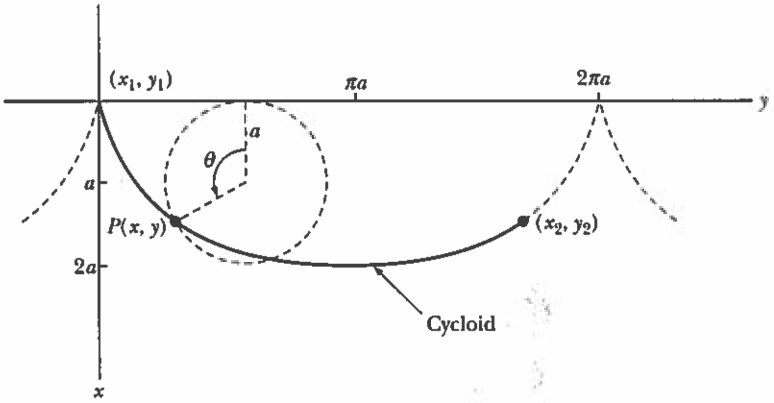

FIGURE 6-4 Example 6.2. The solution of the brachistochrone problem is a cycloid.

and

$$y = a(\theta - \sin\theta) + \text{constant} \tag{6.25}$$

The parametric equations for a *cycloid** passing through the origin are

$$\left.\begin{aligned} x &= a(1 - \cos\theta) \\ y &= a(\theta - \sin\theta) \end{aligned}\right\} \tag{6.26}$$

which is just the solution found, with the constant of integration set equal to zero to conform with the requirement that (0, 0) is the starting point of the motion. The path is then as shown in Figure 6-4, and the constant a must be adjusted to allow the cycloid to pass through the specified point (x_2, y_2). Solving the problem of the brachistochrone does indeed yield a path the particle traverses in a *minimum* time. But the procedures of variational calculus are designed only to produce an extremum—either a minimum or a maximum. It is almost always the case in dynamics that we desire (and find) a minimum for the problem.

EXAMPLE 6.3

Consider the surface generated by revolving a line connecting two fixed points (x_1, y_1) and (x_2, y_2) about an axis coplanar with the two points. Find the equation of the line connecting the points such that the surface area generated by the revolution (i.e., the area of the surface of revolution) is a minimum.

Solution. We assume that the curve passing through (x_1, y_1) and (x_2, y_2) is revolved about the y-axis, coplanar with the two points. To calculate the total area of the surface of revolution, we first find the area dA of a strip. Refer to Figure 6-5.

*A cycloid is a curve traced by a point on a circle rolling on a plane along a line in the plane. See the dashed sphere rolling along $x = 0$ in Figure 6-4.