A surprising consequence of length contraction: Simultaneity is relative

In the search for a logical contradiction to the special relativity idea that the speed of light $c$ is always measured to be the same value, we look at the consequences of length contraction.

The train that Alice rides on is 200 m long. Bob’s platform is actually inside the train station, and is 175 m long. The station also has doors that lower over the tracks at either end.

Here is a description of the experiment that Alice and Bob try. Alice’s train moves with a speed $v = 0.6c = 3c/5$ so $\sqrt{1 - v^2/c^2} = 4/5 = 0.8$. Bob knows that the proper length of the train is $L_{\text{train}_0} = 200$ m, so at that speed if Bob measures the length of the train it will only be

$$L_{\text{train}_B} = L_{\text{train}_0} \sqrt{1 - v^2/c^2} = 4/5 \times 200 \text{ m} = 160 \text{ m}.$$ 

So Bob arranges to have two of his assistants put themselves at the doors at either end of the station. The assistants are instructed to lower the doors when the train is inside the station, and then to raise them again to allow the train to proceed through the station without needing to stop.

From Bob’s point of view, or as measured in Bob’s frame of reference, the length of the train is only 160 m long, and he will have no difficulty closing both doors at the station simultaneously.

From Alice’s point of view, the train station has a length

$$L_{\text{station}_A} = \sqrt{1 - v^2/c^2}L_{\text{station}_0} = 4/5 \times 175 = 140 \text{ m},$$

and Alice is worried—the doors in the station cannot possibly be closed simultaneously on her 200 m long train!

So what “really” happens? Are the doors closed simultaneously or not?

The answer depends upon the frame of reference. In Bob’s frame once the back of the train goes through the front door, both are closed simultaneously. They have successfully demonstrated that the station is bigger than the train. Then the back door is opened so that Alice’s train can continue through the station without crashing into the door or slowing down. But, from Bob’s point of view the doors were both closed simultaneously.

The only way to avoid a logical paradox, is to conclude that from Alice’s point of view, the door at the back of the station is closed at first, then it is opened up just before the front of the train reaches the door, and finally the front door is closed behind the back of the train after the back enters the train station. Thus Alice’s 200 m train could run through the 140 m long station, because the doors were not closed simultaneously in her frame of reference.