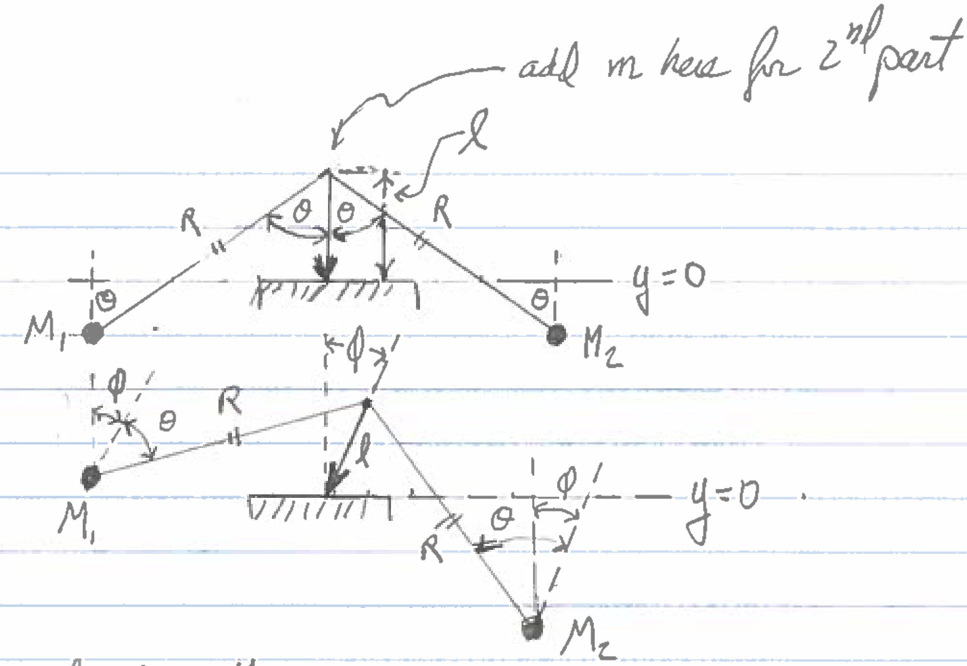
Teeter Toy



Vertical positions of M_1 & M_2

$$y_1 = l\cos\varphi - R\cos(\theta+\varphi) \qquad y_2 = l\cos\varphi - R\cos(\theta-\varphi)$$

$$\therefore\ U(\varphi) = Mgy_1 + Mgy_2 = Mg(y_1+y_2)$$

$$= Mg\left[2l\cos\varphi - R\left\{\underbrace{\cos(\theta+\varphi)+\cos(\theta-\varphi)}_{2\cos\theta\cos\varphi}\right\}\right]$$

$$= 2Mg\cos\varphi\left[l - R\cos\theta\right]$$

(1) Equilibrium: $\partial U(\varphi)/\partial\varphi = 0 = -2Mg\sin\varphi\left[l - R\cos\theta\right]$

$\Rightarrow \varphi = 0$ at equilibrium

(2) Stability: $\partial^2 U/\partial\varphi^2 = -2Mg\cos\varphi\,(l - R\cos\theta) > 0$

$\Rightarrow l - R\cos\theta < 0 \Rightarrow$ $\boxed{R\cos\theta > l}$ & masses are below pivot point!

(3) Add m at top

$$U(\varphi) = 2Mg\cos\varphi\left[l - \cos\theta\right] + mgl\cos\varphi = 2Mg\cos\varphi\left\{l\left(1+\frac{m}{2M}\right) - R\cos\theta\right\}$$

Equilibrium at $\varphi = 0$, stability for $\boxed{R\cos\theta > l\left(1+\frac{m}{2M}\right)}$

$\Rightarrow$ smaller θ for stability!