

# DIMENSIONAL ANALYSIS OF TURKEY BAKING TIME

→ Rate limited by heat conduction thru turkey

$t$  = time to cook turkey  $[T]$

$A$  = surface area of turkey  $[L^2]$

$\alpha$  = thermal diffusivity  $[L^2 T^{-1}]$

$T_0$  = temp on surface  $[\theta]$

$T$  = Temp distribution thru turkey  $[\theta]$

(turkey cooked when certain temp dist. reached)

$$n=5 \quad j=3 \rightarrow k=2$$

$$F = f(t, A, \alpha, T_0)$$

## HEAT TRANSFER NOTES:

Thermal diffusivity = response time of a material (ie turkey meat)

$$\alpha = \frac{k}{\rho c}$$

where  $k$  = thermal conductivity

\* higher  $k$  = faster rate of heat spread

↑  $\alpha$  = fast response

$$\pi_2 = t A^a \alpha^b T^c = T^0 L^0 \theta^0$$

$$[T][L^2][L^2 T^{-1}]^b [\theta]^c = 1$$

$$T: 1 - b = 0 \quad b = 1$$

$$L: 2a + 2b = 0 \quad a = -1$$

$$\theta: c = 0$$

$$\pi_2 = \frac{t \alpha}{A} \leftarrow \text{Fourier \# } (F_0)$$

$$\pi_1 = T A^a \alpha^b T_0^c = T^0 L^0 \theta^0$$

$$[\theta][L^2][L^2 T^{-1}]^b [\theta]^c = 1$$

$$T: -b = 0 \quad b = 0$$

$$L: 2a + 2b = 0 \quad a = 0$$

$$\theta: 1 + c = 0 \quad c = -1$$

$$\pi_1 = \frac{T}{T_0}$$

$$\boxed{\frac{T}{T_0} = f(F_0)}$$

\* TO ACHIEVE SAME TEMP DIST IN DIFFERENT SIZE BODIES (MODEL/PROTOTYPE)  $(F_0)_m$  must equal  $(F_0)_p$

$$\left(\frac{t\alpha}{A}\right)_m = \left(\frac{t\alpha}{A}\right)_p$$

$\alpha = \text{const}$  for turkey meat

$$\rightarrow \frac{t_m}{A_m} \propto \frac{t_p}{A_p} \Rightarrow \frac{t_m}{t_p} \propto \frac{A_m}{A_p}$$

PROBLEM: WE DON'T BUY TURKEYS BY SURFACE AREA!

$$m = \rho V \rightarrow m = \text{const } V$$

$$m = V = f(A)$$

$$[L^3] = [L^2]^a \quad a = 3/2 \quad V = \text{const } A^{3/2}$$

$$m \propto A^{3/2}$$

THEN WE CAN SAY

$$\Rightarrow \boxed{\frac{t_m}{t_p} \propto \left(\frac{m_m}{m_p}\right)^{2/3}}$$