$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0$$

let $\frac{d}{dt} \equiv D$

$$\Rightarrow \frac{dx}{dt} = Dx \quad \text{and} \quad \frac{d^2x}{dt^2} = D^2x$$

$$\Rightarrow D^2x + 2\beta Dx + \omega_0^2 x = 0$$

$$\Rightarrow \left(D^2 + 2\beta D + \omega_0^2\right)x = 0$$

Factorizing the term in the ~~paren~~ parentheses:

$$\Rightarrow \left(D + \beta - \sqrt{\beta^2 - \omega_0^2}\right)\left(D + \beta + \sqrt{\beta^2 - \omega_0^2}\right)x = 0$$

$$\Rightarrow \left(D + \beta - \sqrt{\beta^2 - \omega_0^2}\right)x = 0 \quad \text{and} \quad \rightarrow (1)$$

$$\left(D + \beta + \sqrt{\beta^2 - \omega_0^2}\right)x = 0 \quad \rightarrow (2)$$

Solving (1) :

$$\frac{dx}{dt} + \left(\beta - \sqrt{\beta^2 - \omega_0^2}\right)x = 0$$

$$\Rightarrow \frac{dx}{x} = -\left(\beta - \sqrt{\beta^2 - \omega_0^2}\right)dt$$

$$\Rightarrow \ln x = -\left(\beta - \sqrt{\beta^2 - \omega_0^2}\right)t + c_1$$

$$\Rightarrow x = c_2 e^{-\left(\beta - \sqrt{\beta^2 - \omega_0^2}\right)t}$$

$$\Rightarrow x = c_2 e^{-(\beta - \sqrt{\beta^2-\omega_0^2})t}$$

if $\beta > \omega_0$ then $\beta - \sqrt{\beta^2-\omega_0^2} > 0$

let $c_3 = \beta - \sqrt{\beta^2-\omega_0^2} \Rightarrow x = c_2 e^{-c_3 t}$

$\Rightarrow x$ just goes down exponentially with time and you do not have oscillating solutions.

if $\beta < \omega_0$

$$\Rightarrow \sqrt{\beta^2-\omega_0^2} = i\sqrt{\omega_0^2-\beta^2} \quad \text{where } i = \sqrt{-1}$$

$$\Rightarrow x = c_2 e^{-\beta t} e^{i\sqrt{\omega_0^2-\beta^2}\,t}$$

$$\left[e^{i\theta} = \cos\theta + i\sin\theta\right]$$

$$\Rightarrow x = c_2 e^{-\beta t}\left[\cos(\sqrt{\omega_0^2-\beta^2})t + i\sin(\sqrt{\omega_0^2-\beta^2})t\right]$$

Also:

$$\left(D + \beta + \sqrt{\beta^2-\omega_0^2}\right)x = 0$$

$$\Rightarrow x = c_4 e^{-\beta t}\left[\cos(\sqrt{\omega_0^2-\beta^2})t - i\sin(\sqrt{\omega_0^2-\beta^2})t\right]$$

Full solution:

$$x = e^{-\beta t}\left[(c_2+c_4)\cos(\sqrt{\omega_0^2-\beta^2})t + i(c_2-c_4)\sin\sqrt{\omega_0^2-\beta^2}\,t\right]$$

$$\Rightarrow x = e^{-\beta t}\left[c_5\cos\omega' t + c_6\sin\omega' t\right] \text{ where}$$

$$\omega' = \sqrt{\omega_0^2-\beta^2}$$

$$\Rightarrow x = A e^{-\beta t} \cos(\omega' t + \varphi)$$

For an RLC circuit:

$$\beta = \frac{R}{2L} \quad , \quad \omega_0^2 = \frac{1}{LC}$$

$$\Rightarrow \omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\Rightarrow q = q_m e^{-\frac{Rt}{2L}} \cos(\omega' t + \varphi)$$