

HOMEWORK C

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Due: February 2

HW 1: Read Harris 2.5 first. The Doppler effect is used in many devices especially in tracking moving bodies. Consider a satellite moving with velocity \vec{v} at \vec{r} from a ground radar. The satellite sends out EM signal of frequency f_o (proper frequency). The detector on the ground would detect Doppler shifted frequency f_D . Since the frequency f_o is known, the ground station would measure the beat frequency $f_D - f_o$.

(1) Show that $f_D - f_o \approx f_o \left(1 + \frac{v}{c} \cos \theta\right)$. Here θ is the angle between $-\vec{r}$ and \vec{v} . *The ground detector is at the origin and the satellite is at \vec{r} .*

(2) The radial velocity of the satellite is $\frac{dr}{dt} = \hat{r} \cdot \vec{v}$. Then you can calculate the total radial distance of travel between times t_a and t_b by simply counting the number of cycles of beat frequency (N_{ba}):

$$\Delta r = r_b - r_a = -\lambda_o N_{ba},$$

where $\lambda_o = c/f_o$ is the proper wavelength of the satellite signal. Derive this relation.

HW 2: Harris 2-57

HW 3: Harris 2-59

HW 4: In frame S , event B occurs $2 \mu\text{s}$ after event A and $\Delta x = 1.5 \text{ km}$ away from event A .

(1) How fast an observer must move along the positive x direction so that the event A and B occur simultaneously.

(2) Is it possible for event B to precede event A for some observer? Explain.

HW 5: An electron of rest energy $mc^2 = 0.511 \text{ MeV}$ moves respect to the laboratory at speed $u = 0.6c$. Find p in MeV/c , E , and E_k .

HW 6: Harris 2-76

HW 7: An electron of total energy 4.0 MeV moves perpendicular to a uniform magnetic field along a circular path whose radius is 4.2 cm . (a) What should be the strength of the magnetic field? (b) By what factor does γm_o exceeds the electron rest mass? *We derived the radius of the motion in class. The mass in the expression should be the mass in motion. Then you reach $B = \frac{P}{eR}$. You know what to do to get the relativistic momentum from the total energy!*

HW 8: A spaceship of mass 10^6 kg is coasting through space when suddenly it becomes necessary to accelerate. The ship ejects 10^3 kg of fuel in a very short time at a speed of $0.5c$ relative to the ship.

(a) Neglecting the mass change of the ship, calculate the speed of the ship in the frame in which it was initially at rest (before the acceleration). *Let's call the frame S' . In that reference frame the total linear momentum was 0. This is essentially a collision problem.*

Since the ejection of fuel does not involve any external force, the total momentum should be conserved. You will find that $\beta = v/c \ll 1$ and certainly can use Taylor expansion somewhere!

(b) Do the same calculation based on Newtonian mechanics.

HW 9: A particle of rest mass m_o and speed v collides and sticks to a stationary particle of mass M_o . What is the final speed of the composite particle?

HW 10: In the laboratory frame a particle of rest mass m_o and speed v is moving toward a particle of mass m_o at rest. What is the speed of the inertial frame in which the total momentum of the system is zero?

Back-of-the-Envelop Physics A spinning skater beautifully demonstrates the angular momentum conservation. When a 60 kg skater was spinning at the angular velocity $\omega = \pi$ rad with her both arms stretched out. How fast can she spin when her arms are down beside her body? Will it be $\omega \sim 5\pi$ or $\sim 50\pi$?

In back of the envelop estimation, you should not be afraid of simplifying the geometry. In this case, you can consider the skater as a cylinder of mass 60 kg and a reasonable radius (≈ 30 cm?) and each arm as a long rod. Make an educated guess on the mass and length of each arm.