

HOMEWORK E

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Due: March 2, 2017

HW 1: Harris 4-48

HW 2: Harris 4-58

HW 3: Harris 4-65.

HW 4: Harris 4-68

HW 5: One can estimate the zero-point energy of a quantum harmonic oscillator using the uncertainty principle. Consider a harmonic oscillator whose energy is given by

$$E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2.$$

Here we use the conventional notation for the mass (m) and natural frequency (ω) of the oscillator. Classically, the ground state energy (minimum energy) is zero meaning that the particle is frozen at the bottom of the potential. In quantum mechanics, the uncertainty principle forbids this and allows an uncertainty in position as $\Delta x = \delta$. Then the momentum uncertainty would be from $\Delta x \Delta p \sim \hbar$. For the lowest momentum one can argue that $p \approx \Delta p$ (why?). You can then write the total energy of the ground state in terms of δ and minimize E to estimate the lowest energy. Follow this procedure to extract painlessly the ground state energy (zero-point energy) of an harmonic oscillator.

HW 6: We made a heuristic derivation of the Schrödinger equation for a nonrelativistic quantum particle. Knowing the relativistic energy $E^2 = p^2c^2 + m_o^2c^4$, derive a wave equation for a relativistic quantum particle of rest mass m_o .

HW 7: Show that the wave function $\Psi(x) = A \sin(kx - \omega t)$ does not satisfy the time-dependent Schrödinger equation. But $\Psi(x) = Ae^{i(kx - \omega t)}$ does satisfy the equation.

HW 8: In a region of space, a particle has has a wave function given by $\psi(x) = Ae^{-x^2/2L^2}$ and energy $\hbar^2/2mL^2$, where L is a length. Find the potential energy as a function of x .
What is the Schrödinger equation, again?

HW 9: Harris 5-11.

HW 10: Harris 5-17.

Back-of-the-Envelop Physics The peak in Sun's spectrum is at $\lambda \approx 500$ nm. Assuming that solar radiation is monochromatic at the peak wavelength mentioned above, how many photons enter in your eyes per second?

A photon with a certain wavelength carries a specific energy in quantum world. What is the approximate area of your pupil?