PHY3063 Spring 2015

HOMEWORK A

Instructor: Yoonseok Lee

1: We have discussed the drag force on a moving object (sphere) in fluid. Let us perform the dimensional analysis with a slightly different set of governing variables:

$$F_d = f(D, v, \rho, \mu)$$

where D, v, ρ, and μ represent the diameter and speed of the sphere, fluid density, and viscosity, respectively.

(1) The motion of a viscous fluid is governed by the Navier-Stokes equation (equation of motion for fluid):

$$\rho\frac{\partial\vec{v}}{\partial t} + \rho\left(\vec{v}\cdot\nabla\right)\vec{v} = -\nabla p + \mu\nabla^2\vec{v}.$$

Here, $\vec{v}$ is the velocity of fluid component. Using this equation, extract the dimension of viscosity μ. This $\vec{v}$ has nothing to do with v, the object speed.

(2) Are the four governing variables dimensionally independent? Why?

(3) Construct a dimensionless variable Π_μ from μ using other three variables.

(4) Show that the drag force can be rewritten the completely dimensionless form:

$$\frac{F_d}{\rho D^2 v^2} = \Phi\left(\Pi_\mu\right),$$

where $\Phi(x)$ is an unknown function of a dimensionless variable x.

It is important to note that the above equation implies that all flows with the same Π_μ are dynamically similar. $Re = \Pi_\mu^{-1}$ is called the Reynolds number, which is an important dimensionless number that determines the character of a flow such as laminar or turbulent flow. If you do not have the dimensionless form of the drag force, you will have to conduct experiments to determine F_d vs D keeping all other variables and then F_d vs v keeping other variables, and so on. However, the dimensionless form extracted through the dimensional analysis has effectively two variables, the reduced drag force and Π_μ. You do not have to vary viscosity and density. You can extract the above dependence in a single wind tunnel experiment.

HW 2: Perform the Taylor expansion of $tanh(x)$ to the order of x^3 around $x = 0$.

HW 3: Using Taylor expansion, express $f(\beta) = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}$ for $\beta \ll 1$.

HW 4: A particle of mass m is moving in 1-dimension under potential energy given by

$$U(x) = -\frac{1}{3}x^3 + \frac{3}{2}ax^2 - 2a^2x + \frac{1}{3}a^3 \quad (a > 0).$$

(1) Without using the aid of graphing calculator, plot $U(x)$.

(2) You will find a minimum in $U(x)$. The particle is oscillating around the minimum point. What is the oscillation frequency f_o?

HW 5: Tipler 1-10.

HW 6: Consider an inelastic collision in which two particles collide and scatter into two other particles: $A + B \to C + D$. m_A, m_B, m_C, and m_D represent the mass of each particle. Show that the law of conservation of *classical* linear momentum is invariant under the Galilean transformation if and only if total mass is conserved.