This is the graph for the velocity of the rocket for the parameters given in problem 9-60, using the formula:

\[ v = -gt + u \ln \left( \frac{m_0}{m} \right) \]  

(1)

I said in class that if \( v < 0 \) i.e. \( gt > u \ln \left( \frac{m_0}{m} \right) \) for the initial period as is the case for this problem, then this formula is valid only when \( v \) becomes greater than zero. Actually we cannot use this formula at all if \( v < 0 \) for the initial period. So the way to do this problem is: show that \( v < 0 \) for \( t \) less than about 48 seconds. Then we know we cannot use this formula for our velocity calculations. The reason is that the rocket never reaches the negative velocities because of the launch pad and hence the subsequent motion is also different. What we can do with the equation 1 is to find the acceleration of the rocket as a function of time.

From equation 1 we get:

\[ v = -gt + u \ln m_0 - u \ln m \]

\[ \Rightarrow \ a = \frac{dv}{dt} = -g - \frac{u}{m} \left( \frac{dm}{dt} \right) \quad \text{(because \( u \) and \( m_0 \) are constants)} \]

\[ \Rightarrow \ a = -g + \frac{uc}{m} \quad \text{(because \( \frac{dm}{dt} = -\alpha \)} \]

The rocket will remain on the launch pad as long as the acceleration (and not the velocity) is negative i.e. \( g > \frac{uc}{m} \). The rocket starts moving up when \( a \geq 0 \) i.e. \( g = \frac{uc}{m} \).

\[ g = \frac{uc}{m} \]

\[ \Rightarrow \ g = \frac{uc}{m_0 - \alpha t} \]

\[ \Rightarrow \ t = \frac{m_0}{\alpha} - \frac{u}{g} \]

(2)

This will give us the correct time. This method works because in our derivation of equation 1, we started with the equation for total external force \( F_{\text{ext}} = \frac{dp}{dt} \), which accounts for accelerations and not velocities. The velocity was obtained by integrating this equation.

You can also get equation 2 by putting the thrust (-u dm/dt) equal to the weight of the rocket:

\[ -u \frac{dm}{dt} = mg \]

\[ \Rightarrow \ u\alpha = (m_0 - \alpha t)g \]

\[ \Rightarrow \ t = \frac{m_0}{\alpha} - \frac{u}{g} \]

Once \( a > 0 \), you can start using equation 1 again but you have to replace \( m_0 \) by \( m_0 - \alpha t_0 \) and \( t \) by \( t - t_0 \), where \( t_0 \) is the time when \( a = 0 \). So the modified equation 1 will be:

\[ v = -g(t-t_0) + u \ln \left( \frac{(m_0 - \alpha t_0)}{m} \right) \]

You can check that at \( t = t_0 \), this equation will give us \( v = 0 \) and \( v > 0 \) for \( t > t_0 \).