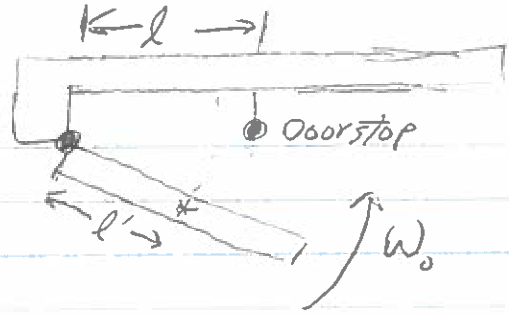
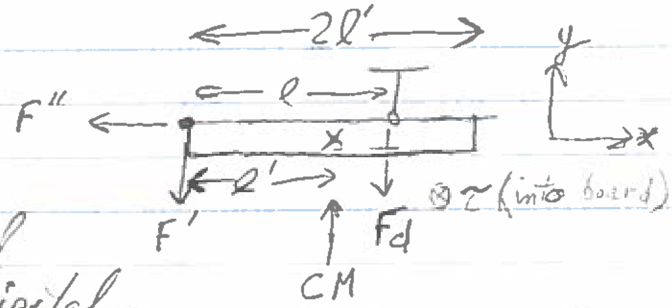
The doorstop problem.




minimize hinge wear by proper choice of $l$

Forces during impact




$F_d$ due to stop

$F''$ due to hinge is small radial force that provides centripetal acceleration

$F'$ on hinge and $F_d$ are the large impact forces

Minimize $F'$ ($\tau \equiv$ torque)

$L_{final} - L_{initial} = \int_{t_i}^{t_f} \tau\, dt$ since $dL = \tau\, dt$

$L_{final} = 0$ & $L_i = I\omega_0$ where $I = I_{CM} + Ml'^2 = \frac{M(2l')^2}{12} + Ml'^2 = \frac{4}{3}Ml'^2$

Torque on door during collision $\tau = -l F_d$

$* \therefore I\omega_0 = -\int_{t_i}^{t_f} \tau\, dt = l\int_i^f F_d\, dt$

$P_i = MV_y = Ml'\omega_0$ where, $l'$ = hinge to CM distance, $\vec{dp} = \vec{F}dt$

$P_f = 0$ with $F_y = -(F' + F_d)$

$\therefore P_f - P_i = \int F_y\, dt \Rightarrow P_i = -\int F_y\, dt = \int (F' + F_d)\, dt = \overbrace{Ml'\omega_0}^{P_i}$

$\Rightarrow \int F' dt + \frac{I\omega_0}{l} = Ml'\omega_0 \Rightarrow \int F' dt = \left(Ml' - \frac{I}{l}\right)\omega_0 \simeq 0$ {To minimize impact!

$* \Rightarrow l = \frac{I}{Ml'} = \frac{\frac{4}{3}Ml'^2}{Ml'} = \frac{4}{3}l' = \frac{2}{3}L$ ✓✓