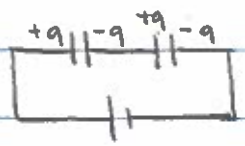
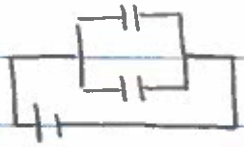
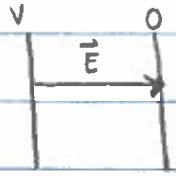
E/M Exam Study Session Notes

parallel-plate capacitor, $C = \epsilon_0 \frac{A}{d}$

Series	Parallel
$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$	$C = C_1 + C_2$
Charge is the same	Voltage is the same
	
Voltage is different	Charge is different

$E \rightarrow \frac{\text{Volts}}{m}$



Properties of solutions to Laplace's equation!

ex) max. or minima?

Work and Energy! $W = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$

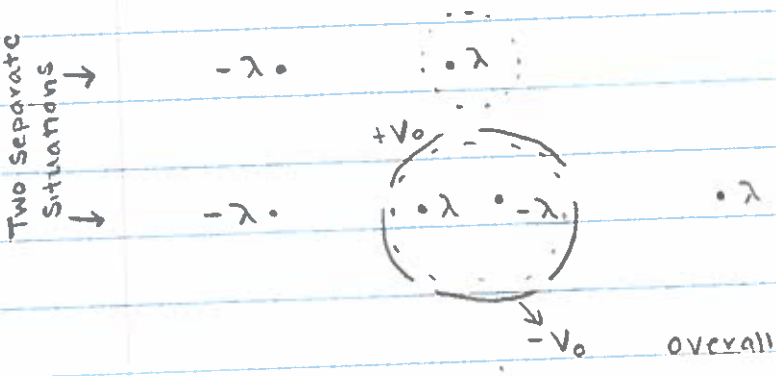
What is work done by internal force or by the electric field?

$W_{ext} > 0$ because you put in work to get these charges closer

$W_E > 0$ because charge is moving in direction of force

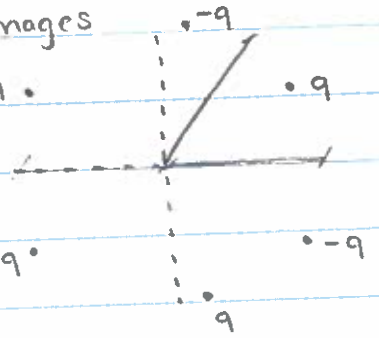
P 3.40 → lines of charged

Eq. potential is a circle, not centered at the point.



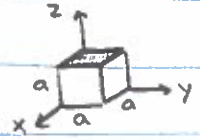
overall potential around the circle is zero

* Mirror-Images



If you have a cube w/ six sides, all w/ potential V_0, they each contribute $\frac{V_0}{6}$ to the center. If they have varying potentials, they will contribute different amounts but still add up to V_0

$$V_1 + V_2 + V_3 + V_4 + V_5 + V_6 = V_0$$



$V = 0$ @ $x = 0, x = a, y = 0, y = a, z = 0$

$V = V_0$ @ $z = a$

$$\frac{d^2X}{dx^2} = -k^2X \qquad \frac{d^2Y}{dy^2} = -l^2Y \qquad \frac{d^2Z}{dz^2} = (k^2 + l^2)Z$$

$X = A\sin kx + \cancel{B}^{\,0}\cos kx$ $\qquad k = \frac{n\pi}{a}$

$Y = C\sin ly + \cancel{D}^{\,0}\cos ly$ $\qquad l = \frac{m\pi}{a}$

$Z = Ee^{\sqrt{k^2+l^2}\,z} + Fe^{-\sqrt{k^2+l^2}\,z}$ $\qquad E = -F$

$\hookrightarrow \quad Z = E\sinh\sqrt{k^2+l^2}\,z$

$$V(x,y,z) = \sum_n \sum_m C_{n,m} \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{a} \sinh\frac{\pi}{a}\sqrt{n^2+m^2}\,z$$

$$V_0 = V(x,y,a) = \sum_n \sum_m C_{n,m} \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{a} \sinh\pi\sqrt{n^2+m^2}$$

$$V_0 \int_0^a \sin\frac{n'\pi x}{a}\,dx \int_0^a \sin\frac{m'\pi y}{a}\,dy$$

$$= \sum_n \sum_m C_{n,m} \sinh\pi\sqrt{n^2+m^2} \int_0^a \sin\frac{n\pi x}{a} \sin\frac{n'\pi x}{a}\,dx \cdot \int_0^a \sin\frac{m\pi y}{a} \sin\frac{m'\pi y}{a}\,dy$$

On RHS, only $n' = n$ and $m' = m$ terms survive, so...

$$V_0 \int_0^a\int_0^a \sin\frac{n'\pi x}{a}\,dx \sin\frac{m'\pi y}{a}\,dy$$

$$= \sum_n \sum_m C_{n,m} \sinh\pi\sqrt{n^2+m^2} \underbrace{\int_0^a \sin^2\frac{n\pi x}{a}\,dx}_{a/2} \underbrace{\int_0^a \sin^2\frac{m\pi y}{a}\,dy}_{a/2}$$

LHS: $V_0 \left[\frac{-\cos\frac{n\pi x}{a}}{n\pi/a}\right]_0^a \left[\frac{-\cos\frac{m\pi y}{a}}{m\pi/a}\right]_0^a$

$$= \frac{V_0 a^2}{mn\pi^2}\left[1-\cos n\pi\right]\left[1-\cos m\pi\right]$$

$$\frac{V_0 \cancel{a^2}}{mn\pi^2}\underbrace{(1-\cos n\pi)}_{0 \text{ or } 2}\underbrace{(1-\cos m\pi)}_{0 \text{ or } 2} = C_{nm}\frac{\cancel{a^2}}{4}\sinh\pi\sqrt{n^2+m^2}$$

$$C_{m,n} = \frac{16V_0}{mn\pi^2\sinh\pi\sqrt{m^2+n^2}}$$

$$V(r,\theta) = \sum_l \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$$

- Plug back into equation and you will get zero.
- $V \to 0, \quad r \to \infty$

$\sum_l A_l r^l P_l(\cos\theta)$ for $r < R$

$\sum_l \frac{B_l}{r^{l+1}} P_l(\cos\theta)$ for $r > R$

continuity: $\sum_l A_l R^l P_l(\cos\theta) = \sum_l \frac{B_l}{r^{l+1}} P_l(\cos\theta)$

orthogonality relation: $A_l R^l \frac{2}{2l+1} = \frac{B_l}{R^{l+1}} \frac{2}{2l+1}$

$$B_l = A_l R^{2l+1}$$

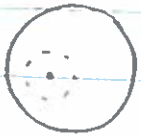
$\vec{E}$ field is discontinuous:

$$\frac{dV_{out}}{dr} - \frac{dV_{in}}{dr}\bigg|_{r=R} = \frac{-\sigma_0(\theta)}{\epsilon_0}$$

What does the mean value theorem mean?

What are the physical implications?

↳ Laplace equation applies on the inside of a chosen spherical surface, so does mean value theorem



Avg of values of potential on surface of hypothetical sphere will equate to the potential at the center of that sphere.

1st uniqueness theorem - no metal

2nd uniqueness theorem - has metal or can have metal?

No RC-circuits. Only capacitors.