Chapter 23

Geometrical Optics: Mirrors and Lenses and other Instruments
You stand two feet away from a plane mirror. How far is it from you to your image?

a. 2.0 ft  b. 3.0 ft  c. 4.0 ft  d. 5.0 ft
The 100 inch (2.5 m) Hooker reflecting telescope at Mount Wilson Observatory near Los Angeles, California.
International Year of Astronomy, Public Lecture, Fri. April 3rd 7.20pm, NPB 1002

"100 HOURS OF ASTRONOMY"
Unlocking the Mysteries of the Heavens: From Galileo's Telescope to the Future Generation of Giant Telescopes

Professors Rafael Guzman and Elizabeth Lada

The lecture will be preceded by a telescope assembly workshop in the Physics lobby beginning at 6:30pm. Anyone with a telescope who would like help with assembly should bring it to the lobby before the talk. After the talk, a public night will be held at the Campus Teaching Observatory, weather permitting. Those with their own telescopes are encouraged to bring them for observing assistance and training during the public observing night beginning at 8:30pm.

http://www.astro.ufl.edu/IYA/home.html
A *spherical mirror* has the shape of a segment of a sphere.

A *concave spherical mirror* has the silvered surface of the mirror on the inner, or concave, side of the curve.

A *convex spherical mirror* has the silvered surface of the mirror on the outer, or convex, side of the curve.
Concave Mirror, Notation

- The mirror has a *radius of curvature* of $R$.
- Its *center of curvature* is the point $C$.
- Point $V$ is the center of the spherical segment.
- A line drawn from $C$ to $V$ is called the *principle axis* of the mirror.
Spherical Aberration

- Rays are generally assumed to make small angles with the mirror.
- When the rays make large angles, they may converge to points other than the image point.
- This results in a blurred image.
- This effect is called spherical aberration.
Image Formed by a Concave Mirror

- Geometry can be used to determine the magnification of the image.

- $h'$ is negative when the image is inverted with respect to the object.
Image Formed by a Concave Mirror

- Geometry shows the relationship between the image and object distances.

- This is called the mirror equation.
Focal Length

- If an object is very far away, then \( p = \infty \) and \( 1/p = 0 \)
- Incoming rays are essentially parallel
- In this special case, the image point is called the focal point
- The distance from the mirror to the focal point is called the focal length
  - The focal length is \( \frac{1}{2} \) the radius of curvature
Focal Point and Focal Length, cont

- The focal point is dependent solely on the curvature of the mirror, not by the location of the object

- \( f = \frac{R}{2} \)

- The mirror equation can be expressed as
Focal Length Shown by Parallel Rays
Convex Mirrors

- A convex mirror is sometimes called a **diverging** mirror.
- The rays from any point on the object diverge after reflection as though they were coming from some point behind the mirror.
- The image is virtual because it lies behind the mirror at the point where the reflected rays appear to originate.
- In general, the image formed by a convex mirror is upright, virtual, and smaller than the object.
Image Formed by a Convex Mirror

Front

Back

O I F C

p q

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### Sign Conventions for Mirrors

**TABLE 23.1**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>In Front</th>
<th>In Back</th>
<th>Upright Image</th>
<th>Inverted Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>Object location</td>
<td>$p$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Image location</td>
<td>$q$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Focal Length</td>
<td>$f$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Image height</td>
<td>$h'$</td>
<td></td>
<td></td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Magnification</td>
<td>$M$</td>
<td></td>
<td></td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

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Ray Diagrams

- A *ray diagram* can be used to determine the position and size of an image.
- They are graphical constructions which tell the overall nature of the image.
- They can also be used to check the parameters calculated from the mirror and magnification equations.
Drawing A Ray Diagram

- To make the ray diagram, you need to know
  - The position of the object
  - The position of the center of curvature
- Three rays are drawn
  - They all start from the same position on the object
- The intersection of any two of the rays at a point locates the image
  - The third ray serves as a check of the construction
The Rays in a Ray Diagram

- Ray 1 is drawn parallel to the principle axis and is reflected back through the focal point, F
- Ray 2 is drawn through the focal point and is reflected parallel to the principle axis
- Ray 3 is drawn through the center of curvature and is reflected back on itself
Notes About the Rays

- The rays actually go in all directions from the object.
- The three rays were chosen for their ease of construction.
- The image point obtained by the ray diagram must agree with the value of q calculated from the mirror equation.
The object is outside the center of curvature of the mirror \((p > R/2)\)
- The image is real \((q > 0)\)
- The image is inverted \((m > 0)\)
- The image is smaller than the object \((|m| < 1)\)
Ray Diagram for a Concave Mirror, $p < f$

- The object is between the mirror and the focal point
- The image is virtual
- The image is upright
- The image is larger than the object
algebra

- \( q = \frac{pf}{(p-f)} \)

\( \Rightarrow q > 0 \) if \( p > f \) \( \Rightarrow \) the image is real \((q > 0)\), inverted \((m < 0)\) and smaller \((|m| < 1)\) if ..

\( q/p < 1 \) if \( p > 2f = R \)

If \( p < f, q < 0, m > 0 \), image is virtual, straight and larger, \( m > 1 \) \((f/(f-p) > 1)\)
Ray Diagram for a Convex Mirror

- The object is in front of a convex mirror
- The image is virtual
- The image is upright
- The image is smaller than the object
Notes on Images

- With a concave mirror, the image may be either real or virtual
  - When the object is outside the focal point, the image is real
  - When the object is at the focal point, the image is infinitely far away
  - When the object is between the mirror and the focal point, the image is virtual

- With a convex mirror, the image is always virtual and upright
  - As the object distance increases, the virtual image gets smaller
Images Formed by Refraction

- Rays originate from the object point, O, and pass through the image point, I.
- When $n_2 > n_1$, ______ ______
- Real images are formed on the side opposite from the object.
## Sign Conventions for Refracting Surfaces

### TABLE 23.2

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>In Front</th>
<th>In Back</th>
<th>Upright Image</th>
<th>Inverted Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>Object location</td>
<td>$p$</td>
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<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Image location</td>
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<td></td>
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<tr>
<td>Radius</td>
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<td>-</td>
<td>+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Image height</td>
<td>$h'$</td>
<td></td>
<td></td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

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The image formed by a flat refracting surface is on the same side of the surface as the object.

- The image is virtual.
- The image forms between the object and the surface.
- The rays bend away from the normal since $n_1 > n_2$. 

$n_1 > n_2$
Atmospheric Refraction

- There are many interesting results of refraction in the atmosphere
  - Sunsets
  - Mirages
Atmospheric Refraction and Sunsets

- Light rays from the sun are bent as they pass into the atmosphere.
- It is a gradual bend because the light passes through layers of the atmosphere:
  - Each layer has a slightly different index of refraction.
- The Sun is seen to be above the horizon even after it has fallen below it.
Atmospheric Refraction and Mirages

- A mirage can be observed when the air above the ground is warmer than the air at higher elevations.
- The rays in path B are directed toward the ground and then bent by refraction.
- The observer sees both an upright and an inverted image.
Thin Lenses

- A thin lens consists of a piece of glass or plastic, ground so that each of its two refracting surfaces is a segment of either a sphere or a plane.
- Lenses are commonly used to form images by refraction in optical instruments.
Thin Lens Shapes

- These are examples of *converging* lenses
- They have positive focal lengths
- They are thickest in the middle
More Thin Lens Shapes

- These are examples of *diverging* lenses.
- They have negative focal lengths.
- They are thickest at the edges.

Biconcave  Convex-concave  Plano-concave
Focal Length of Lenses

- The focal length, $f$, is the image distance that corresponds to an infinite object distance
  - This is the same as for mirrors
- A thin lens has two focal points, corresponding to parallel rays from the left and from the right
  - A thin lens is one in which the distance between the surface of the lens and the center of the lens is negligible
Focal Length of a Converging Lens

- The parallel rays pass through the lens and converge at the focal point.
- The parallel rays can come from the left or right of the lens.
Focal Length of a Diverging Lens

- The parallel rays diverge after passing through the diverging lens.
- The focal point is the point where the rays appear to have originated.
Lens Equations

- The geometric derivation of the equations is very similar to that of mirrors.
Lens Equations

- The equations can be used for both converging and diverging lenses
  - A converging lens has a positive focal length
  - A diverging lens has a negative focal length
Table 23.3

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>In Front</th>
<th>In Back</th>
<th>Convergent</th>
<th>Divergent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Object location</td>
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<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Image location</td>
<td>$q$</td>
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<td>+</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Lens Radii</td>
<td>$R_1, R_2$</td>
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<td>-</td>
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</tr>
<tr>
<td>Focal Length</td>
<td>$f$</td>
<td></td>
<td></td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>
Focal Length for a Lens

- The focal length of a lens is related to the curvature of its front and back surfaces and the index of refraction of the material.

- This is called the lens maker’s equation.
Ray Diagrams for Thin Lenses

- Ray diagrams are essential for understanding the overall image formation.
- Three rays are drawn:
  - The first ray is drawn parallel to the first principle axis and then passes through (or appears to come from) one of the focal lengths.
  - The second ray is drawn through the center of the lens and continues in a straight line.
  - The third ray is drawn from the other focal point and emerges from the lens parallel to the principle axis.
- There are an infinite number of rays, these are convenient.
Ray Diagram for Converging Lens, $p > f$

- The image is real
- The image is inverted
Ray Diagram for Converging Lens, $p < f$

- The image is virtual
- The image is upright
Ray Diagram for Diverging Lens

- The image is virtual
- The image is upright
Combinations of Thin Lenses

- The image produced by the first lens is calculated as though the second lens were not present.
- The light then approaches the second lens as if it had come from the image of the first lens.
- The image of the first lens is treated as the object of the second lens.
- The image formed by the second lens is the final image of the system.
Combination of Thin Lenses, 2

- If the image formed by the first lens lies on the back side of the second lens, then the image is treated as a virtual object for the second lens.
  - $p$ will be negative.
- The overall magnification is the product of the magnification of the separate lenses.
Combination of Thin Lenses, example

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Lens and Mirror Aberrations

- One of the basic problems is the imperfect quality of the images
  - Largely the result of defects in shape and form
- Two common types of aberrations exist
  - Spherical aberration
  - Chromatic aberration
Spherical Aberration

- Results from the focal points of light rays far from the principle axis are different from the focal points of rays passing near the axis.

- For a mirror, parabolic shapes can be used to correct for spherical aberration.
Chromatic Aberration

- Different wavelengths of light refracted by a lens focus at different points
  - Violet rays are refracted more than red rays
  - The focal length for red light is greater than the focal length for violet light

- Chromatic aberration can be minimized by the use of a combination of converging and diverging lenses
If a virtual image is formed along the principal axis 10 cm from a concave mirror with the focal length 15 cm, what is the object distance from the mirror?

a. 30 cm  

b. 10 cm  

c. 12 cm  

d. 6.0 cm
A small underwater pool light is 1 m below the surface of a swimming pool. What is the radius of the circle of light on the surface, from which light emerges from the water? \( n_{\text{water}} = 1.333 \).

a. 0.57 m  
   b. 0.77 m  
   c. 1.13 m  
   d. 1.43 m  
   e. 2.28 m
Mirrors and Lenses

- What are flat and curved mirrors?
- What are lenses?
- What is a microscope, telescope and other related devices?
- Concepts such as focus, magnification and distortions.
- To other wavelengths
Notation for Mirrors and Lenses

- The *object distance* is the distance from the object to the mirror or lens
  - Denoted by $p$
- The *image distance* is the distance from the image to the mirror or lens
  - Images are formed at the point where rays actually intersect or appear to originate
  - Denoted by $q$
- The *lateral magnification* of the mirror or lens is the ratio of the image height to the object height
  - Denoted by $M$
Types of Images for Mirrors and Lenses

- A *real image* is one in which light actually passes through the image point
  - Real images can be displayed on screens
- A *virtual image* is one in which the light does not pass through the image point
  - The light appears to diverge from that point
  - Virtual images cannot be displayed on screens
More About Images

- To find where an image is formed, it is always necessary to follow at least two rays of light as they reflect from the mirror.
Flat Mirror

- Simplest possible mirror
- Properties of the image can be determined by geometry
- One ray starts at P, follows path PQ and reflects back on itself
- A second ray follows path PR and reflects according to the Law of Reflection
Flat Mirror

- Simplest possible mirror
- Properties of the image can be determined by geometry
- One ray starts at P, follows path PQ and reflects back on itself
- A second ray follows path PR and reflects according to the Law of Reflection
Properties of the Image Formed by a Flat Mirror

- The image is as far behind the mirror as the object is in front
  - $q = p$
- The image is unmagnified
  - The image height is the same as the object height
    - $h' = h$ and $M = 1$
- The image is virtual
- The image is upright
  - It has the same orientation as the object
- There is an apparent left-right reversal in the image
Properties of the Image Formed by a Flat Mirror

- The image is as far behind the mirror as the object is in front
  - \( q = p \) (\( q = -p \))

- The image is unmagnified
  - The image height is the same as the object height
    - \( h' = h \) and \( M = 1 \)
    - \( M = -q/p = 1 \)

- The image is virtual \( q < 0 \)

- The image is upright \( M > 0 \)
  - It has the same orientation as the object

- There is an apparent left-right reversal in the image
Application – Day and Night Settings on Auto Mirrors

- With the daytime setting, the bright beam of reflected light is directed into the driver’s eyes.
- With the nighttime setting, the dim beam of reflected light is directed into the driver’s eyes, while the bright beam goes elsewhere.
If a woman wishes to use a plane mirror on a wall to view both her head and her feet as she stands in front of the mirror, the required length of the mirror:

a. is equal to the height of the woman.
b. is equal to one half the height of the woman.
c. depends on the distance the woman stands from the mirror.
d. depends on both the height of the woman and the distance from the woman to the mirror.
e. Twice the height of the woman.