PHY3063 Spring 2016

HOMEWORK A

Instructor: Yoonseok Lee

Due: January 19, 2016

HW 1: The motion of a viscous fluid is governed by the Navier-Stokes (N-S) equation (equation of motion for fluid):

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \left(\vec{v} \cdot \nabla \right) \vec{v} = -\nabla P + \mu \nabla^2 \vec{v}.$$

Here, $\vec{v}$ is the velocity of the fluid component, P is pressure, and μ is dynamic viscosity.

(1) The N-S equation is a vector equation. Write down the N-S equation for the x-component. $\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$, $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$, and $\nabla^2 = \nabla \cdot \nabla$.

(2) Express the physical dimension of μ in terms of the five base physical dimensions $\{L, M, T, \theta, I\}$. *Dimensional homogeneity of the equation.*

HW 2 We all know a conventional spring follows Hooke's law given by $E = kx$ where k is the spring constant and x represents the displacement from the equilibrium position. An elastic solid can be viewed as a bundle of springs. Consider a bar of length L with a square cross-section of length a. A force F is applied along the length direction and deformed the length by ΔL. In this case the relation between F and ΔL is given by

$$\frac{F}{a^2} = Y \frac{\Delta L}{L}.$$

Here, Y is called Young's modulus, a material specific parameter. For example, Young's modulus of copper is $\approx 1 \times 10^{11}$ pascal. Justify the unit using the equation given through a dimensional consideration.

HW 2: Perform the Taylor expansion of $tanh(x)$ to the order of x^3 around $x = 0$.

HW 3: Using Taylor expansion, express $f(\beta) = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}$ to the order of β^2. Expand $\frac{\cosh x}{1+x}$ around $x = 0$ to the order of x^2.

HW 4: A particle of mass m is moving in 1-dimension under potential energy given by

$$U(x) = -\frac{1}{3}x^3 + \frac{3}{2}ax^2 - 2a^2x + \frac{1}{3}a^3 \quad (a > 0).$$

(1) Without using the aid of graphing calculator, plot $U(x)$.

(2) You will find a minimum in $U(x)$. The particle is oscillating around the minimum point. What is the oscillation frequency f_o?

HW 5: Suppose that you are seated at the center of a huge dark sphere witha radius of 3×10^8 m and with its inner surface highly reflective. A source at the center emits a very short light flash which moves outward through the darkness with uniform intensity as a expanding spherical wave. What would you see during the first 3 seconds if Einstein's 2nd postulate is correct? What would you see if the whole sphere is moving with speed $0.2c$?

HW 6: Consider an inelastic collision in which two particles collide and scatter into two other particles: $A + B \to C + D$. m_A, m_B, m_C, and m_D represent the mass of each particle. In the laboratory frame (S), the linear momentum conservation will give

$$m_A \vec{v}_A + m_B \vec{v}_B = m_C \vec{v}_C + m_D \vec{v}_D.$$

The *classical* linear momentum should also be conserved in another inertial frame (S') moving with velocity $\vec{u}$ relative to S. Show that the law of conservation of *classical* linear momentum leads to conservation of the total mass: $m_A + m_B = m_C + m_D$. (*It is classical, i.e. non-relativistic. Use Galilean transformation.*)