3-12. A simple pendulum consists of a mass $m$ suspended from a fixed point by a weightless, extensionsless rod of length $l$. Obtain the equation of motion $\ddot{\theta}$ in the approx. that $\sin \theta \approx \theta$, show that the natural frequency is $\omega_n = \sqrt{\frac{g}{l}}$, where $g$ is the gravitational field strength. Discuss the motion in the event that the motion takes place in a viscous medium with retarding force $2m\sqrt{g}e^\theta$.

\[
N = I\ddot{\theta} = lF \sin \theta = lmg \sin \theta
\]

\[
I\ddot{\theta} = -lmg \theta
\]

\[
\ddot{\theta} = -\frac{lmg}{ml^2} \theta = -\frac{g}{l} \theta
\]

\[
\omega_n = \sqrt{\frac{g}{l}}
\]

In viscous medium:

\[
I\ddot{\theta} = -lmg \theta - 2m\sqrt{g}e^\theta
\]

\[
\ddot{\theta} = -\frac{g}{l} \theta - \frac{2\sqrt{g}e^\theta}{ml^2} \theta
\]

\[
\ddot{\theta} + 2\sqrt{\frac{g}{l}} \dot{\theta} + \frac{g}{l} \theta = \ddot{\theta} + 2\omega_n \dot{\theta} + \omega_n^2 \theta = 0
\]

$\beta = \omega_n \rightarrow$ critically damping

\[
\theta(t) = (A + Bt)e^{-\omega_n t}
\]