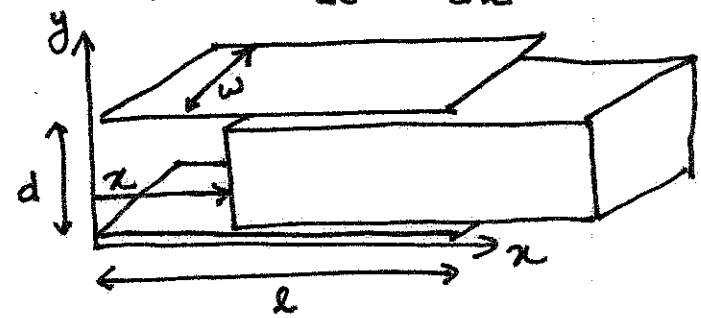
Force on a dielectrics

Constant charge

$$U = \frac{Q^2}{2C} \quad \text{(Potential energy)} \quad \Rightarrow \quad \frac{dU}{dx} = -\frac{Q^2}{2C^2} \cdot \frac{dC}{dx}$$

$$\left(\frac{dU}{dC} \cdot \frac{dC}{dx}\right)$$



$$C = \frac{\epsilon_0 w}{d}\left(\epsilon_r l - \chi_e x\right)$$

$$\Rightarrow \frac{dC}{dx} = -\frac{\epsilon_0 w \chi_e}{d}$$

$$\Rightarrow \frac{dU}{dx} = \left(-\frac{Q^2}{2C^2}\right) \cdot \left(-\frac{\epsilon_0 w \chi_e}{d}\right) = \frac{Q^2 \epsilon_0 w \chi_e}{2C^2 d}$$

$$Q = CV \quad \Rightarrow \quad \frac{dU}{dx} = \frac{V^2}{2} \frac{\epsilon_0 w \chi_e}{d}$$

$$\vec{F} = -\nabla U \quad \Rightarrow \quad \vec{F} = -\frac{dU}{dx}\hat{x} = -\frac{V^2}{2}\frac{\epsilon_0 w \chi_e}{d}\hat{x}$$

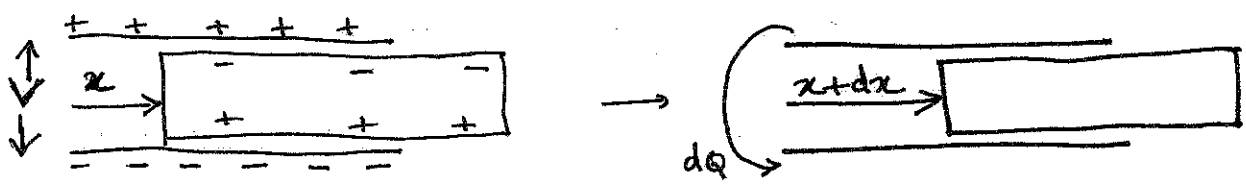
So the force points in the $-\hat{x}$ direction $\Rightarrow$ the dielectric attracted back inside the capacitor.

Constant Voltage

$$U = \frac{1}{2}CV^2 \Rightarrow \frac{dU}{dx} = \frac{dU}{dC}\cdot\frac{dC}{dx} = \frac{V^2}{2}\cdot\left(-\frac{\epsilon_0 w \chi_e}{d}\right)$$

$$\Rightarrow \vec{F} = -\frac{dU}{dx}\hat{x} = +\frac{V^2}{2}\frac{\epsilon_0 w \chi_e}{d}\hat{x}$$

So at first it appears that the force points in the $+\hat{x}$ direction i.e. the dielectric is repelled out of the capacitor! But that's because we did not include the work done by the battery in keeping V constant. As x increases, C goes down and since $Q = CV$, too keep V constant Q has to go down.



The change in U due to $+dQ$ going from the +ive plate to the negative plate $= VdQ$

$$\Rightarrow dU = -Fdx + VdQ \Rightarrow F = \underbrace{-\frac{dU}{dx}}_{\text{positive}} + \underbrace{V\frac{dQ}{dx}}_{\text{negative since Q goes down with increasing } x.}$$

$$Q = CV \Rightarrow \frac{dQ}{dx} = V\frac{dC}{dx}$$

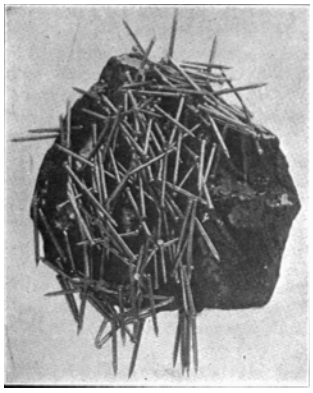
$$\Rightarrow F = -\frac{dU}{dx} + V\frac{dQ}{dx} = -\frac{dU}{dC}\cdot\frac{dC}{dx} + V^2\frac{dC}{dx}$$

$$= \left(-\frac{V^2}{2} + V^2\right)\frac{dC}{dx} = +\frac{V^2}{2}\frac{dC}{dx}$$

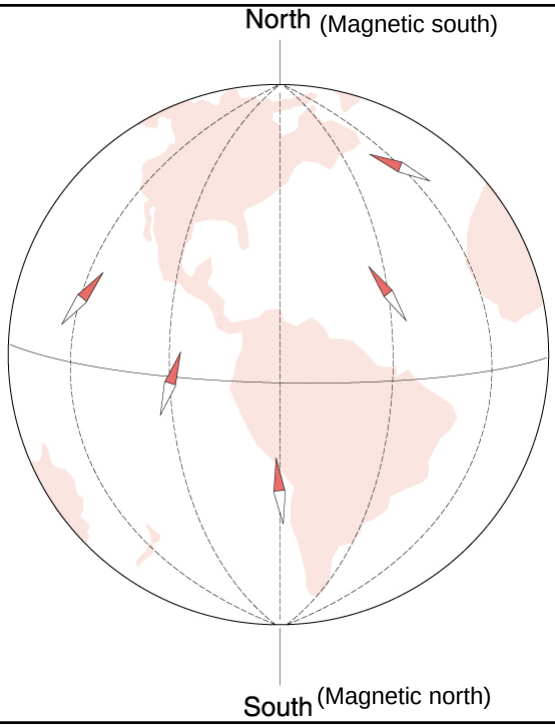
$$= -\frac{V^2}{2}\frac{\epsilon_0 w \chi_e}{d}$$

which is again an attractive force pulling the dielectric into the capacitor.

Magnetic forces

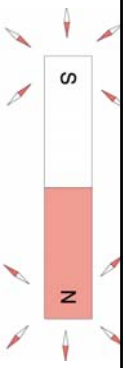


http://en.wikipedia.org/wiki/Lodestone



Some applications

Observations about magnetic fields



Poles of a magnet are the ends where objects are most strongly attracted

- Two poles, called *north* and *south*

Like poles repel each other and unlike poles attract each other

- Similar to electric charges

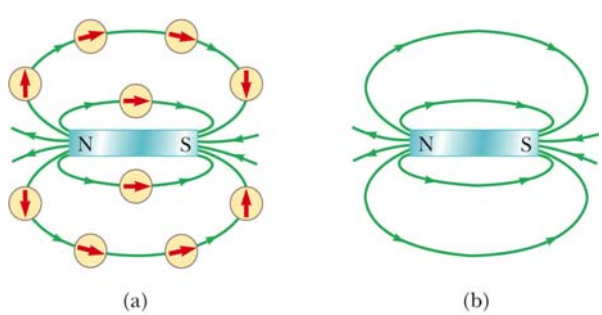
Magnetic poles cannot be isolated

- If a permanent magnetic is cut in half repeatedly, you will still have a north and a south pole
- This differs from electric charges
- There is some theoretical basis for monopoles, but none have been detected

Slightly more quantitative

- A vector quantity
- Symbolized by $\vec{B}$
- Direction is given by the direction a *north pole* of a compass needle points in that location
- *Magnetic field lines* can be used to show how the field lines, as traced out by a compass, would look

Magnetic Field Lines, sketch



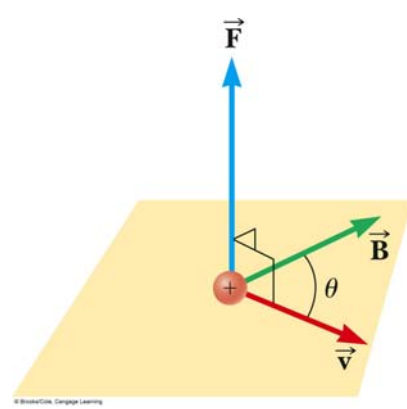
- A compass can be used to show the direction of the magnetic field lines (a)
- A sketch of the magnetic field lines (b). Direction and strength just like E-field

Magnetic Fields

- When a charged particle is moving through a magnetic field, a magnetic force acts on it
  - This force has a maximum value when the charge moves perpendicularly to the magnetic field lines
  - This force is zero when the charge moves along the field lines

Finding the Direction of Magnetic Force

- Experiments show that the direction of the magnetic force is always perpendicular to both $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$
- F_{max} occurs when $\vec{\mathbf{v}}$ is perpendicular to $\vec{\mathbf{B}}$
- F = 0 when $\vec{\mathbf{v}}$ is parallel to $\vec{\mathbf{B}}$
- Proportional to the magnitude of the $\vec{\mathbf{v}}$, $\vec{\mathbf{B}}$, and charge q.



Magnetic Fields

- One can define a magnetic field in terms of the magnetic force exerted on a test charge moving in the field with velocity $\vec{\mathbf{v}}$
  - Similar to the way electric fields are defined
- $$B \equiv \frac{F}{qv\sin\theta}$$

Units of Magnetic Field

- The SI unit of magnetic field is the *Tesla* (T)

$$T = \frac{Wb}{m^2} = \frac{N}{C\cdot(m/s)} = \frac{N}{A\cdot m}$$

  - Wb is a Weber
- The cgs unit is a *Gauss* (G)
  - 1 T = 10^4 G

A Few Typical B Values

- Conventional laboratory magnets
  25000 G or 2.5 T
- Superconducting magnets
  200000 G or 20 T
- Magnet lab (Tallahassee)
  about 1000000 G or 100 T
- Earth's magnetic field
  0.5 G or 5×10^{-5} T