• This is an open notes test lasting 1 hour and 50 minutes.

• There are 4 problems. Problem 1 has 3 parts and problem 4 has 3 parts. The points for each part are marked. The total is 60 points.

• Begin each problem on a fresh sheet of paper. Use only one side of a sheet of paper.

• Put your name, the problem number, and the page number on each sheet.

• To receive partial credit you must explain what you are doing. For the problems where you have to prove a formula, you have to show your calculations to receive full credit.

• Try to draw a box around important results.

There are 3 pages including this page. Page 3 also has some useful integrals and formulae. Do not forget to look at all parts of the problems.
1. A driven, damped oscillator has a $Q = 6$. Recall the velocity vs $\omega$ resonance curve for such an oscillator.

   a) Show that $\beta^2 = \frac{\omega_0^2}{146}$

   b) Find the maximum value of the magnitude of the velocity $\dot{x}$ in the velocity resonance curve. Let this value be $x_{\text{max}}$

   c) If $\omega_1$ and $\omega_2$ are the angular frequencies for which $\dot{x} = \frac{x_{\text{max}}}{\sqrt{2}}$, then show that

   $$|\omega_1 - \omega_2| = 0.166\omega_0 \approx \frac{\omega_0}{6}$$

2. Obtain the Fourier series representing the function (half-wave rectifier output):

   $$F(t) = \begin{cases} \sin \omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < 2\frac{\pi}{\omega} \end{cases}$$

   (Note: when calculating $b_n$, you will encounter a term of the form $\frac{\sin(1-n)\pi}{1-n}$.

   Remember that for $n=1$, this term is of the form $\frac{0}{0}$. To determine this term refer to tables attached on page 3)

3. A point mass $m$ is located a distance $D$ from the nearest end of a thin rod of mass $M$ and length $L$ along the axis of the rod. Find the gravitational force exerted on the point mass by the rod.

4. A thin disk of mass $M$ and radius $R$ lies in the $(x, y)$-plane with the $z$-axis passing through the center of the disk. Calculate:  

   (continued on next page)
a) the gravitational potential $\Phi(z)$ along the z-axis (5 points)

b) Plot the approximate shape of the $\Phi(z)$ on both sides of the disk i.e. for positive and negative $z$. Is it symmetric around $z=0$? Why/why not? (5 points)

c) the gravitational field $g(z) = -\nabla \Phi(z)$ along the z-axis. (5 points)

**TABLES**

### E.1 Algebraic Functions

\[
\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right), \quad \left| \tan^{-1} \left( \frac{x}{a} \right) \right| < \frac{\pi}{2} \quad (E.1)
\]

\[
\int \frac{x dx}{a^2 + x^2} = \frac{1}{2} \ln(a^2 + x^2) \quad (E.2)
\]

\[
\int \frac{dx}{x(a^2 + x^2)} = \frac{1}{2a^2} \ln \left( \frac{x^2}{a^2 + x^2} \right) \quad (E.3)
\]

\[
\int \frac{dx}{a^2 x^2 - b^2} = \frac{1}{2ab} \ln \left( \frac{ax - b}{ax + b} \right) \quad (E.4a)
\]

\[
= -\frac{1}{ab} \coth^{-1} \left( \frac{ax}{b} \right), \quad a^2 x^2 > b^2 \quad (E.4b)
\]

\[
= -\frac{1}{ab} \tanh^{-1} \left( \frac{ax}{b} \right), \quad a^2 x^2 < b^2 \quad (E.4c)
\]

\[
\int \frac{dx}{\sqrt{a + bx}} = \frac{2}{b} \sqrt{a + bx} \quad (E.5)
\]

\[
\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2}) \quad (E.6)
\]

\[
2 \sin A \cos B = \sin(A + B) + \sin(A - B)
\]

\[
2 \sin A \sin B = \cos(A - B) - \cos(A + B)
\]

\[
2 \cos A \cos B = \cos(A + B) + \cos(A - B)
\]

\[
\lim_{x \to 0} \frac{\sin ax}{x} = a
\]

\[
\nabla \Phi = i \frac{\partial \Phi}{\partial x} + j \frac{\partial \Phi}{\partial y} + k \frac{\partial \Phi}{\partial z}
\]