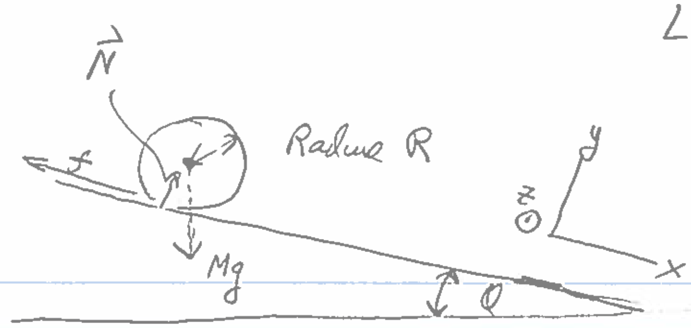


Disk on incline!

rotation + translation ; no slipping

$I\vec{\dot{\omega}} = \vec{\Gamma}_{ext}$ ; $M\ddot{\vec{R}}_{CM} = \vec{F}_{ext} = \sum_i' f_i^{ext}$

$I\ddot{\phi} = \vec{\Gamma}_{ext} = \sum_i' r_i \times \vec{f}_i^{ext}$ (need to have good choice of origin & where force is acting; angular momentum depends on choice)

Rotation:

(i) Choose origin at center of disk

(ii) Identify forces that will generate Torque

$N$? no since $\vec{r} \parallel -\vec{N}$

$Mg$? no since $\vec{r} = 0$

$\vec{f}$ yes $\vec{r} \perp \vec{f}$ $\Rightarrow$ $\vec{\Gamma}_{ext} = fR(-\hat{z})$ note - sign

$fR = I_{CM}\alpha = I_{CM}\dot{\omega} = \frac{1}{2}MR^2\dot{\omega}$

$\Rightarrow \boxed{f = \frac{M}{2}R\dot{\omega}}$ has $\left[\frac{ML}{t^2}\right]$ dimensions.

Translation:

$\vec{F}_{ext} = (Mg\sin\phi - f)\hat{x} = M\ddot{\vec{R}}_{CM} = M\dot{\vec{v}}_x$

$\Rightarrow \boxed{f = Mg\sin\phi - M\dot{v}_x}$

Constraint of no slipping, $\boxed{v_x = \omega R}$, connects ($\dot{v}_x$ & $\dot{\omega}$ hence) the two $f$'s

i.e. $f = \frac{M}{2}R\dot{\omega} = \frac{M}{2}R\frac{\dot{v}_x}{R} = \frac{M}{2}\dot{v}_x = Mg\sin\theta - M\dot{v}_x$

$\Rightarrow \boxed{\dot{v}_x = \frac{2}{3}g\sin\theta}$ Notice how M cancels out!

cf, M on frictionless surface $\dot{v}_x = g\sin\theta$ (no rotation)
from $Mg\sin\theta = M\dot{v}$