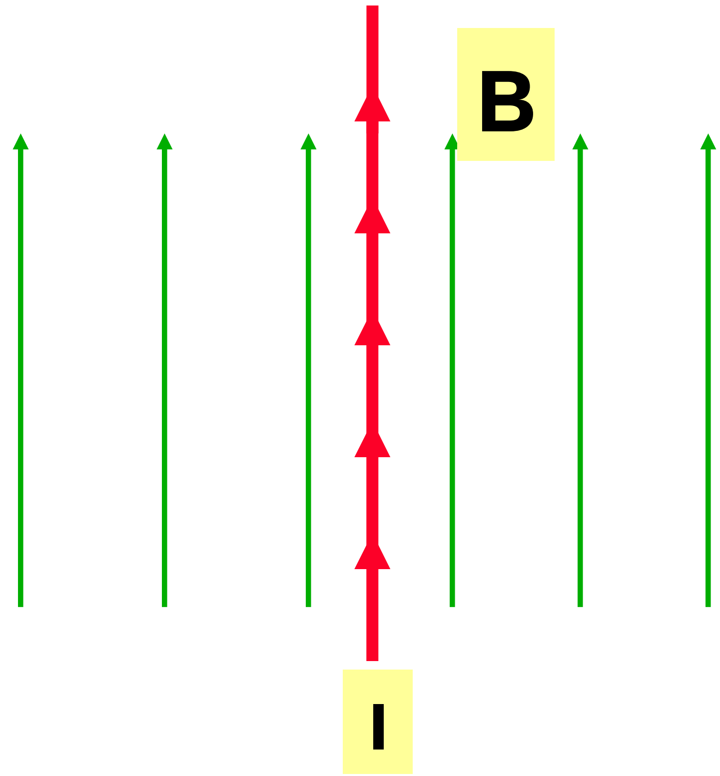
Magnetic Force

➔ A vertical wire carries a current and is in a vertical magnetic field. What is the direction of the force on the wire?

- (a) left
- (b) right
- (c) zero
- (d) into the page
- (e) out of the page



I is parallel to B, so no magnetic force

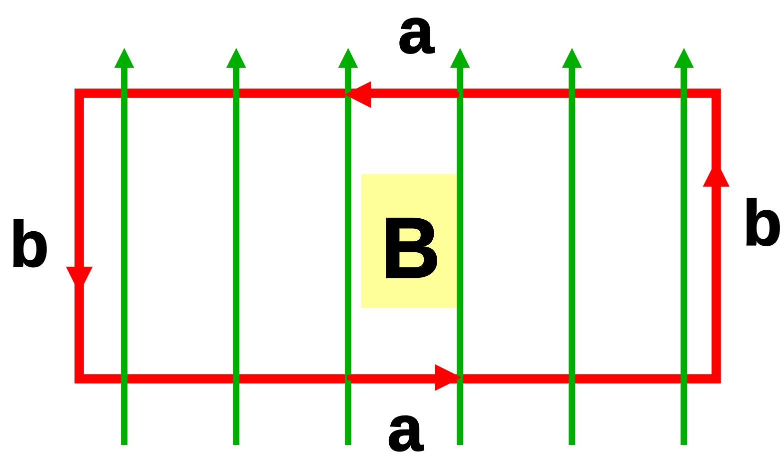
Torque on Current Loop

➔ Consider rectangular current loop
- Forces in left, right branches = 0
- Forces in top/bottom branches cancel
- No net force! (true for any shape)

➔ But there is a net torque!
- Bottom side up, top side down (RHR)
- Rotates around horizontal axis

$$\tau = Fd = (iBa)b = iBab = iBA$$

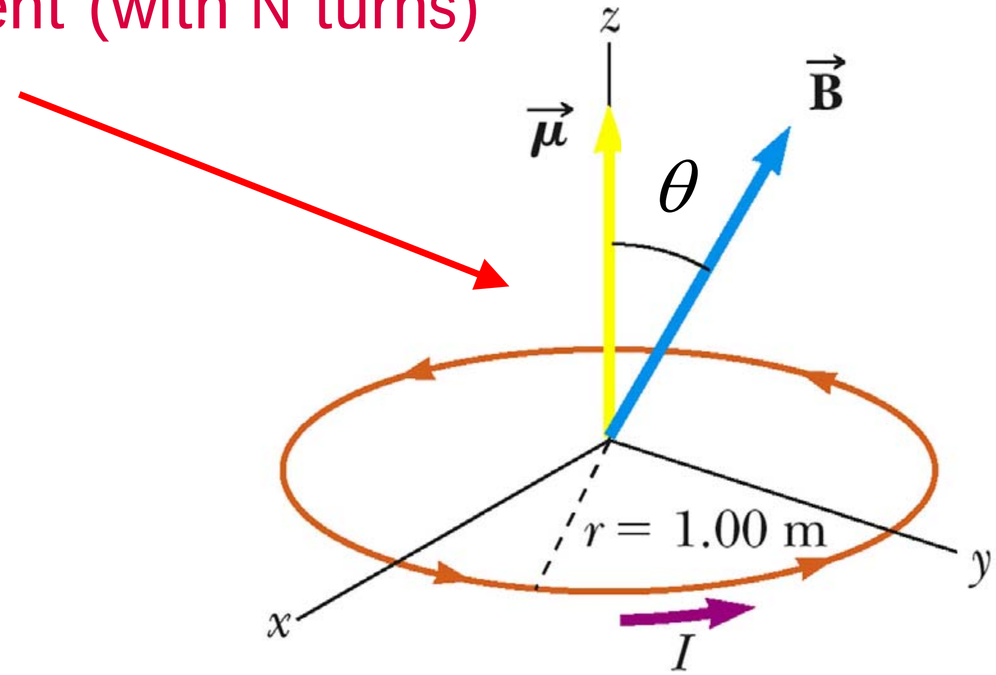
➔ μ = NiA ⇒ "magnetic moment" (N turns)
- True for any shape!!
- Direction of μ given by RHR
- Fingers curl around loop and thumb points in direction of μ



Plane normal is ⊥ B here

General Treatment of Magnetic Moment, Torque

- ➔ μ = NiA is magnetic moment (with N turns)
  - ◆ Direction of μ given by RHR

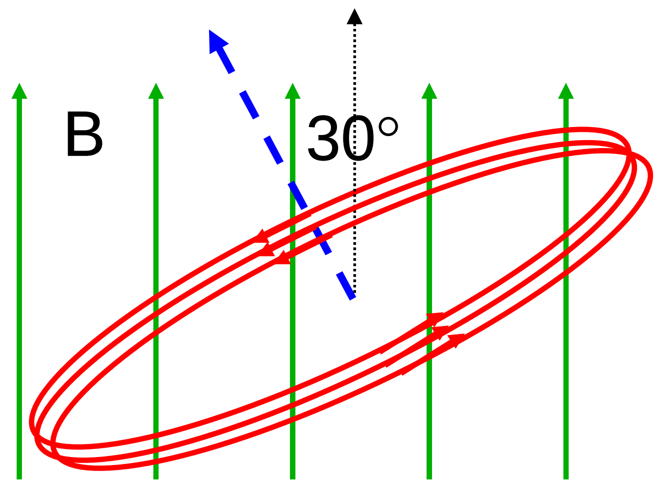


- ➔ Torque depends on angle θ between μ and B

$$\tau = \mu B \sin\theta$$

Torque Example

➔ A 3-turn circular loop of radius 3 cm carries 5A current in a B field of 2.5 T. Loop is tilted 30° to B field.



➔ $\mu = NiA = 3i\pi r^2 = 3\times 5\times 3.14\times (0.03)^2 = 0.0339\,\mathrm{A\cdot m^2}$

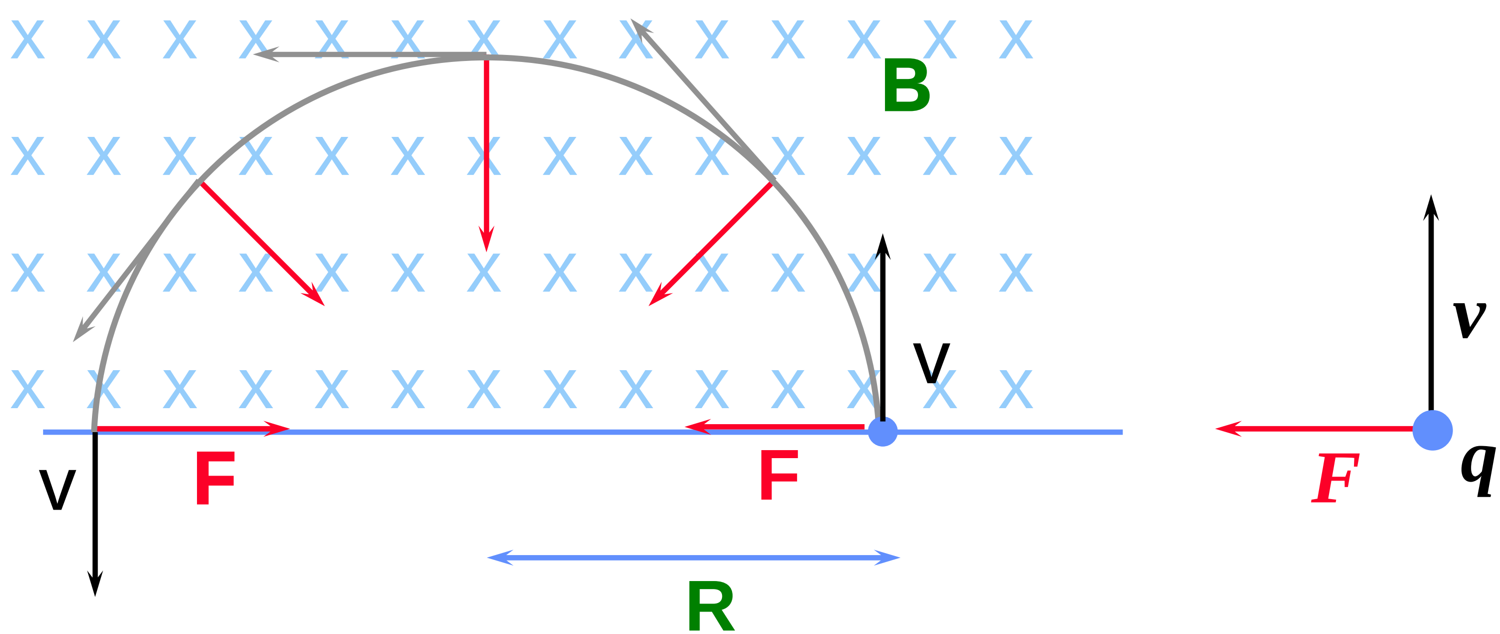
➔ $\tau = \mu B \sin 30° = 0.0339\times 2.5\times 0.5 = 0.042\,\mathrm{N\cdot m}$

➔ Rotation *always* in direction to align μ with B field

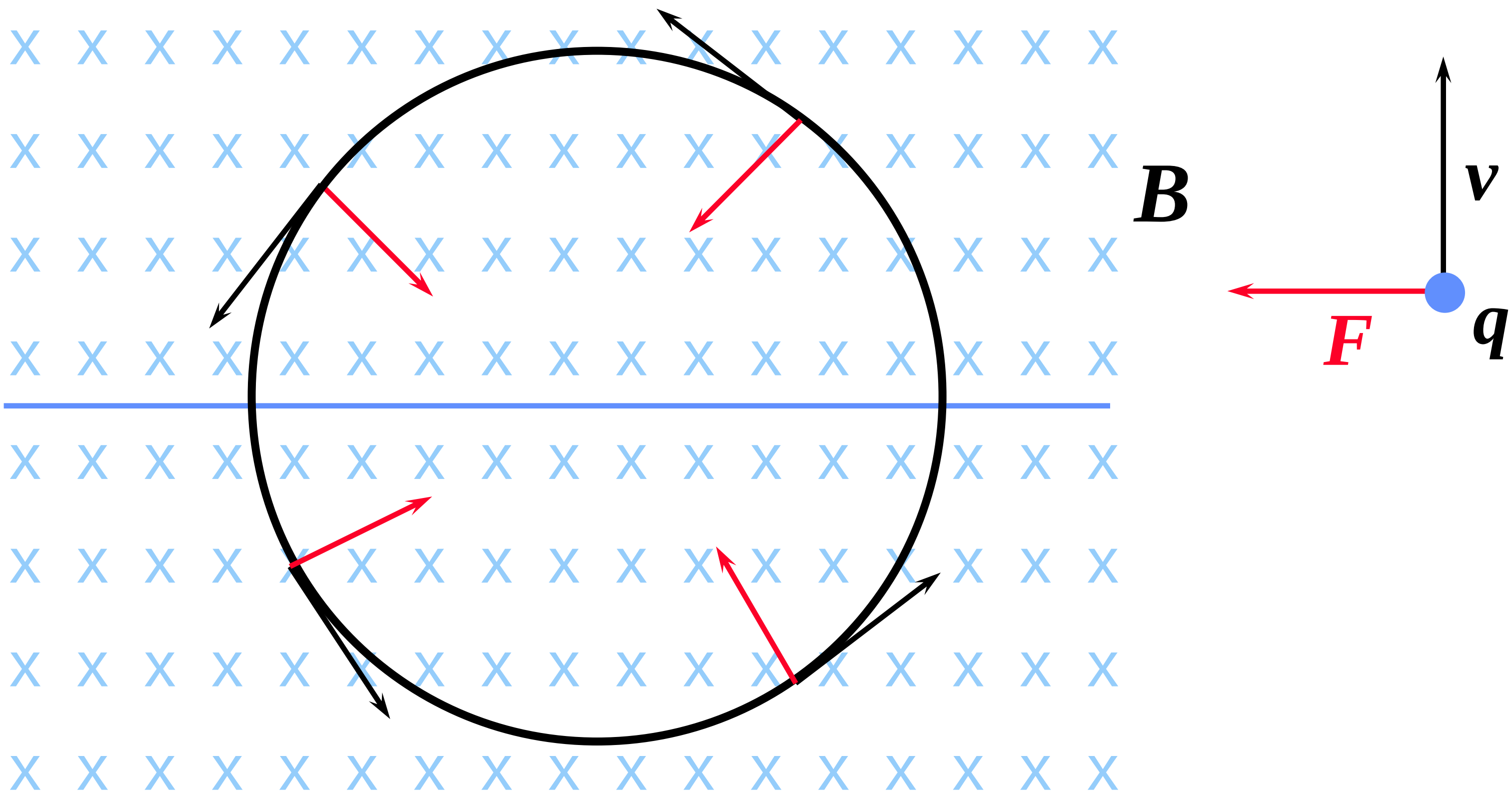
Trajectory in a Constant Magnetic Field

➔ A charge q enters B field with velocity v perpendicular to B. What path will q follow?

- Force is always ⊥ velocity and ⊥ B
- Path will be a circle. F is the centripetal force needed to keep the charge in its circular orbit. Let's calculate radius R



Circular Motion of Positive Particle



$$\frac{mv^2}{R} = qvB \quad \Rightarrow \quad \boxed{R = \frac{mv}{qB}}$$

Cosmic Ray Example

➔Protons with energy 1 MeV move ⊥ earth B field of 0.5 Gauss or B = 5 × 10^{-5} T. Find radius & frequency of orbit.

$$K = \tfrac{1}{2} m v^2 \Rightarrow \quad v = \sqrt{\frac{2K}{m}}$$

$$K = \left(10^6\right)\left(1.6\times10^{-19}\right) = 1.6\times10^{-13}\ \text{J}$$

$$m = 1.67\times10^{-27}\ \text{kg}$$

$$R = \frac{mv}{eB} = \frac{\sqrt{2mK}}{eB}$$

$$R = 2900\,\text{m}$$

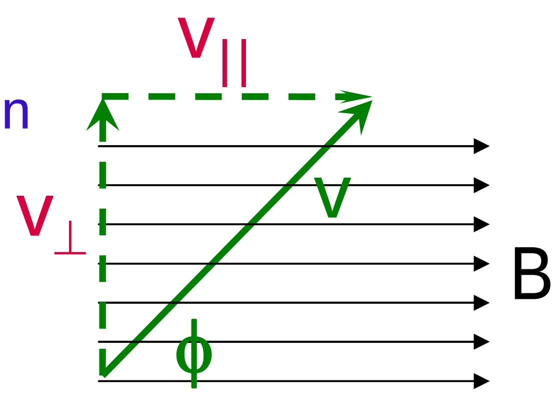
$$f = \frac{1}{T} = \frac{v}{2\pi R} = \frac{v}{2\pi\left(mv/eB\right)} = \frac{eB}{2\pi m}$$

$$f = 760\,\text{Hz}$$

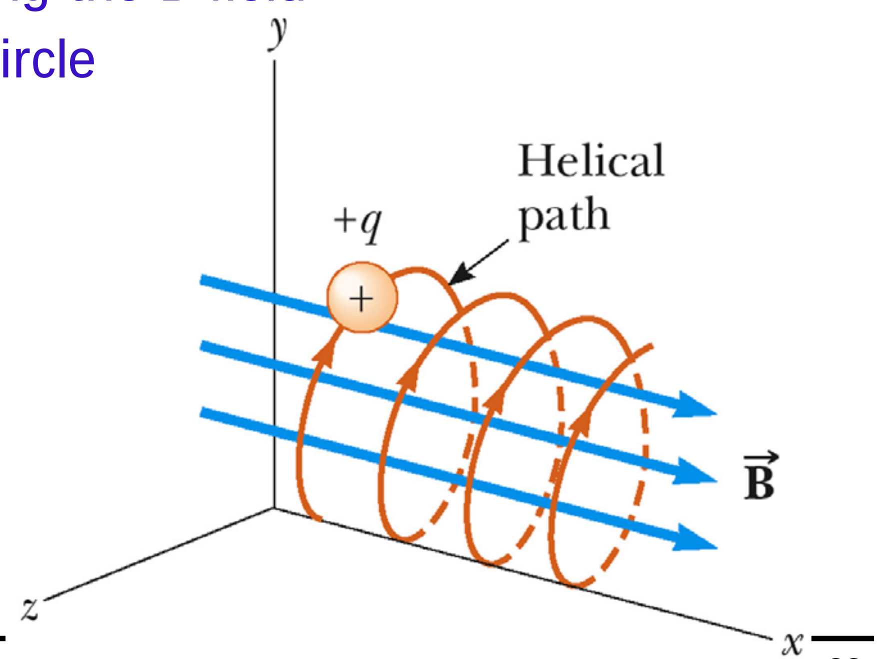
Frequency is independent of v!

Helical Motion in B Field

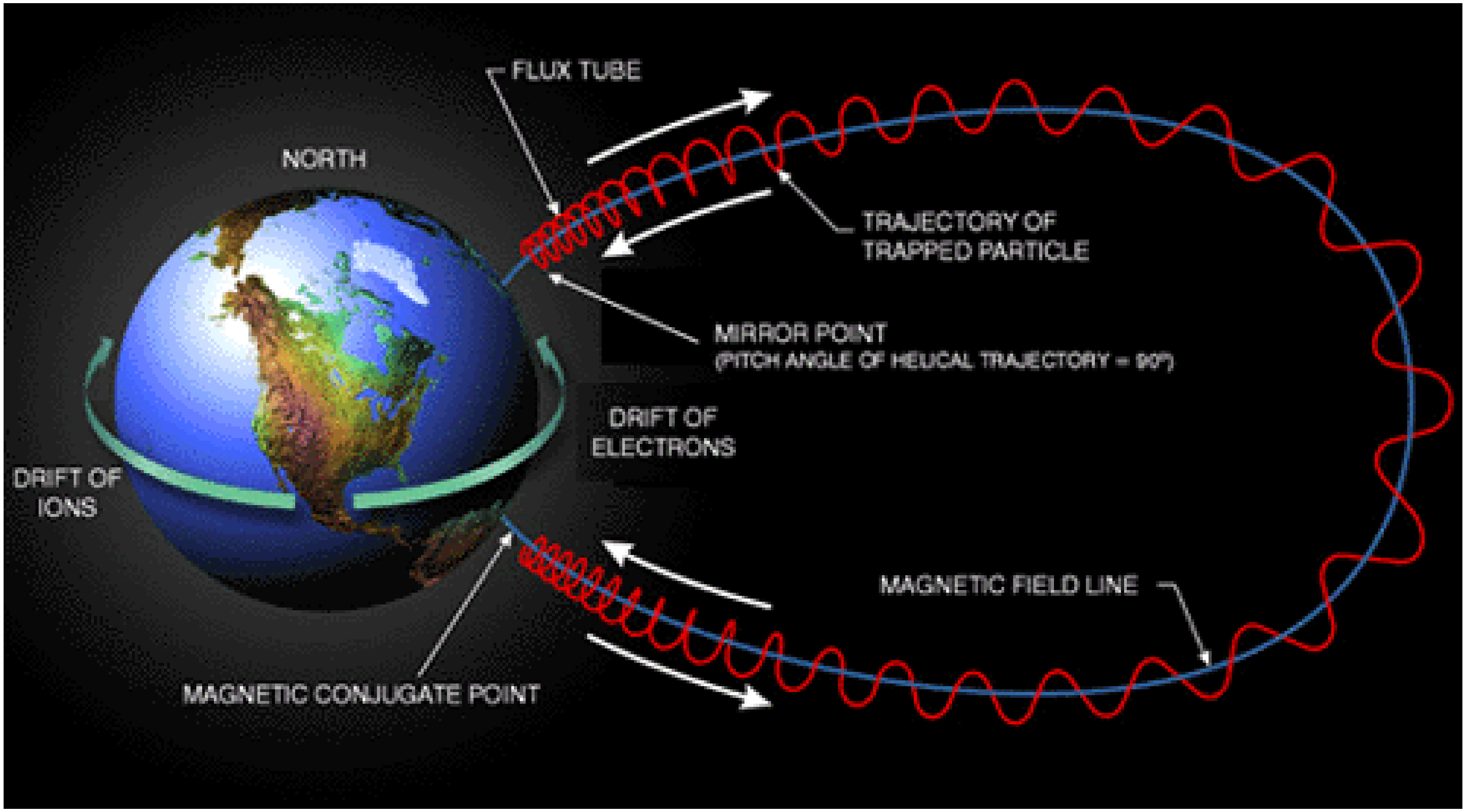
- ➔Velocity of particle has 2 components
  - $\vec{v} = \vec{v}_{||} + \vec{v}_{\perp}$ (parallel to B and perp. to B)
  - Only $v_{\perp} = v \sin\phi$ contributes to circular motion
  - $v_{||} = v \cos\phi$ is unchanged
- ➔So the particle moves in a helical path
  - $v_{||}$ is the constant velocity along the B field
  - $v_{\perp}$ is the velocity around the circle



$$R = \frac{mv_{\perp}}{qB}$$

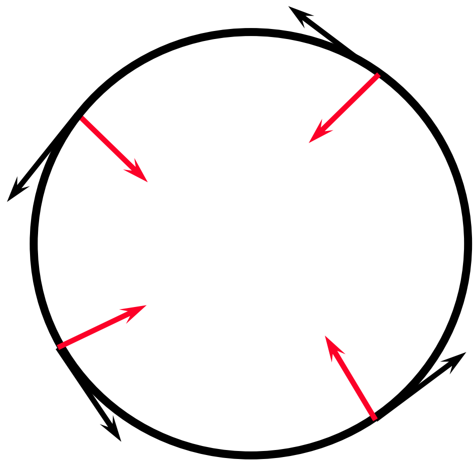


Helical Motion in Earth's B Field



Magnetic Field and Work

- ➔ Magnetic force is *always* perpendicular to velocity
  - ◆ Therefore B field does no work!
  - ◆ Why? Because $\Delta K = \vec{F} \cdot \Delta\vec{x} = \vec{F} \cdot \left(\vec{v}\Delta t\right) = 0$
- ➔ Consequences
  - ◆ Kinetic energy does not change
  - ◆ Speed does not change
  - ◆ Only direction changes
  - ◆ Particle moves in a circle (if $\vec{v} \perp \vec{B}$)



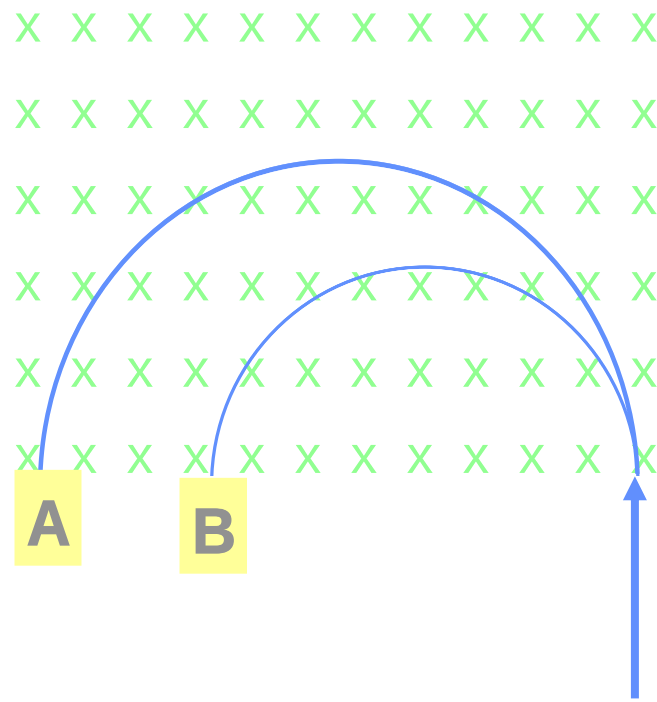
Magnetic Force

➔Two particles of the same charge enter a magnetic field with the same speed. Which one has the bigger mass?

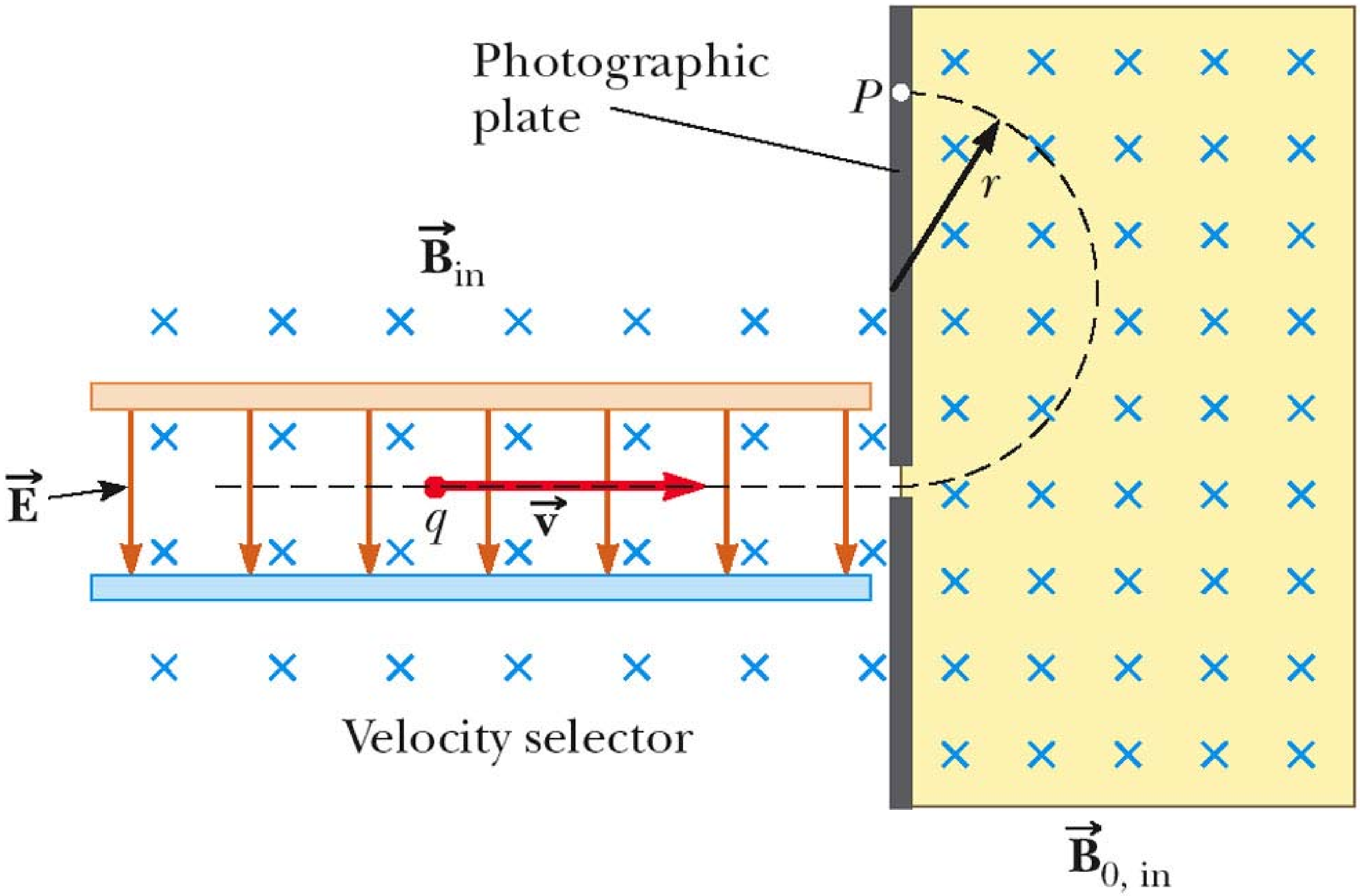
- A
- B
- Both masses are equal
- Cannot tell without more info

$$R = \frac{mv}{qB}$$

Bigger mass means bigger radius



Mass Spectrometer



Mass Spectrometer Operation

➔Positive ions first enter a "velocity selector" where E ⊥ B and values are adjusted to allow only undeflected particles to enter mass spectrometer.

- Balance forces in selector ⇒ "select" v

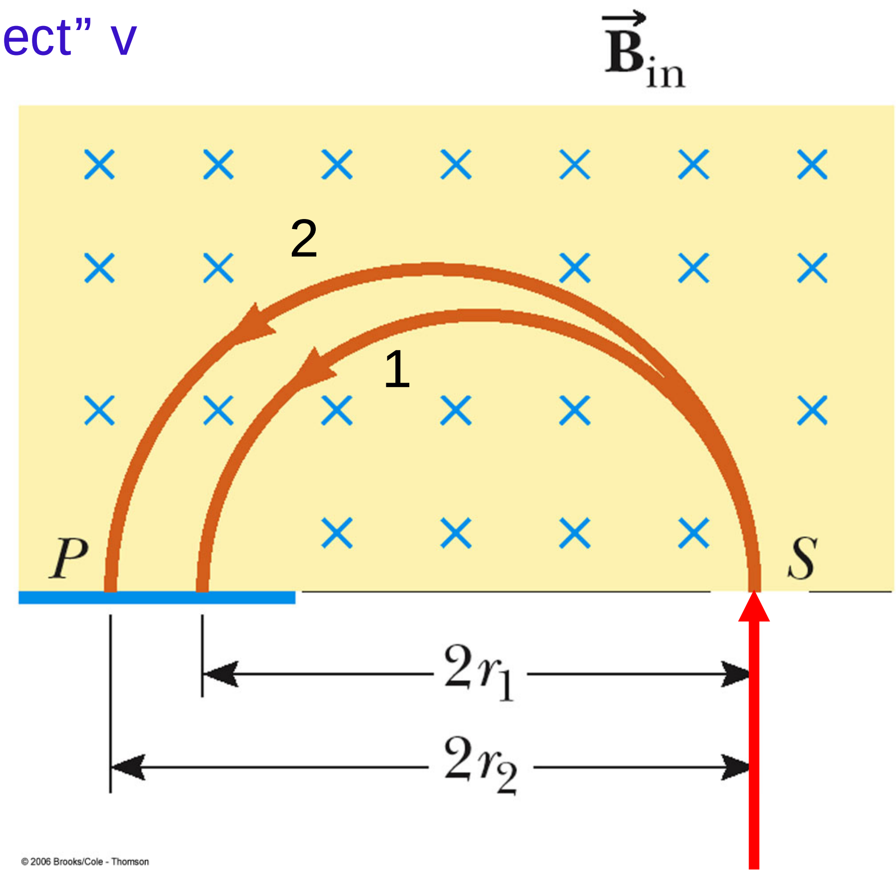
$$qE = qvB$$

$$v = E / B$$

- Spectrometer: Determine mass from v and measured radius r

$$r_1 = \frac{m_1 v}{qB}$$

$$r_2 = \frac{m_2 v}{qB}$$



Mass Spectrometer Example

➔ A beam of deuterons travels right at v = 5 x 10^5 m/s

- What value of B would make deuterons go undeflected through a region where E = 100,000 V/m pointing up vertically?

$$eE = evB$$

$$B = E / v = 10^5 / 5\times10^5 = \boxed{0.2\,\mathrm{T}}$$

- If the electric field is suddenly turned off, what is the radius and frequency of the circular orbit of the deuterons?

$$\frac{mv^2}{R} = evB \quad \Rightarrow R = \frac{mv}{eB} = \frac{\left(3.34\times10^{-27}\right)\left(5\times10^5\right)}{\left(1.6\times10^{-19}\right)(0.2)} = \boxed{5.2\times10^{-2}\ \mathrm{m}}$$

$$f = \frac{1}{T} = \frac{v}{2\pi R} = \frac{5\times10^5}{(6.28)\left(5.2\times10^{-2}\right)} = \boxed{1.5\times10^6\ \mathrm{Hz}}$$

Quiz: Work and Energy

➔ A charged particle enters a uniform magnetic field. What happens to the kinetic energy of the particle?

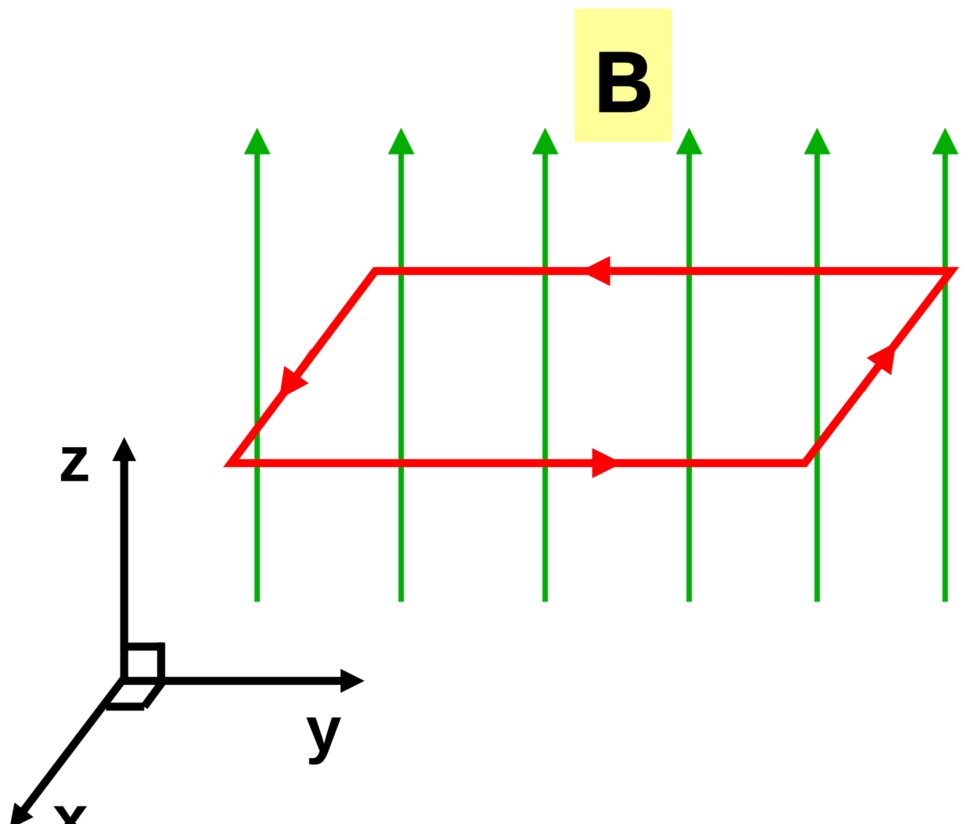
- (1) it increases
- (2) it decreases
- (3) it stays the same
- (4) it changes with the direction of the velocity
- (5) it depends on the direction of the magnetic field

Magnetic field does no work, so K is constant

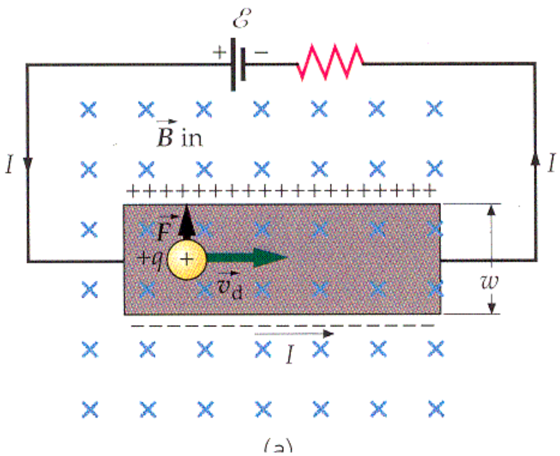
Magnetic Force

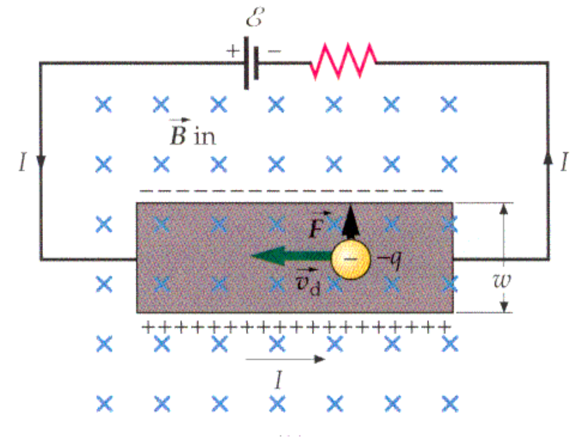
- A rectangular current loop is in a uniform magnetic field. What direction is the net force on the loop?
  - (a) +x
  - (b) +y
  - (c) zero
  - (d) –x
  - (e) –y

Forces cancel on opposite sides of loop



Hall Effect: Do + or – Charges Carry Current?





- ➔ + charges moving counter-clockwise experience upward force
- ➔ Upper plate at *higher* potential

- ➔ – charges moving clockwise experience upward force
- ➔ Upper plate at *lower* potential

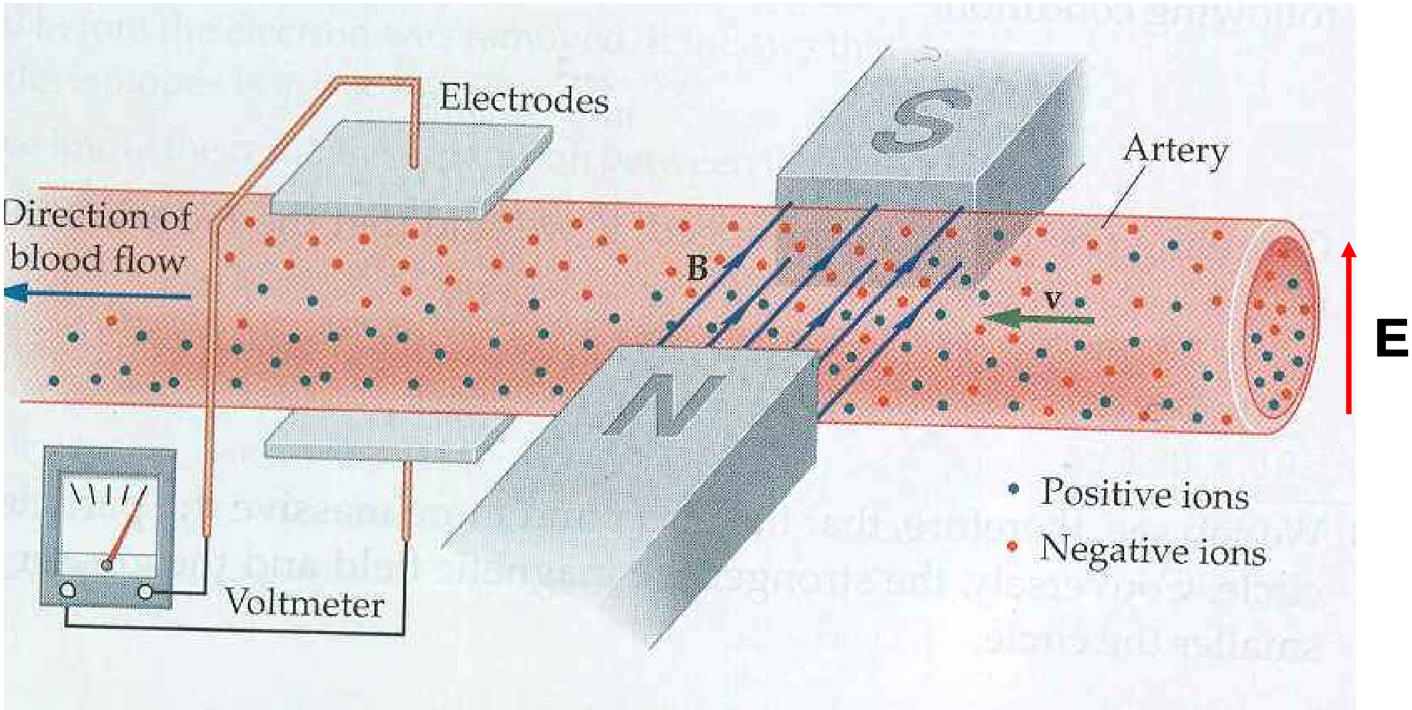
Equilibrium between magnetic (up) & electrostatic forces (down):

$$F_{\text{up}} = qv_{\text{drift}}B \qquad F_{\text{down}} = qE_{\text{induced}} = q\frac{V_H}{w}$$

$$V_H = v_{\text{drift}}Bw = \text{"Hall voltage"}$$

This type of experiment led to the discovery (E. Hall, 1879) that current in conductors is carried by negative charges

Electromagnetic Flowmeter



- Moving ions in the blood are deflected by magnetic force
- Positive ions deflected down, negative ions deflected up
- This separation of charge creates an electric field E pointing up
- E field creates potential difference V = Ed between the electrodes
- The velocity of blood flow is measured by v = E/B

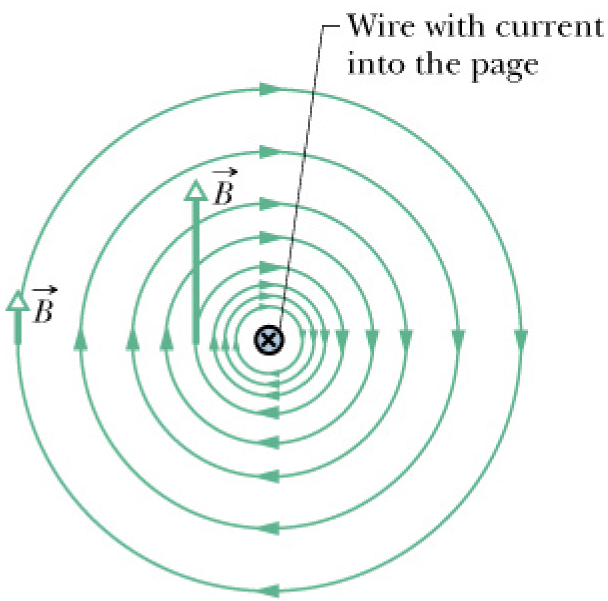
Creating Magnetic Fields

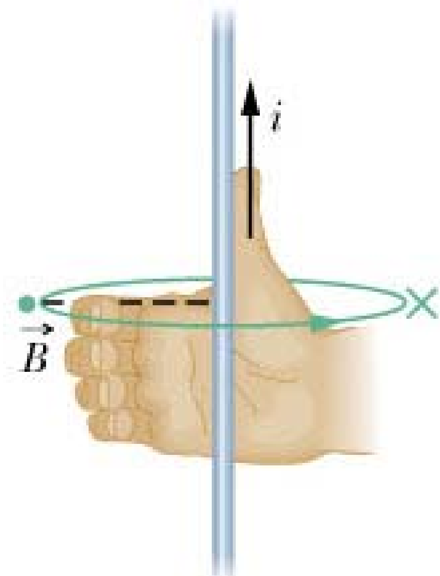
- Sources of magnetic fields
  - Spin of elementary particles (mostly electrons)
  - Atomic orbits ($L > 0$ only)
  - Moving charges (electric current)
- Currents generate the most intense magnetic fields
  - Discovered by Oersted in 1819 (deflection of compass needle)
- Three examples studied here
  - Long wire
  - Wire loop
  - Solenoid

B Field Around Very Long Wire

➔Field around wire is circular, intensity falls with distance

- ◆Direction given by RHR (compass follows field lines)



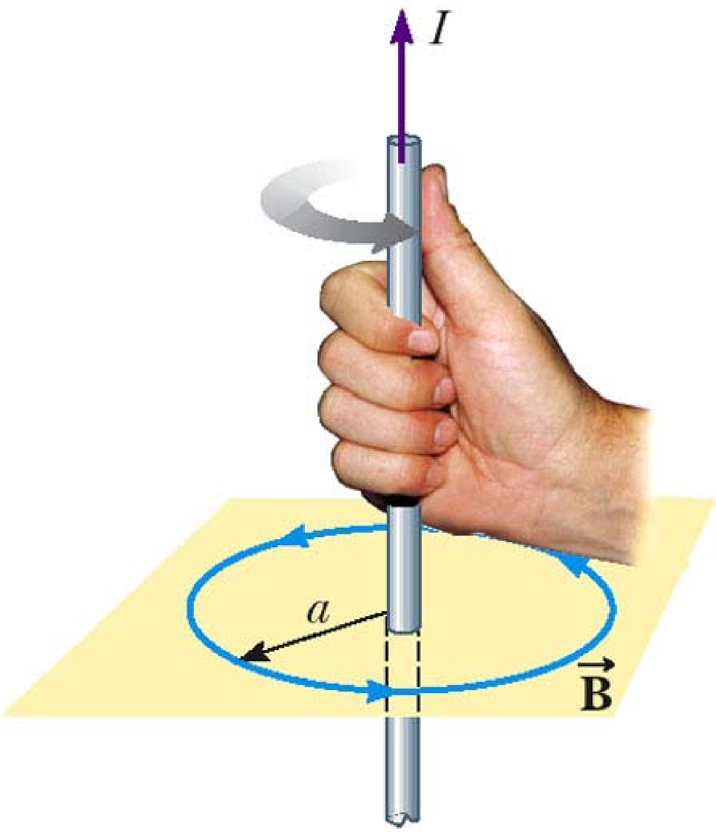


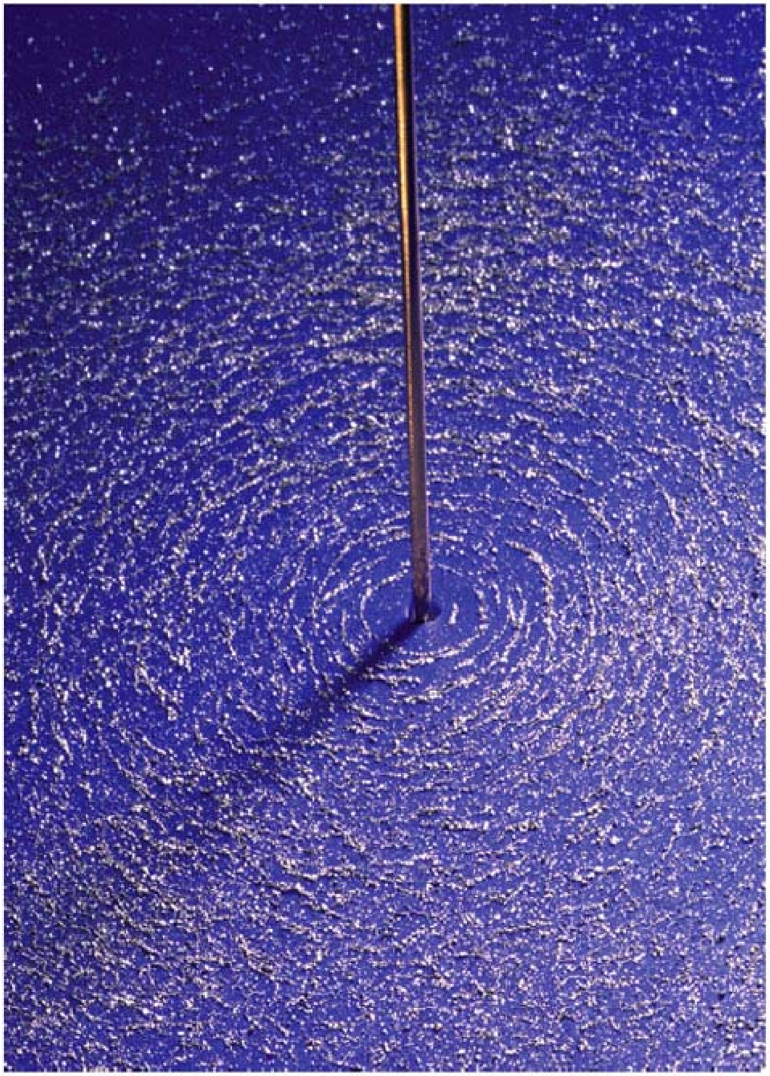
$$B = \frac{\mu_0 i}{2\pi r}$$

Right Hand Rule #2

$\mu_0 = 4\pi \times 10^{-7}$ μ_0 = "Permeability of free space"

Visual of B Field Around Wire

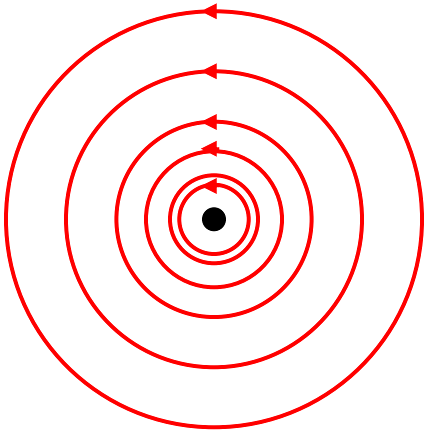




B Field Example

- I = 500 A toward observer. Find B vs r
  - RHR $\Rightarrow$ field is counterclockwise

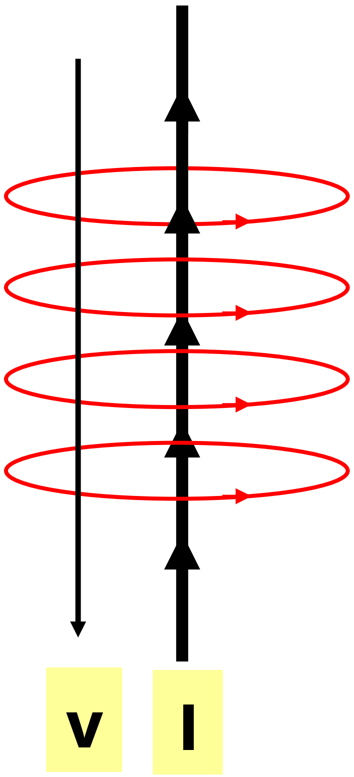
$$B = \frac{\mu_0 i}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\right)500}{2\pi r} = \frac{10^{-4}}{r}$$



  - r = 0.001 m B = 0.10 T = 1000 G
  - r = 0.005 m B = 0.02 T = 200 G
  - r = 0.01 m B = 0.010 T = 100 G
  - r = 0.05 m B = 0.002 T = 20 G
  - r = 0.10 m B = 0.001 T = 10 G
  - r = 0.50 m B = 0.0002 T = 2 G
  - r = 1.0 m B = 0.0001 T = 1 G

Charged Particle Moving Near Wire

- Wire carries current of 400 A upwards
  - Proton moving at v = 5 × 10^6 m/s downwards, 4 mm from wire
  - Find magnitude and direction of force on proton
- Solution
  - Direction of force is to left, *away* from wire
  - Magnitude of force at r = 0.004 m



$$F = evB = ev\left(\frac{\mu_0 I}{2\pi r}\right)$$

$$F = \left(1.6\times10^{-19}\right)\left(5\times10^{6}\right)\left(\frac{2\times10^{-7}\times400}{0.004}\right)$$

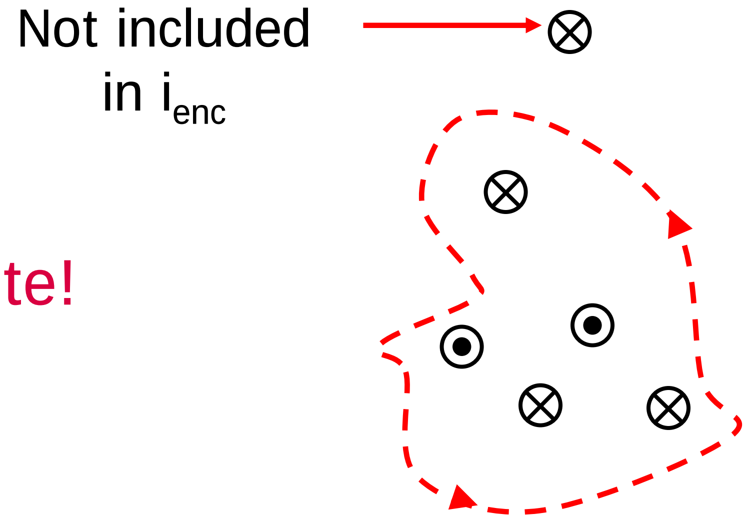
$$F = 1.6\times10^{-14}\text{ N}$$

Ampere's Law

➔Take arbitrary path around set of currents

- Let i_{enc} be total enclosed current (+ up, – down)
- Let $B_{||}$ be component of B along path

$$\sum_i B_{||} \Delta s = \mu_0 i_{\text{enc}}$$



➔Only currents inside path contribute!

- 5 currents inside path (included)
- 1 outside path (not included)

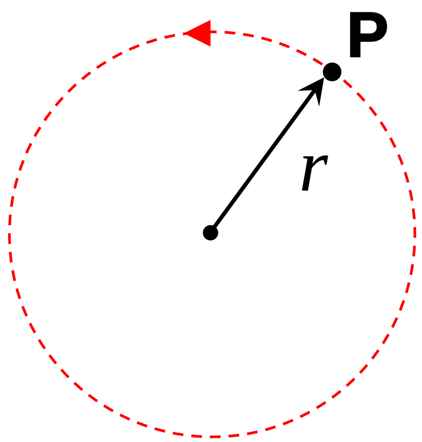
Ampere's Law For Straight Wire

➔Let's try this for long wire. Find B at distance at point P

- Use circular path passing through P (center at wire, radius r)
- From symmetry, B field must be circular

$$\sum_i B_{||}\Delta s = B(2\pi r) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r}$$



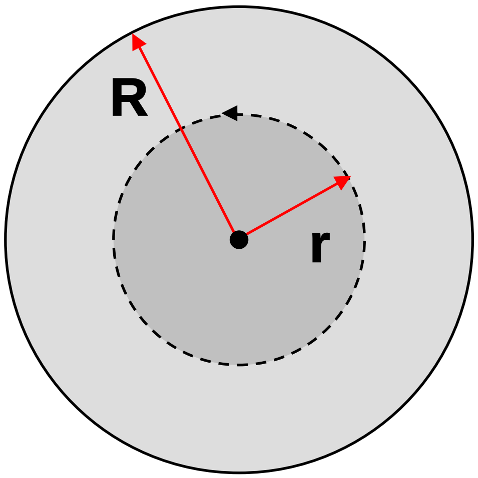
➔An easy derivation

Useful Application of Ampere's Law

- ➔ Find B field inside long wire, assuming uniform current
  - ◆ Wire radius R, total current i
  - ◆ Find B at radius r = R/2

$$\sum_i B_{\parallel} \Delta s = \mu_0 i_{\text{enc}}$$

- ➔ Key fact: enclosed current ∝ area



$$i_{\text{enc}} = i \times \frac{A_{\text{enc}}}{A_{\text{tot}}} = i \times \left(\frac{\pi r^2}{\pi R^2} \right) = \frac{i}{4} \qquad r = R/2$$

$$\sum_i B_{\parallel} \Delta s = B \left(2\pi \frac{R}{2} \right) = \mu_0 \frac{i}{4}$$

$$B = \frac{1}{2} \frac{\mu_0 i}{2\pi R}$$

$$B = \frac{\mu_0 i}{2\pi R}$$ On surface

Ampere's Law (cont)

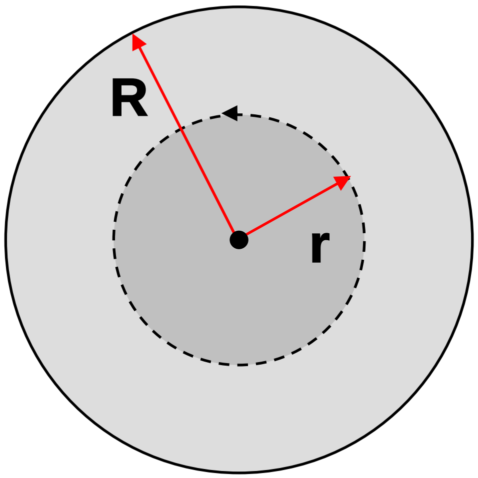
➔Same problems: use Ampere's law to solve for B at *any* r

- Wire radius R, total current i

$$\sum_i B_{\parallel}\Delta s = \mu_0 i_{\text{enc}}$$

$$i_{\text{enc}} = i \times \frac{A_{\text{enc}}}{A_{\text{tot}}} = i \times \left(\frac{\pi r^2}{\pi R^2}\right) = i\frac{r^2}{R^2} \qquad (r \le R)$$

$$= i \qquad (r \ge R)$$



$$\sum_i B_{\parallel}\Delta s = B(2\pi r) = \mu_0 i\left(\frac{r^2}{R^2}\right) \text{ or } \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi R}\frac{r}{R} \qquad (r \le R)$$

$$B = \frac{\mu_0 i}{2\pi r} \qquad r \ge R$$

Force Between Two Parallel Currents

➔Force on I_2 from I_1

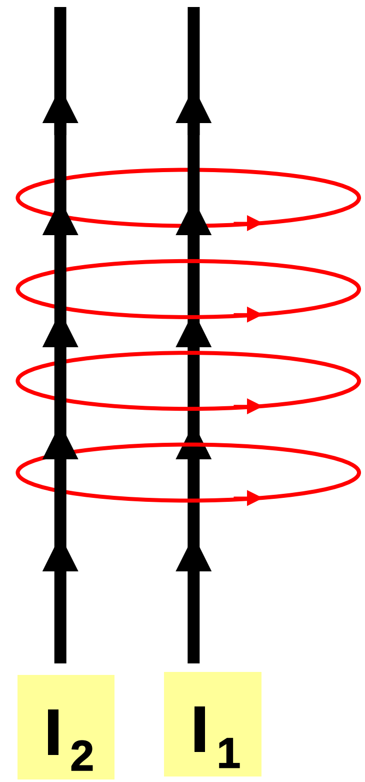
$$F_2 = I_2 B_1 L = I_2\left(\frac{\mu_0 I_1}{2\pi r}\right)L = \frac{\mu_0 I_1 I_2}{2\pi r}L$$

- ◆RHR $\Rightarrow$ Force towards I_1

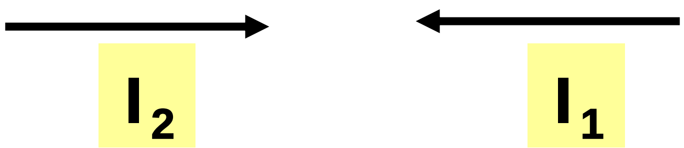
➔Force on I_1 from I_2

$$F_1 = I_1 B_2 L = I_1\left(\frac{\mu_0 I_2}{2\pi r}\right)L = \frac{\mu_0 I_1 I_2}{2\pi r}L$$

- ◆RHR $\Rightarrow$ Force towards I_2



➔Magnetic forces *attract* two parallel currents



Force Between Two Anti-Parallel Currents

➔Force on I_2 from I_1

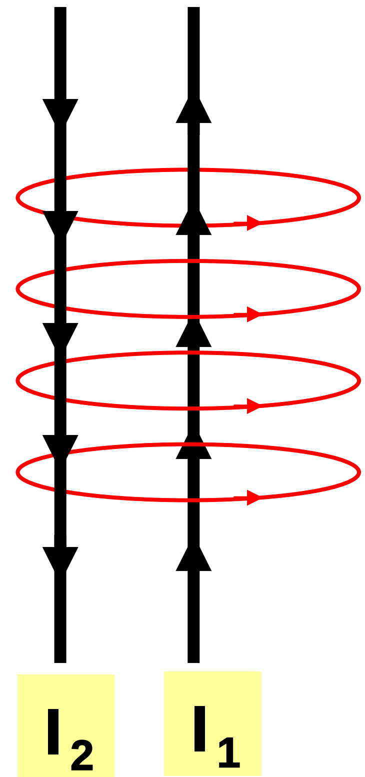
$$F_2 = I_2 B_1 L = I_2\left(\frac{\mu_0 I_1}{2\pi r}\right)L = \frac{\mu_0 I_1 I_2}{2\pi r}L$$

- RHR ⇒ Force away from I_1

➔Force on I_1 from I_2

$$F_1 = I_1 B_2 L = I_1\left(\frac{\mu_0 I_2}{2\pi r}\right)L = \frac{\mu_0 I_1 I_2}{2\pi r}L$$

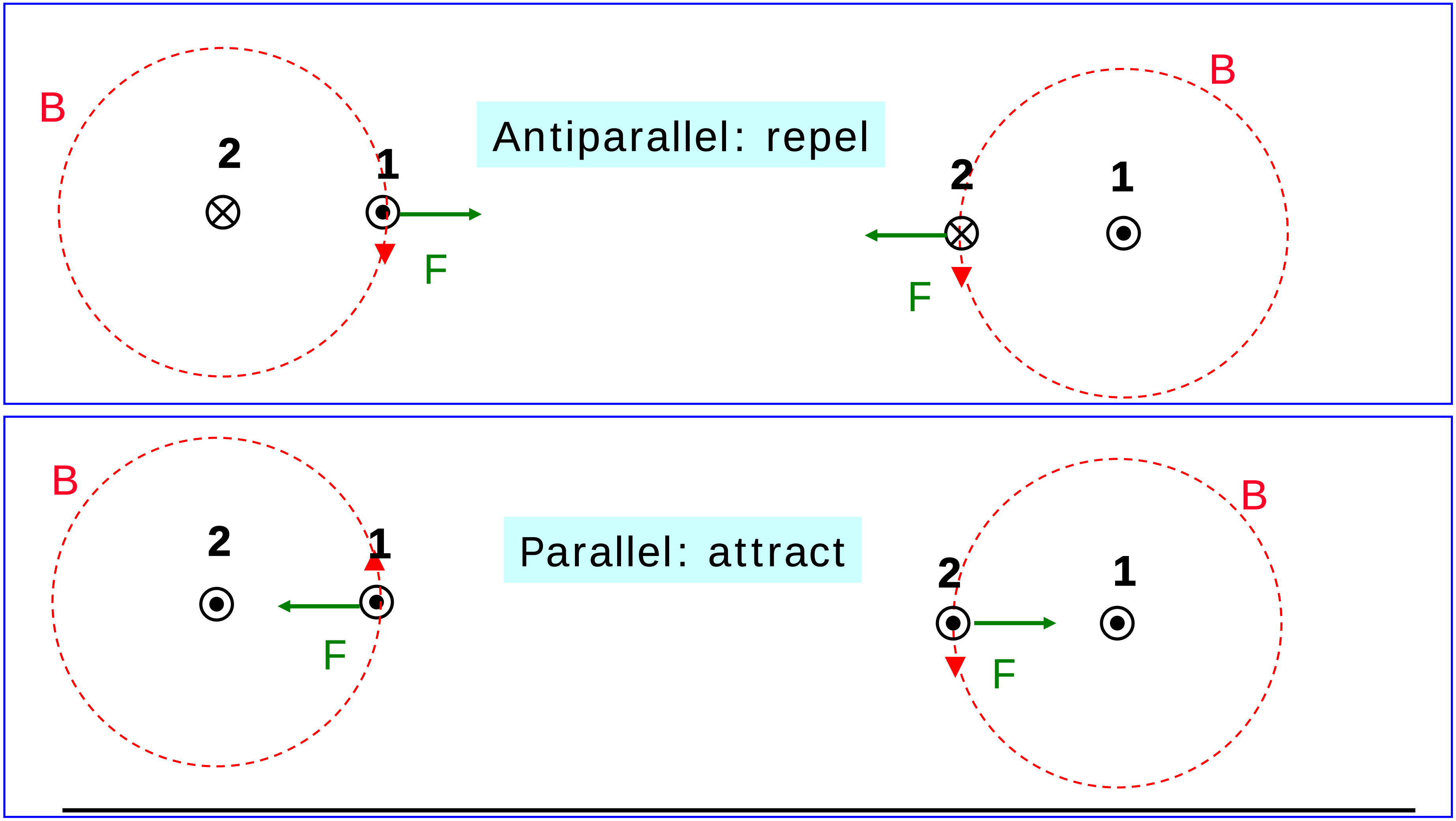
- RHR ⇒ Force away from I_2



➔Magnetic forces *repel* two antiparallel currents

Parallel Currents (cont.)

➔Look at them edge on to see B fields more clearly



B Field @ Center of Circular Current Loop

➔Radius R and current i: find B field at center of loop

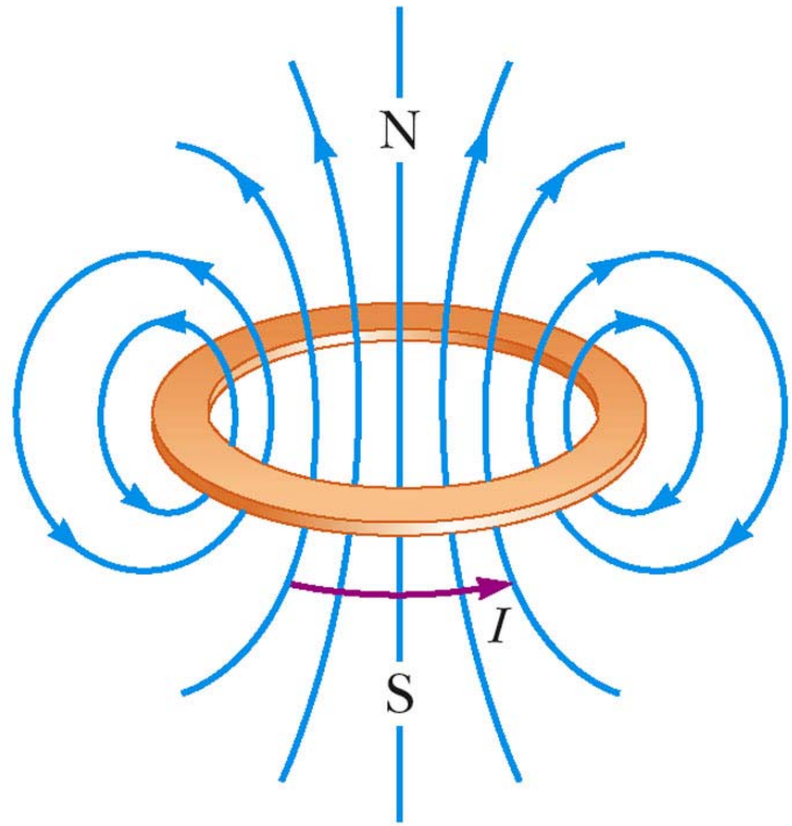
$$B = \frac{\mu_0 i}{2R}$$

From calculus

◆ Direction: RHR #3 (see picture)

➔If N turns close together

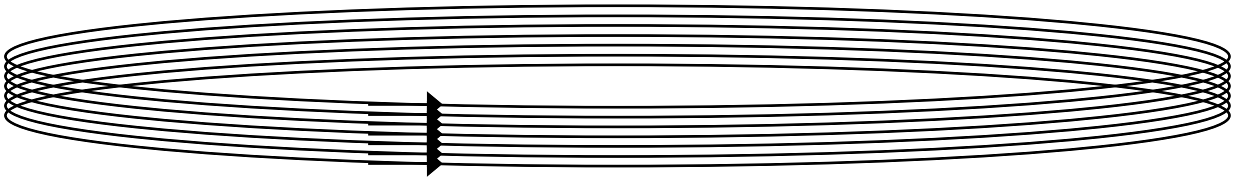
$$B = \frac{N \mu_0 i}{2R}$$



Current Loop Example

➔i = 500 A, r = 5 cm, N=20

$$B = N\frac{\mu_0 i}{2r} = \frac{(20)\left(4\pi\times10^{-7}\right)500}{2\times0.05} = 1.26\,\mathrm{T}$$



B Field of Solenoid

➔Formula found from Ampere's law

- i = current
- n = turns / meter

$$B = \mu_0 i n$$

- B ~ constant inside solenoid
- B ~ zero outside solenoid
- Most accurate when $L \gg R$

➔Example: i = 100A, n = 10 turns/cm

- n = 1000 turns / m

$$B = \left(4\pi \times 10^{-7}\right)(100)\left(10^{3}\right) = 0.13\text{T}$$