

HOMEWORK C

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Due: February 18

HW 1: An electron of total energy 4.0 MeV moves perpendicular to a uniform magnetic field along a circular path whose radius is 4.2 cm. (a) What should be the strength of the magnetic field? (b) By what factor does γm_o exceeds the electron rest mass? *We derived the radius of the motion in class. The mass in the expression should be the mass in motion. Then you reach $B = \frac{P}{eR}$. You know what to do to get the relativistic momentum from the total energy!*

HW 2: A spaceship of mass 10^6 kg is coasting through space when suddenly it becomes necessary to accelerate. The ship ejects 10^3 kg of fuel in a very short time at a speed of $0.5c$ relative to the ship.

(a) Neglecting the mass change of the ship, calculate the speed of the ship in the frame in which it was initially at rest (before the acceleration). *Let's call the frame S' . In that reference frame the total linear momentum was 0. This is essentially a collision problem. Since the ejection of fuel does not involve any external force, the total momentum should be conserved. You will find that $\beta = v/c \ll 1$ and certainly can use Taylor expansion somewhere!*

(b) Do the same calculation based on Newtonian mechanics.

HW 3: A particle of rest mass m_o and speed v collides and sticks to a stationary particle of mass M_o . What is the final speed of the composite particle?

HW 4: In the laboratory frame a particle of rest mass m_o and speed v is moving toward a particle of mass m_o at rest. What is the speed of the inertial frame in which the total momentum of the system is zero?

HW 5: In class we derived the the angle dependence of a photon energy $E(\theta)$ scattered off a particle of mass m_o at rest:

$$E(\theta) = \frac{E_o}{1 + \left(\frac{E_o}{m_o c^2}\right) (1 - \cos \theta)},$$

where E_o is the initial photon energy. Using the Planck's energy-wavelength relation, show that the above equation can be expressed in terms of photon wavelength by

$$\lambda - \lambda' = \frac{h}{m_o c} (1 - \cos \theta),$$

where $\frac{h}{m_o c}$ is called the Compton wavelength. *Hint: Start with $E(\theta)^{-1}$.*

HW 6: Kirchoff argued that the radiation in the cavity in thermal equilibrium should be isotropic and homogeneous. If that is the case, the emissive power per unit area $R(\lambda, T)$ is connected with the energy density inside the blackbody cavity $u(\lambda, T)$ through a universal

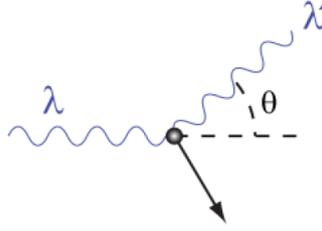


FIG. 1:

relation:

$$u(\lambda, T) = \frac{4R(\lambda, T)}{c}.$$

The derivation of the above relation will give you a clear picture of how the two quantities are related. Check the dimensional homogeneity of the relation.

Hint: Consider a blackbody cavity depicted as a large cube with an opening of area δA at the bottom where the radiation is emitted. The cavity is in thermal equilibrium at T and the inside has a uniform energy density $u(\lambda, T)$. The small cube represents a volume element in the polar coordinate $dV = r^2 \sin \theta dr d\theta d\phi$ located at (r, θ, ϕ) . The energy contained in this volume element is $u(\lambda, T)dV$. Since the radiation in the cavity is isotropic, the volume element will radiate isotropically. Then the radiation energy going through the opening is

$$d\psi = \frac{\delta A \cos \theta}{4\pi r^2} u(\lambda, T) dV.$$

In δt of time, all the volume elements inside the hemisphere of radius $r = c\delta t$ will contribute to the total radiation energy passing through the opening.

$$R(\lambda, T) = \frac{\psi}{\delta A \delta t}.$$

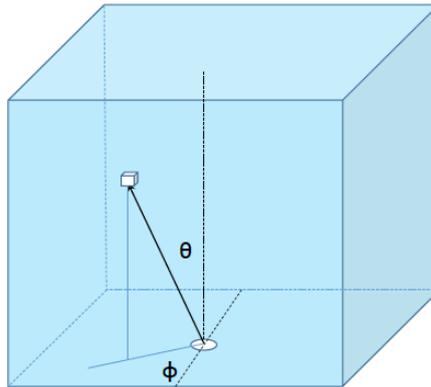


FIG. 2:

HW 7: Harris 3.13

HW 8: Harris 3.23