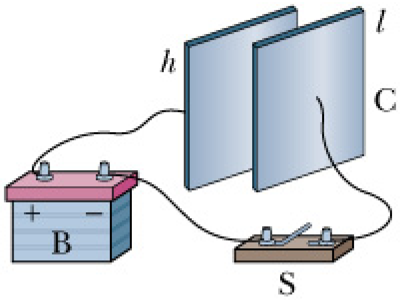
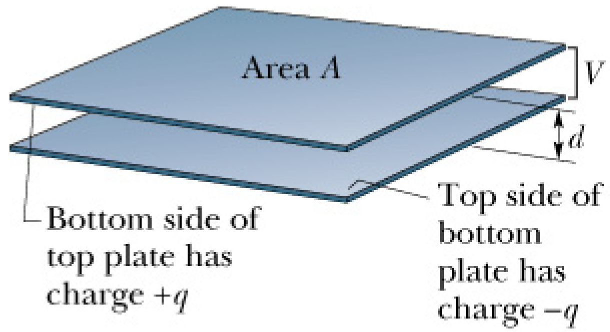
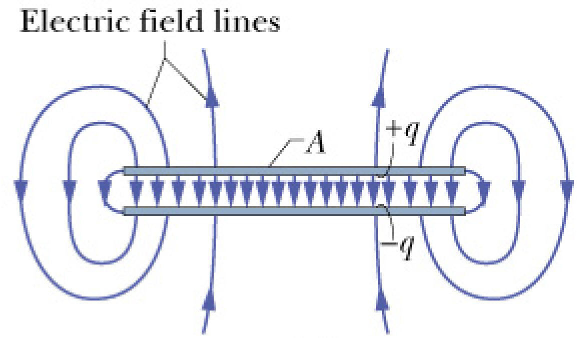
Capacitance



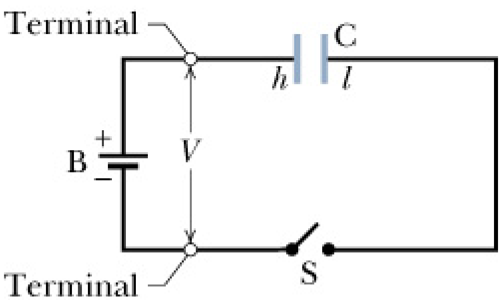
(a)



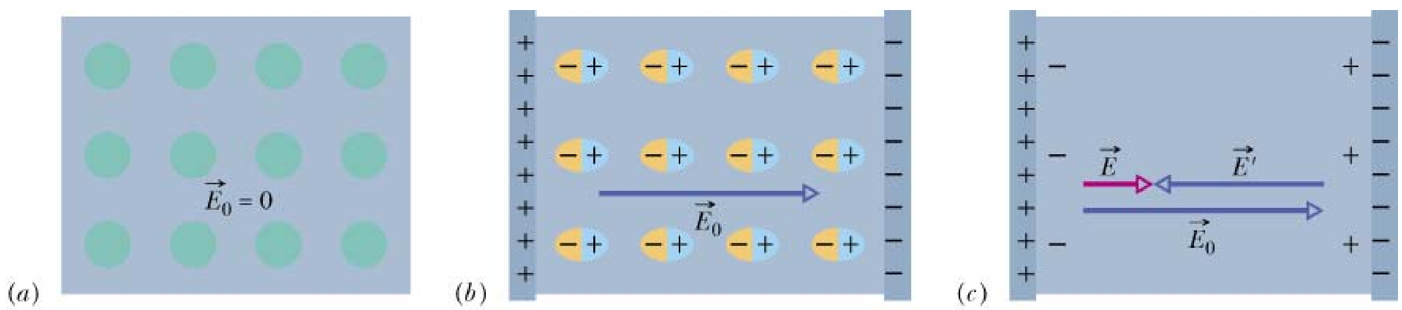
(a)



(b)



(b)



Purpose of Capacitors

- ➔Storage of charge: Q = CV
  - ◆Used in DC and AC circuits
- ➔Storage of energy
  - ◆Can provide energy to circuits
- ➔Used in DC and AC circuits
  - ◆Timing in DC circuits
  - ◆Resonance in AC circuits
  - ◆(Later in course)

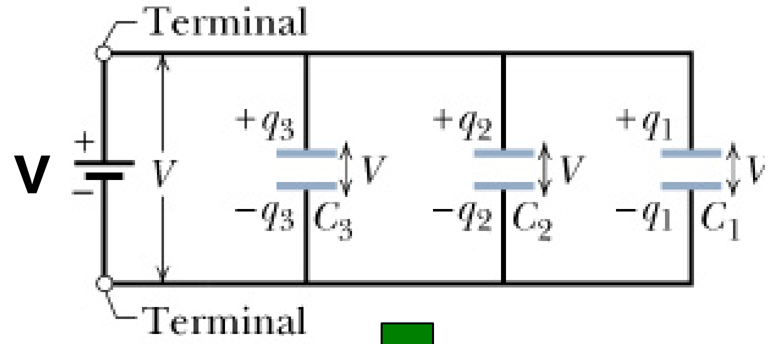
$$U = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2$$

Capacitors in Parallel

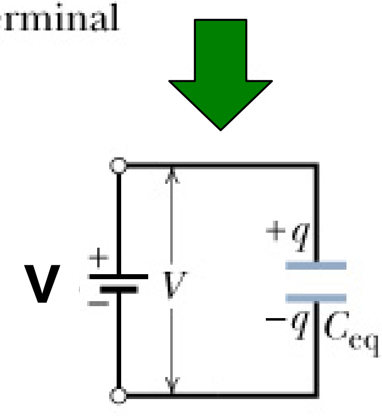
➔$V_1 = V_2 = V_3$ (same potential top and bottom)

➔Total charge: $q_{tot} = q_1 + q_2 + q_3$

➔$C_{eq}V = C_1V + C_2V + C_3V$



$$C_{eq} = C_1 + C_2 + C_3$$



(b)

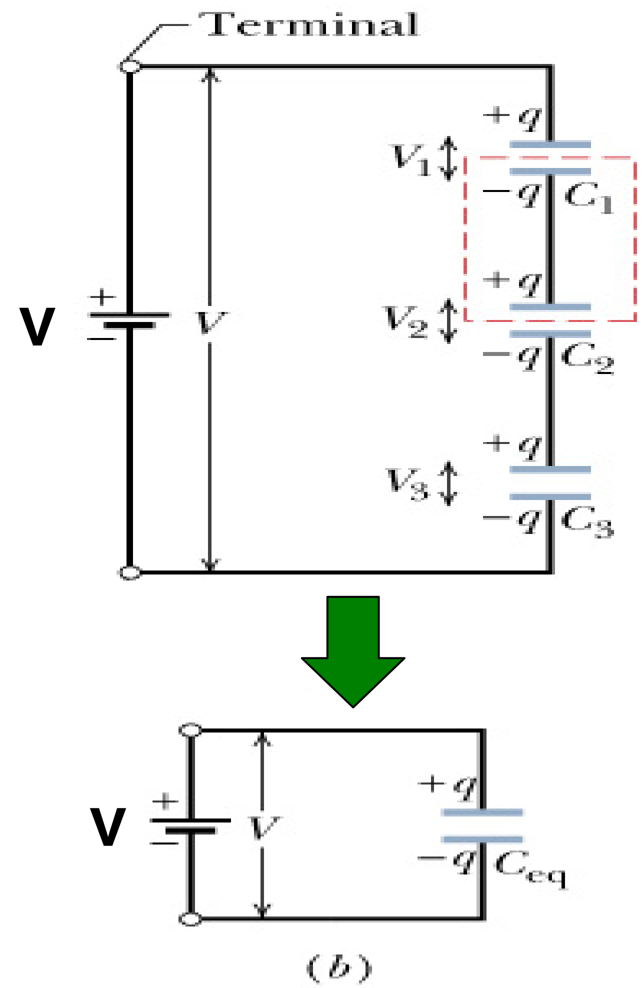
- ➢ **Basic law for combining capacitors in parallel**
- ➢ **Works for N capacitors**

Capacitors in Series

- ➔ $q_1 = q_2 = q_3$ (same current charges all capacitors)
- ➔ Total potential: $V = V_1 + V_2 + V_3$
- ➔ $q/C_{eq} = q/C_1 + q/C_2 + q/C_3$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

- ➢ **Basic law for combining capacitors in series**
- ➢ **Works for N capacitors**



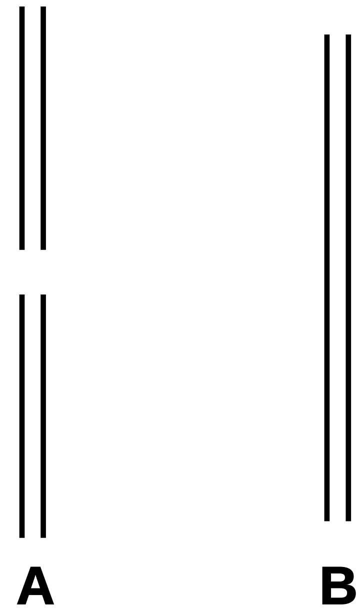
ConcepTest

➔ Two identical parallel plate capacitors are shown in an end-view in Figure A. Each has a capacitance of C.

If the two are joined together at the edges as in Figure B, forming a single capacitor, what is the final capacitance?

- (a) C/2
- (b) C
- (c) 2C
- (d) 0
- (e) Need more information

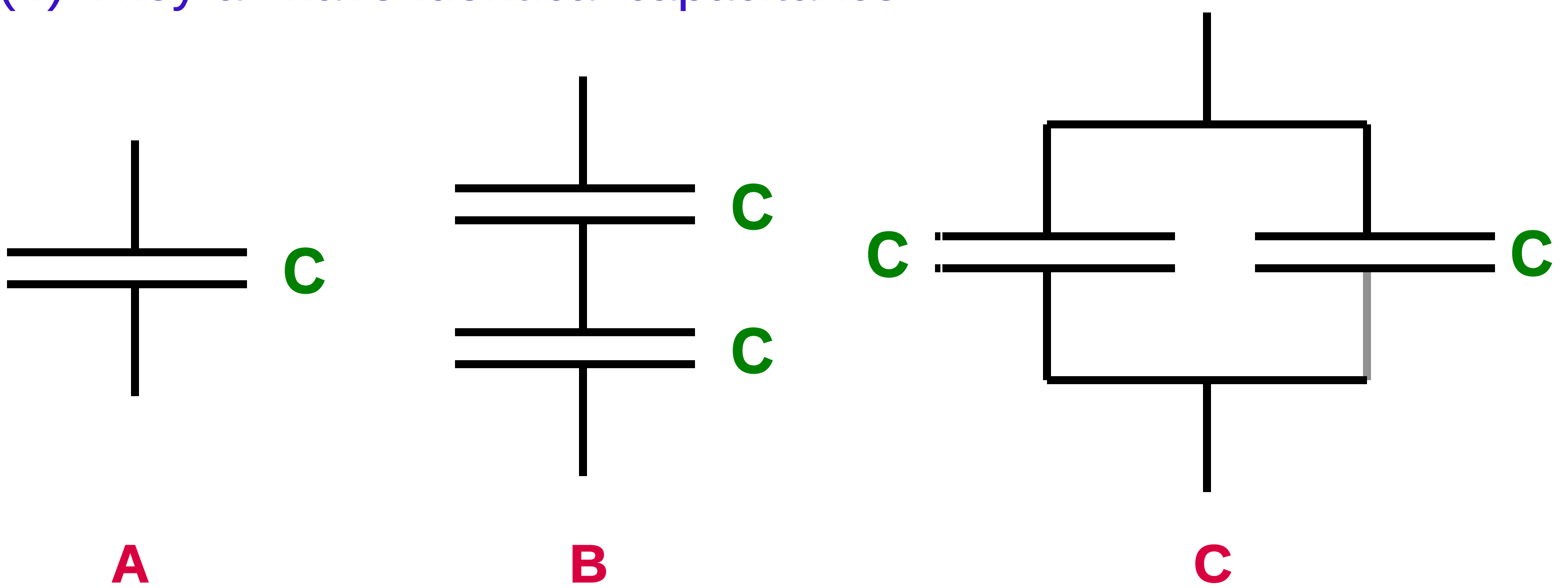
Area is doubled



ConcepTest

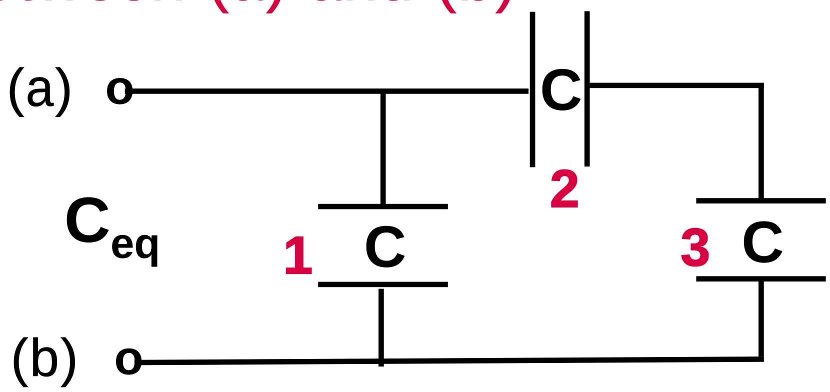
➔Each capacitor is the same in the three configurations. Which configuration has the lowest equivalent capacitance?

- (1) A
- (2) B C/2 (series)
- (3) C
- (4) They all have identical capacitance



Capacitors in Circuits

- ➔Find total capacitance C_{eq} between (a) and (b)
  - ◆Use multi-step process



- ➔C_{23} = 2 + 3 in series
  - ◆$1/C_{23} = 1/C + 1/C = 2/C$
  - ◆$C_{23} = C/2$
- ➔$C_1 + C_{23}$ in parallel
  - ◆$C_{eq} = C + C/2 = 3C/2$

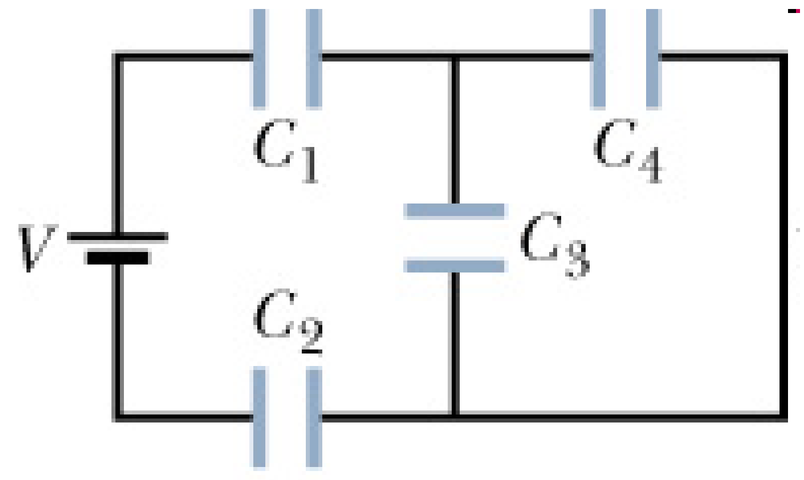
Another Example

➔Assume all capacitors = 10 μF. Find total capacitance

- C_3 and C_4 in parallel: C_{34} = 10 + 10 = 20 μF
- C_1, C_{34}, C_2 in series

$$\frac{1}{C_{\mathrm{eq}}} = \frac{1}{10} + \frac{1}{20} + \frac{1}{10} = \frac{5}{20}$$

$$C_{\mathrm{eq}} = 4.0\,\mu\mathrm{F}$$



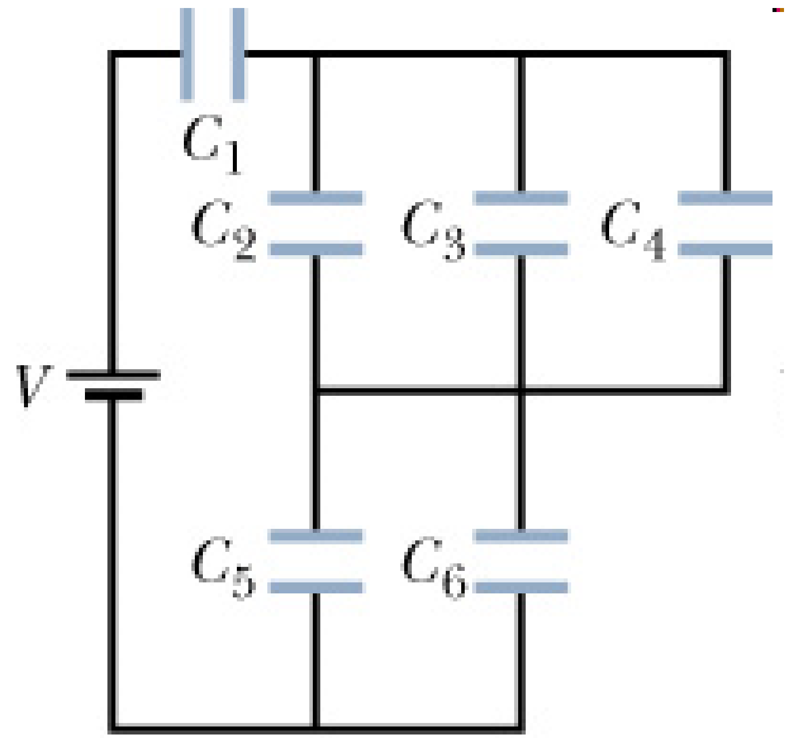
➔How much charge provided by battery to fully charge capacitors? Assume V = 20.

- Q = C_{eq} x V = 4 x 20 = 80 μC

Another Example

➔Assume all capacitors = 10 μF. Find total capacitance

- ◆ C_{234} = 30 μF (parallel)
- ◆ C_{56} = 20 μF (parallel)
- ◆ C_1, C_{234}, C_{56} (series)



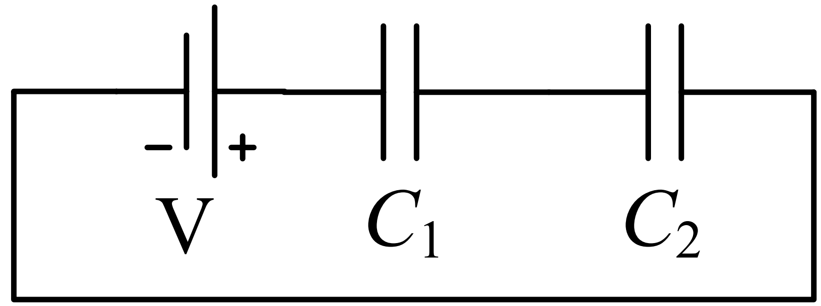
$$\frac{1}{C_{\text{eq}}} = \frac{1}{10} + \frac{1}{30} + \frac{1}{20} = \frac{11}{60}$$

$$C_{\text{eq}} = 5.45\,\mu\text{F}$$

➔How much charge provided by battery to fully charge capacitors? Assume V = 20.

- ◆ Q = C_{eq} x V = 5.45 x 20 = 109 μC

Find Charges on Series Capacitors



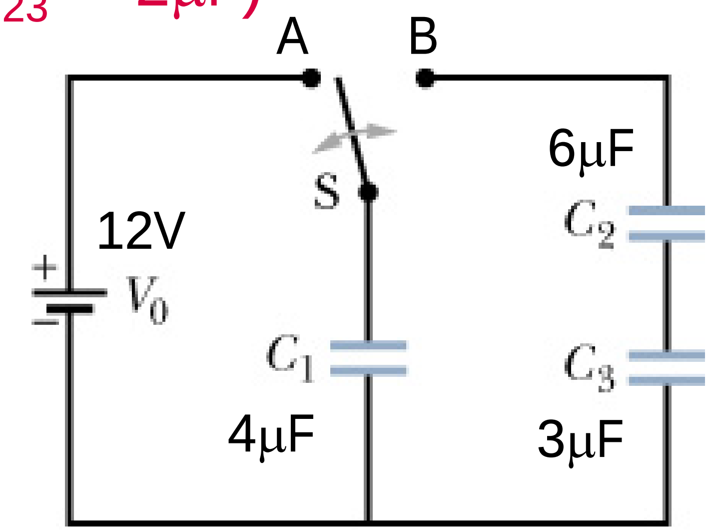
- ➔ Let V = 10, C_1 = 6μF, C_2 = 12μF
  - ◆ Find charges, voltages on C_1, C_2
- ➔ Combine series capacitances
  - ◆ This is what battery "sees"!

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{72}{18} = 4\mu\text{F}$$

- ➔ Find q_{eq}, then use $q_{eq} = q_1 = q_2$
  - ◆ $q_{eq} = C_{eq} V = 4 \times 10 = 40\ \mu C$
- ➔ Find V_1, V_2
  - ◆ $V_1 = q_1 / C_1 = 40 / 6 = 6.67$ V
  - ◆ $V_2 = q_2 / C_2 = 40 / 12 = 3.33$ V
  - ◆ $V_1 + V_2 = 10$, as expected

Example: Find q_i and V_i on All Capacitors

- ➔ C_1 is charged in position A, then S is thrown to B position
  - ◆ Initial voltage across C_1: $V_0 = 12$
  - ◆ Initial charge on C_1: $q_{10} = 12 \times 4 = 48\mu C$
- ➔ After switch is thrown to B:
  - ◆ Charge flows from C_1 to C_2 and C_3
  - ◆ $V_1 = V_{23}$ (parallel branches)
- ➔ q_2 and q_3 in series: $q_2 = q_3 = q_{23}$ ($C_{23} = 2\mu F$)
  - ◆ Charge conservation: $q_{10} = q_1 + q_{23}$
  - ◆ $48 = C_1V_1 + C_{23}V_1$ ($V_1 = V_{23}$)
  - ◆ Find V_1: $V_1 = 48 / (C_1 + C_{23}) = 8$ V
  - ◆ Find q_1: $q_1 = C_1V_1 = 32\mu C$
  - ◆ $q_{23} = 48 - 32 = q_2 = q_3 = 16\mu C$
  - ◆ $V_2 = q_2 / C_2 = 2.67$ V
  - ◆ $V_3 = q_3 / C_3 = 5.33$ V

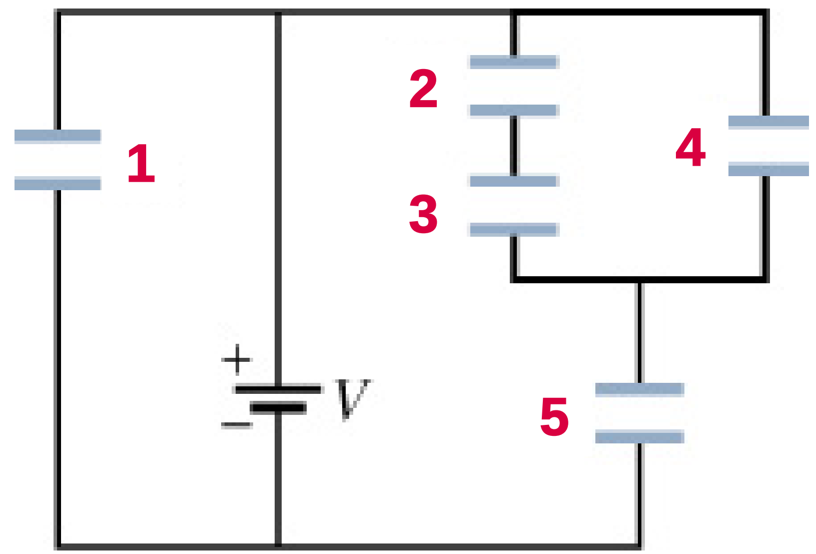


Another Example

➔ Each capacitor has C = 10μF. Find the total capacitance

➔ Do it in stages

- 2 & 3 $\Rightarrow C_{23} = 5\ \mu F$
- Add 4 $\Rightarrow C_{234} = 15\ \mu F$
- Add 5 $\Rightarrow C_{2345} = 6\ \mu F$
- Add 1 $\Rightarrow C_{12345} = 16\ \mu F$

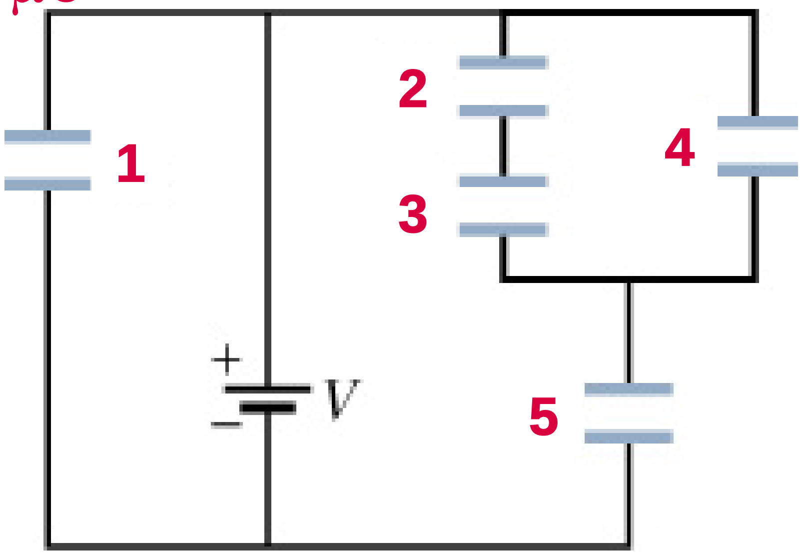


➔ Charge supplied by battery (20V)

- $q_{tot} = C_{12345} \times V = 16 \times 20 = 320\ \mu C$

Find Charges, Voltages on All Capacitors

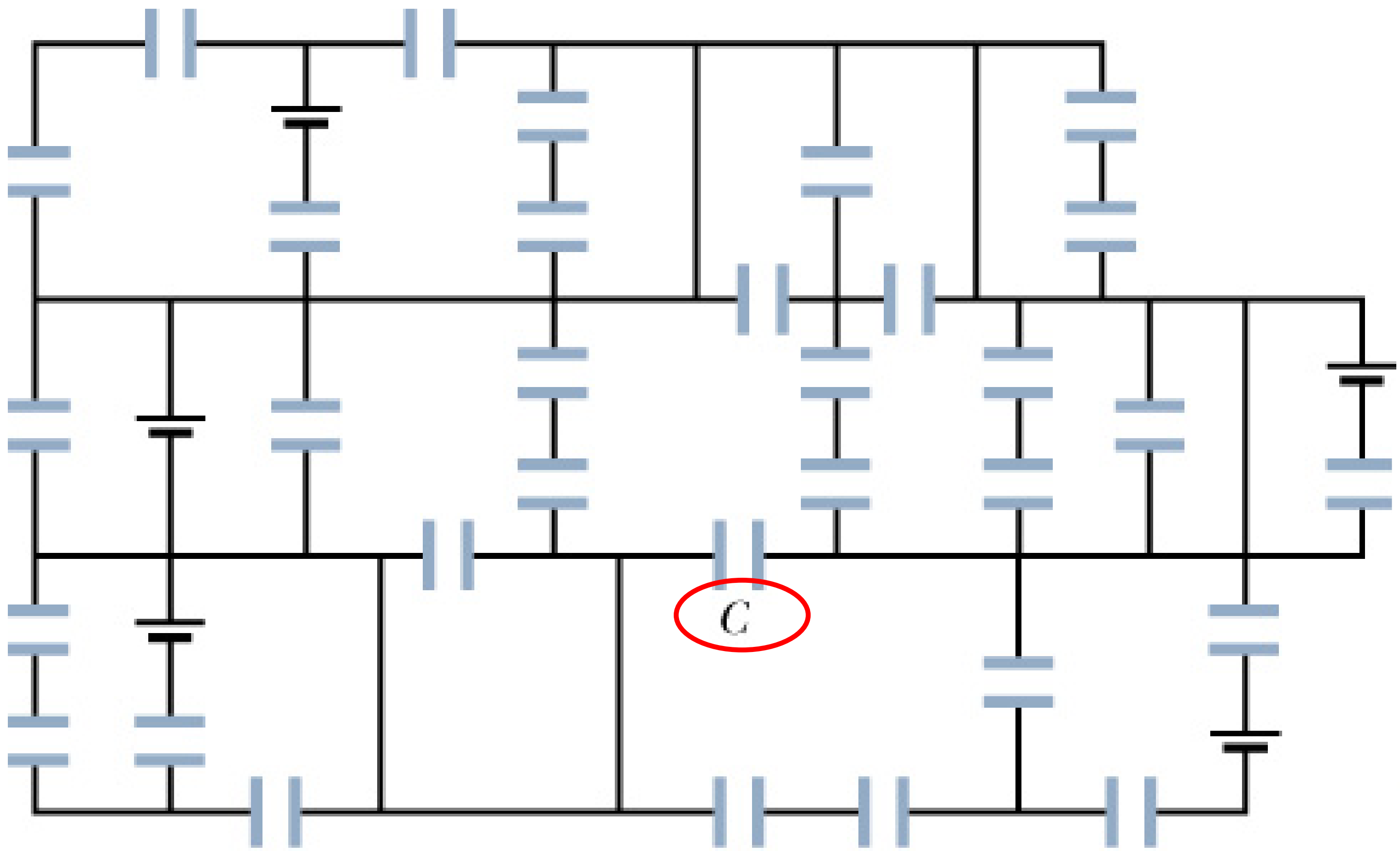
- ➔Each capacitor has capacitance 10μF. V = 20 volts
  - ◆ $q_1 = C_1V = 10 \times 20 = 200\mu C$
- ➔ $C_{2345} = 6\mu F$, $q_{2345} = 6 \times 20 = 120\ \mu C$
  - ◆ $q_{2345} = q_{234} = q_5 = 120\mu C$ (series)
  - ◆ $V_5 = q_5 / C_5 = 120 / 10 = 12$
- ➔Find q_4, $V_4 = V_{23}$
  - ◆ $V_{234} = V_4 = 20 - 12 = 8$
  - ◆ $q_4 = C_4V_4 = 10 \times 8 = 80\mu C$
- ➔Find q_2, q_3, V_2, V_3 ($C_{23} = 5\mu F$)
  - ◆ $q_2 = q_3 = q_{23} = C_{23} \times V_{23} = 5 \times 8 = 40\mu C$
  - ◆ $V_2 = q_2 / C_2 = 40 / 10 = 4$
  - ◆ $V_3 = q_3 / C_3 = 40 / 10 = 4$



Check: $V_2 + V_3 = 8$ (= V_{23})

Capacitor Monster

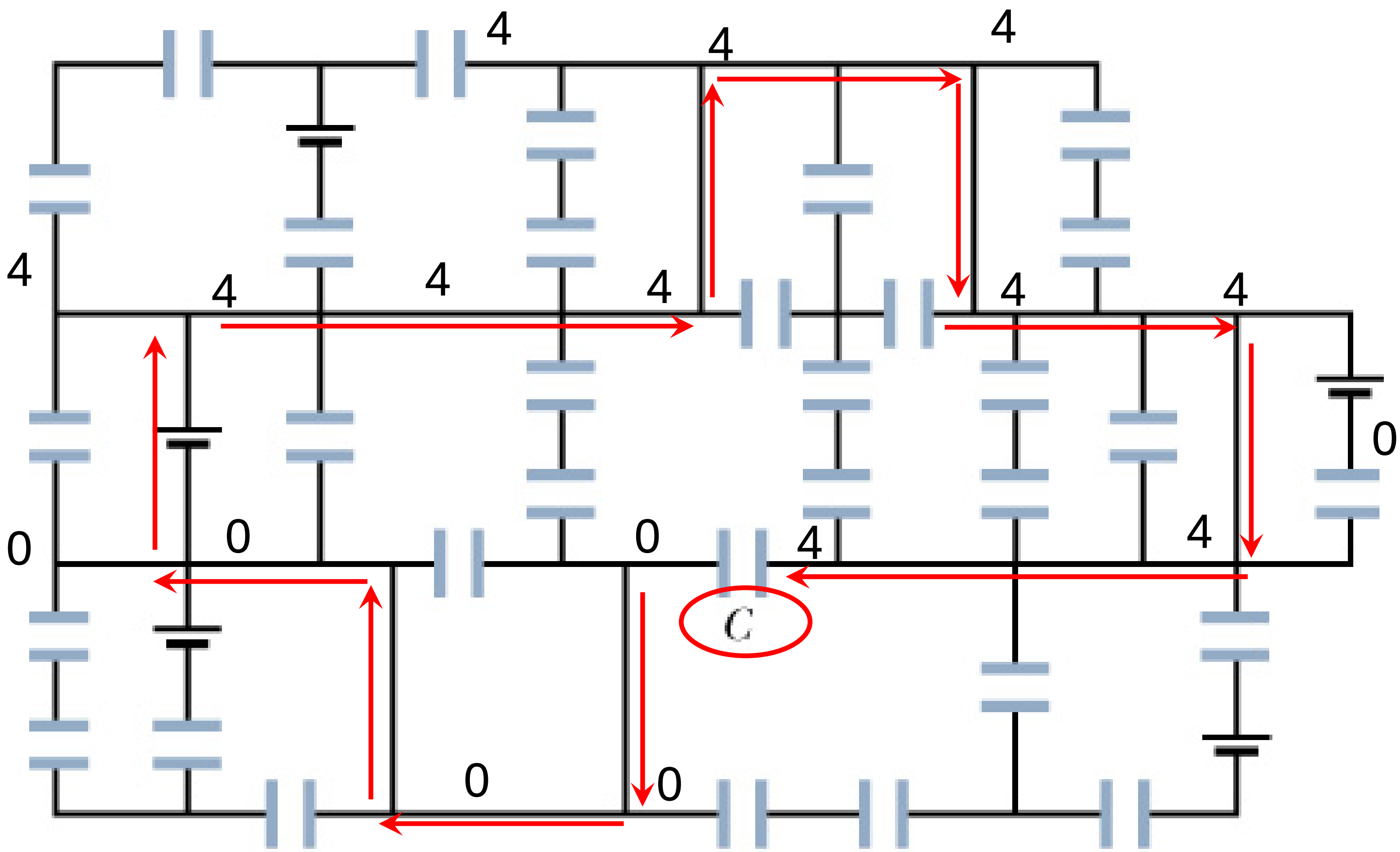
All voltages = 4V, all capacitors = 2μF. What is the charge on C? Can you find the charge on all capacitors?



Capacitor Monster

All voltages = 4V, all capacitors = 2μF. What is the charge on C?

q = CV = 2 x 4 = 8μC



Energy in a Capacitor

- ➔ Capacitors have energy associated with them
  - ◆ Grab a charged capacitor with two hands and find out!
- ➔ Calculation of stored energy
  - ◆ Proof requires simple calculus derivation
  - ◆ Energy = work moving charge from – to + surface

$$U = \frac{Q^2}{2C} = \tfrac{1}{2} C V^2$$

- ➔ Capacitors store and release energy as they acquire and release charge
  - ◆ This energy is available to drive circuits

Example of Capacitor Energy

➔C = 5 μF, V = 200

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\times\left(5\times10^{-6}\right)\times200^2 = 0.1\,\mathrm{J}$$

➔Change V to 20000 (as in demo of large HV capacitor)

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\times\left(5\times10^{-6}\right)\times20000^2 = 1000\,\mathrm{J}$$

Where is the Energy Stored?

- ➔Answer: Energy is stored in the electric field itself!!
- ➔Example: Find energy density of two plate capacitor
  - ◆E field is constant

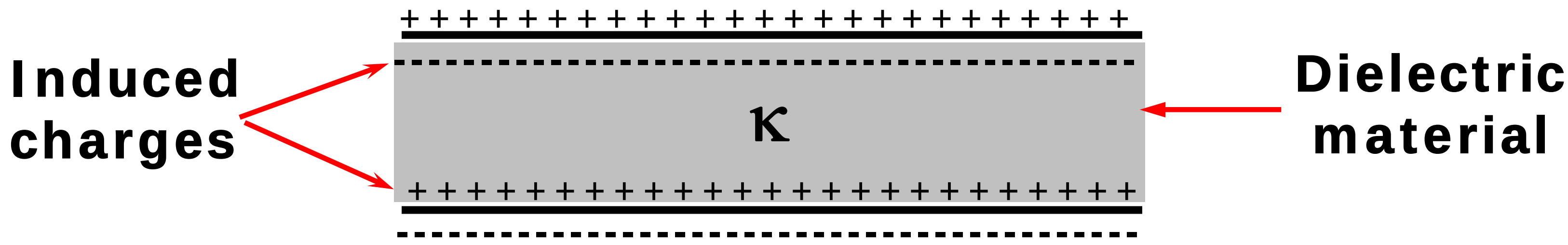
$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad} = \frac{\varepsilon_0 \left(A/d\right)\left(Ed\right)^2}{2Ad} = \tfrac{1}{2}\varepsilon_0 E^2$$

- ➔Energy density depends only on E field!
  - ◆A general result, independent of geometry
  - ◆Can be shown more generally by Maxwell's equations

$$u = \tfrac{1}{2}\varepsilon_0 E^2$$

Dielectric Materials and Capacitors

➔ Insulating material that can be polarized in E field



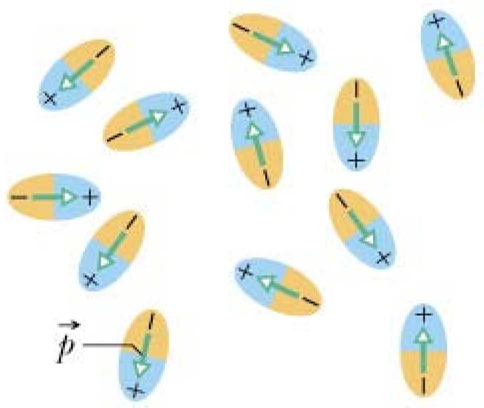
➔ Induced charges at dielectric surface partially cancel E field

- $E \rightarrow E / \kappa$ $\kappa > 1$ is "dielectric constant"
- $V \rightarrow V / \kappa$ (since $V = Ed$)
- $C \rightarrow \kappa C$ (since $C = Q / V$)

➔ But "good" dielectric requires more than high κ value

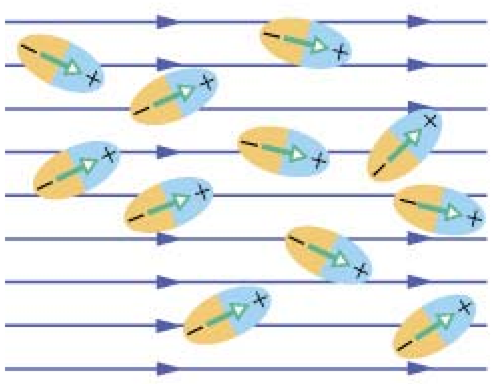
- Good insulator (no charge leakage)
- High breakdown voltage (no arcing at high voltage)
- Low cost (affordable)

Dielectric Mechanism is Due to Polarization



(a)

E = 0, Dipoles randomly aligned



(b)

- **E applied, partially aligns dipoles**
- **Aligned dipoles induce surface charges**
- **Surface charges partially cancel E field**

http://hyperphysics.phy-astr.gsu.edu/hbase/electric/dielec.html

Dielectric Constants of Some Materials

Material	Dielectric constant κ
Vacuum	1.0
Air	1.00059
Bakelite	4.9
Strontium titanate	310
Water	80
Ethanol	24
Mica	5.4
Barium Titanate	100 – 1250
Paper	3.7
Beeswax	2.7 – 3.0
Silica glass	3.8

Example of Dielectric Use

➔Simple capacitor: A = 2m^2, d = 1μm (no dielectric)

- Place 200 volts across C

$$C = \varepsilon_0 \frac{A}{d} = 8.85\times10^{-12}\frac{2}{10^{-6}} = 1.77\times10^{-5} = 17.7\ \mu\text{F}$$

$$q = CV = 17.7\ \mu\text{F}\times 200 = 3540\ \mu\text{C}$$

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(1.77\times10^{-5}\right)\left(200\right)^2 = 0.35\ \text{J}$$

➔Now disconnect C from circuit

- Insert strontium titanate dielectric (κ = 310) into capacitor
- Charge is conserved, calculate new C, V and U

$$C_{\text{new}} = \kappa C = 310\times 17.7 = 5490\ \mu\text{F}$$

$$V_{\text{new}} = V/\kappa = 200/310 = 0.65\ \text{volts}$$

$$U_{\text{new}} = U/\kappa = 0.35/310 = 0.0011\ \text{J}\quad \left(= \tfrac{1}{2}C_{\text{new}}V_{\text{new}}^2\right)$$

Similar Example

- ➔ Same capacitor as before, but this time insert dielectric while C is in the circuit
  - $C_{new} = \kappa C = 5490\ \mu F$
  - V is still 200 volts (maintained by battery)
- ➔ Calculate new q and U in this example

$$q_{\text{new}} = C_{\text{new}} V = 5490\ \mu\text{F} \times 200 = 1.1\ \text{C}$$

$$U_{\text{new}} = \tfrac{1}{2} C_{\text{new}} V^2 = \tfrac{1}{2}\left(5490 \times 10^{-6}\right)\left(200\right)^2 = 110\ \text{J}$$

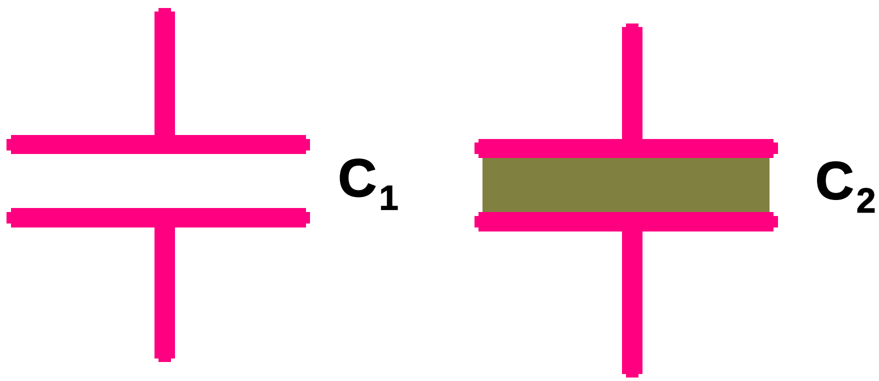
- ➔ Notice how q and U are increased by factor κ

ConcepTest

➔Two identical capacitors are given the same charge Q, then disconnected from a battery.

After C_2 has been charged and disconnected it is filled with a dielectric. Compare the voltages of the two capacitors.

- (1) $V_2 < V_1$
- (2) $V_2 > V_1$
- (3) $V_2 = V_1$

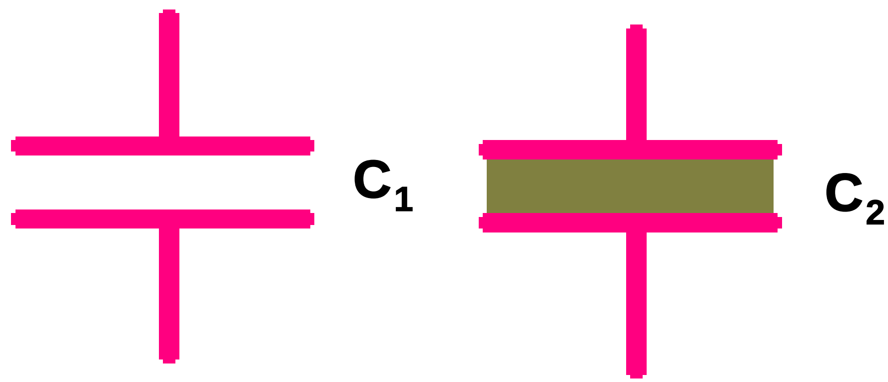


Charge is unchanged, but dielectric reduces E to E/κ and V to V/κ

ConcepTest

➔When we fill the capacitor with the dielectric, what is the amount of work required to fill the capacitor?

- ◆(1) W > 0
- ◆(2) W < 0
- ◆(3) W = 0



Energy is reduced from U to U/κ, so work is negative

If U is total energy in capacitor

- ➢Positive work: One “pushes in” dielectric $\Delta U > 0$
- ➢Negative work: Capacitor “sucks in” dielectric $\Delta U < 0$