Physics 3221  
Fall Term 2004  
Final exam, December 17, 2004

- This is an open notes test lasting 2 hours.
- There are four problems which are divided into subsections. The points for each subsection are marked.
- Begin each problem on a fresh sheet of paper. Use only one side of a sheet of paper.
- Put your name, the problem number, and the page number in the upper right hand corner of each sheet.
- To receive partial credit you must explain what you are doing. Carefully labeled figures are important. Randomly scrawled equations aren't helpful.
- Try to draw a box around important results.

There are 3 pages including this page. Do not forget to look at all parts of the problems.
1. Calculate the Fourier series expansion for the periodic function shown in the figure below i.e.
   a) Write down waveform as a function of t and the constants A and ω. (5 points)
   b) Find the Fourier coefficients for this waveform. (10 points)

![Periodic Function Diagram]

2. A single stage rocket takes off from Earth (it’s a well designed rocket so it doesn’t waste any fuel sitting at the launch pad). The exhaust velocity is u, the initial mass (at time t = 0) is m₀, and the fuel burn rate is a constant α.
   a) What is the velocity of the rocket after a time t? (5 points) [Just write the expression from your notes].
   b) Integrate this relation with respect to time and show that the height yₘ that the rocket reaches by the burnout time tₘ when all the fuel is burnt out is given by: (10 points)

   \[ yₘ = utₘ - \frac{1}{2}gtₘ^2 - \frac{um}{\alpha}ln\frac{m₀}{m} \]

   [Hint: you have to use dm/dt = -α, and boundary condition y=0 and m=m₀ at t=0.]
   c) Assuming a constant g throughout, how much further in height will the rocket go after burnout if the velocity at tₘ = vₘ? (5 points)

3. A lightly damped oscillator consists of a mass m attached to a spring (with spring constant k) and is being driven at its resonance frequency ω₀:
   a) Write down the equation for the displacement x(t) for this damped, driven harmonic oscillator if its oscillating with an amplitude D. (2 points)
   b) What is the total energy of this system in terms of k and D? (3 points)

PROBLEM CONTINUED ON NEXT PAGE
c) How much energy is lost by the system in one time period due to the damping $\beta$? (This energy is of course replenished by the driving force). [Take the light damping approximation] (5 points)

d) Using results from (b) and (c) show that: (5 points)

$$Q \approx 2\pi \times \left( \frac{\text{Total Energy}}{\text{Energy lost in one period}} \right)$$

4. Two separate short problems:

a) If the gravitational field vector $g$ inside a sphere is independent of the distance from the center of the sphere $r$, how does the density $\rho(r)$ of the sphere vary as a function of $r$? (5 points)

b) A particle is dropped into a hole drilled straight through the center of the Earth. Neglecting rotational effects show that the particle’s motion is simple harmonic if the Earth has a uniform density $\rho$ and show that the period of oscillation is $\sqrt{\frac{3\pi}{G\rho}}$. (5 points)

Helpful integrals:

\[ \int \ln x \, dx = x \ln x - x \]

\[ \int_1^2 u \, dv = uv \bigg|_1^2 - \int_1^2 v \, du \]