PHY3063 Spring 2017

HOMEWORK A

Instructor: Yoonseok Lee
Due January 19, 2017

HW 1: Using dimensional analysis estimate the following integral.

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2}\, dx \sim .$$

Compare your estimation with the actual result.

HW 2: We have discussed the drag force on a moving object (sphere) in fluid. Let us perform the dimensional analysis with a slightly different set of governing variables:

$$F_d = f(D, v, \rho, \mu)$$

where D, v, ρ, and μ represent the diameter and speed of the sphere, fluid density, and dynamic viscosity, respectively.

(1) The motion of a viscous fluid is governed by the Navier-Stokes equation (equation of motion for fluid):

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho\, (\vec{v} \cdot \nabla)\, \vec{v} = -\nabla p + \mu \nabla^2 \vec{v}.$$

Here, $\vec{v}$ is the velocity of fluid component. Using this equation (dimensional homogeneity), extract the dimension of dynamic viscosity μ. *$\vec{v}$ is different from the object speed v.*

(2) Are the four governing variables dimensionally independent? Why? *There are only three dimensionally independent variable.*

(3) One can construct two different combinations with the dimension of force. What are those? *Linear drag $F \propto v$ and quadratic drag $F \propto v^2$.*

(4) When an object is falling under gravity, it will eventually reach a terminal velocity. Express the terminal velocities for the two cases of drag force constructed in (3) through dimensional consideration.

HW 3: Perform the Taylor expansion of $tanh(x)$ to the order of x^3 around $x = 0$.

HW 4: Using Taylor expansion, express $f(\beta) = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}$ for $\beta \ll 1$.

HW 5: Expand $\frac{\cosh x}{1+x}$ around $x = 0$ to the order of x^2.

HW 6: A particle of mass m is moving in 1-dimension under potential energy given by

$$U(x) = -\frac{1}{3}x^3 + \frac{3}{2}ax^2 - 2a^2x + \frac{1}{3}a^3 \quad (a > 0).$$

(1) Without using the aid of graphing calculator, plot $U(x)$.

(2) You will find a minimum in $U(x)$. The particle is oscillating around the minimum point. What is the oscillation frequency f_o? *A quadratic potential minimum generates a linear restoring force (Hooke's Law).*