

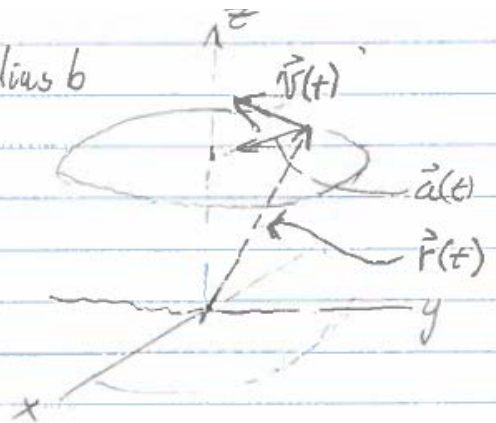
Ex. Particle in circular orbit of radius  $b$

$$\vec{r} = -b \sin \omega t \hat{i} + b \cos \omega t \hat{j} + c \hat{k}$$

(ccw from top)

$$|\vec{r}| = (b^2 \sin^2 \omega t + b^2 \cos^2 \omega t + c^2)^{1/2}$$

$$= (b^2 + c^2)^{1/2} = \text{constant}$$



But direction is changing!

$$\vec{v} = \dot{\vec{r}} = -\omega b \cos \omega t \hat{i} - \omega b \sin \omega t \hat{j} + 0 \hat{k}$$

$$\Rightarrow |\vec{v}| = \sqrt{\omega^2 b^2 (\cos^2 \omega t + \sin^2 \omega t)} = \omega b \quad \text{constant}$$

$$\vec{v} \cdot \vec{r} = (-\omega b \cos \omega t \hat{i} - \omega b \sin \omega t \hat{j}) \cdot (-b \sin \omega t \hat{i} + b \cos \omega t \hat{j})$$

$$= \omega b^2 \cos \omega t \sin \omega t - \omega b^2 \sin \omega t \cos \omega t = 0$$

$$\Rightarrow \vec{v} \perp \vec{r}$$

$$\vec{a} = \ddot{\vec{r}} = \dot{\vec{v}} = +\omega^2 b \sin \omega t \hat{i} - \omega^2 b \cos \omega t \hat{j}$$

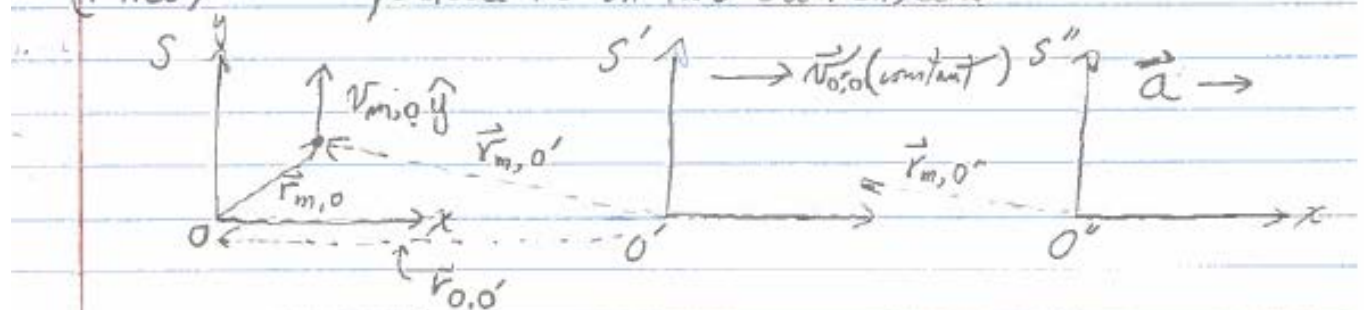
$$\Rightarrow |\vec{a}| = \omega^2 b \quad \text{constant} \quad [b=R \Rightarrow |\vec{a}| = \omega^2 R = \frac{v^2}{R} \text{ since } v=\omega R]$$

$$\vec{v} \cdot \vec{a} = (-\omega b \cos \omega t \hat{i} - \omega b \sin \omega t \hat{j}) \cdot (+\omega^2 b \sin \omega t \hat{i} - \omega^2 b \cos \omega t \hat{j})$$

$$= \omega^3 b^2 (-\cos \omega t \sin \omega t + \sin \omega t \cos \omega t) = 0$$

$$\Rightarrow \vec{v} \perp \vec{a} \quad (\text{see diagram})$$

A subtlety: 1<sup>st</sup> Law only holds in inertial frames of reference  
(T1.26) particle  $m$  in two dimensions



(a) Frame  $S$ :  $\vec{r}_{m,O} = v_{m,O} t \hat{y}$   $\hat{y} = \hat{j}$

ignore  $x$  component.

(b) Frame  $S'$ :  $\vec{r}_{m,O} + \vec{r}_{O',O} = \vec{r}_{m,O'} = \vec{r}_{m,O} - \vec{r}_{O',O}^*$

$\therefore \vec{r}_{m,O'} = \vec{r}_{m,O} - \vec{r}_{O',O} \Rightarrow \boxed{v_{m,O'} = |v_{m,O}| \hat{y} - v_{O',O} \hat{x}}$

$\Rightarrow \vec{r}_{m,O'} = |v_{m,O}| t \hat{y} - v_{O',O} t \hat{x}$  in  $S'$   $\Rightarrow$  velocity is constant

$\Rightarrow \vec{a}_{m,O'} = \dot{\vec{v}}_{m,O'} = 0$  since  $\dot{v}_{O',O} = 0$

(c) Frame  $S''$ :

\*  $\vec{r}_{m,O''} = \vec{r}_{m,O} - \vec{r}_{O'',O}$

$\therefore v_{m,O''} = v_{m,O} \hat{y} - at \hat{x}$  (velocity not constant)  $\Rightarrow$  1<sup>st</sup> Law doesn't hold!

with  $\vec{r}_{m,O''} = v_{m,O} t \hat{y} - \frac{at^2}{2} \hat{x}$  in  $S''$

$\Rightarrow \ddot{\vec{r}}_{m,O''} \neq 0$  &  $S''$  is not inertial frame