

## ELECTROMAGNETISM Exam 3 Review

### First Method

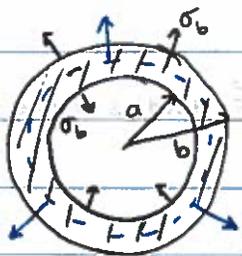
P4.15  $\vec{P} = \frac{k}{r} \hat{r}$

$$\sigma_b = \vec{P} \cdot \hat{n} = \frac{k}{b} \text{ (outer)}$$
$$\sigma_b = -\frac{k}{a} \text{ (inner)}$$



inner surface,  $\hat{n}$  points inward

outer surface,  $\hat{n}$  points outward



Gaussian surface inside the dielectric

$$-\frac{k}{a} 4\pi a^2 \rightarrow \text{surface charge}$$

$$\rho_b = -\nabla \cdot \vec{P} = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{k}{r}) = -\frac{k}{r^2}$$

(not a constant surface charge density)

$$\text{Volume charge enclosed} = -k \int_a^r \frac{1}{r^2} r^2 dr \sin\theta d\theta d\phi$$
$$= -4\pi k(r-a)$$

$$\oint \vec{E} \cdot d\vec{a} = E \cdot 4\pi r^2 = \frac{1}{\epsilon_0} [-4\pi a k - 4\pi k(r-a)]$$

Solving for  $E \rightarrow E = \frac{-k}{\epsilon_0 r}$

Now, because we took radially outward to be  $+\hat{n}$  here,

negative  $E$  means that the electric field is pointing inward.

$$-4\pi k a \rightarrow \text{inner surface}$$

$$+4\pi k b \rightarrow \text{outer surface}$$

So for  $-k \int_a^b \frac{1}{r^2} r^2 dr \sin\theta d\theta d\phi = -4\pi k(b-a)$

$$\sigma_{bi} + \sigma_{bo} + E = 0$$

$$P_b = -\nabla \cdot \vec{P}, \quad q_v = -\int_V (\nabla \cdot \vec{P}) d\tau$$

$$q_{\text{surface}} = \int_V \vec{P} \cdot d\vec{a} = +\int_V (\nabla \cdot \vec{P}) d\tau$$

The sum of  $q_v$  and  $q_{\text{surface}}$  should cancel.

(Here, they do.)

$$\oint \vec{D} \cdot d\vec{a} = q_f = 0 \quad (\text{no free charge})$$

Inside and outside of dielectric material, the  $\vec{E}$  field is zero

$$\text{because } \vec{D} = 0 = \epsilon_0 \vec{E} + \vec{P} \rightarrow \epsilon_0 \vec{E} = -\vec{P}$$

$$\rightarrow \vec{E} = \frac{-\vec{P}}{\epsilon_0} = \frac{-k}{\epsilon_0 r} \hat{r} \quad \text{looks familiar!}$$

$\vec{D}$  field method is much quicker.

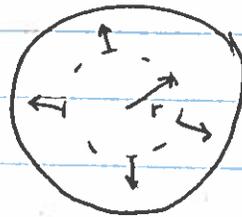
If you have a spherical shape and draw a <sup>spherical</sup> gaussian surface

with the same origin,

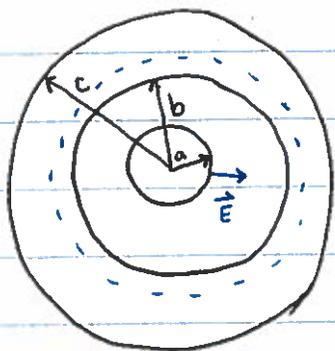
even if there is a  $\vec{D}$  field, around the

gaussian surface, the overall will be zero.

$$(\text{symmetry!}) \quad \oint \vec{D} \cdot d\vec{a} = 0$$



$\oint \vec{E} \cdot d\vec{a}$  and  $\oint \vec{D} \cdot d\vec{a}$  must abide by the same rules of symmetry.

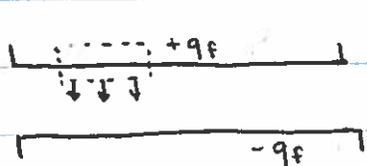


$$\vec{D} = \epsilon \vec{E}$$

$$V = - \int^+ \vec{E} \cdot d\vec{l}$$

This problem will not be on the test.

Parallel-plate capacitors again



$\vec{D}$  field inside a metal at equilibrium has to be zero  
- Metal can't be polarized

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\begin{matrix} \uparrow & \uparrow \\ 0 & 0 \end{matrix}$$

There is a  $\vec{D}$  field in the dielectric material.



$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

pillbox is a volume



$$\oint \vec{D} \cdot d\vec{a} = \sigma_f A$$

because whatever charge is inside the dielectric is not free charge

$$\text{So... } DA = \sigma_f A \rightarrow D = \sigma_f$$

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} = 1 + \chi_e$$

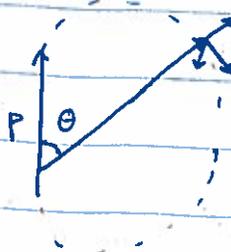
$$\vec{P} = \epsilon_0 \chi_e \vec{E}, \text{ linear dielectric}$$

$$V = \frac{\rho \cos \theta}{4\pi \epsilon_0 r^2}$$

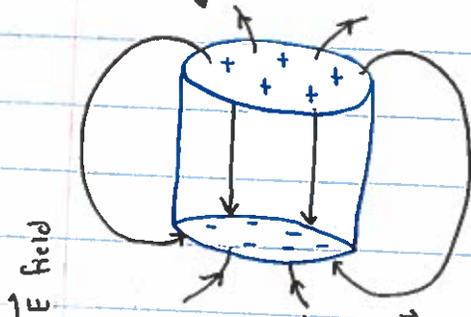
$$\nabla = \hat{r} \frac{d}{dr} + \frac{1}{r} \frac{d}{d\theta} \hat{\theta}$$

$$E_r = \frac{-2\rho \cos \theta}{4\pi \epsilon_0 r^3}$$

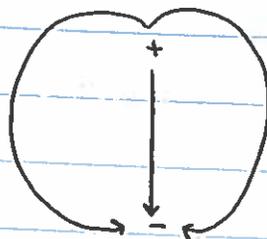
$$E_\theta = \frac{\rho \sin \theta}{4\pi \epsilon_0 r^3}$$



bound charges, not free charges



inside,  $\vec{D}$  and  $\vec{E}$  are opposite directions



for frozen in polarization

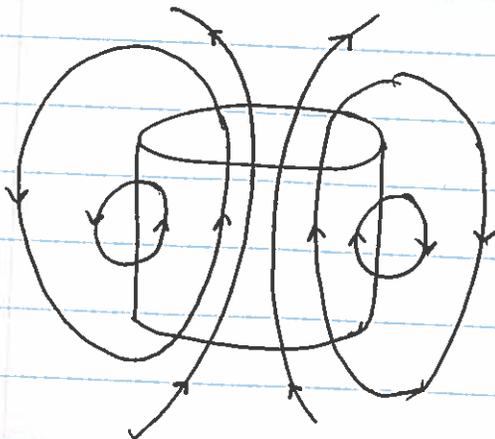
outside,  $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$   
Same direction

inside,  $\epsilon_0 \vec{E}$  and  $\vec{P}$  are in opposite directions but  $\vec{P}$  overwhelms  
So  $\vec{D}$  is opposite  $\vec{E}$

Remember boundary condition?

$$D_{\perp}^{\text{above}} - D_{\perp}^{\text{below}} = \sigma_f \quad (\text{true for all dielectrics})$$

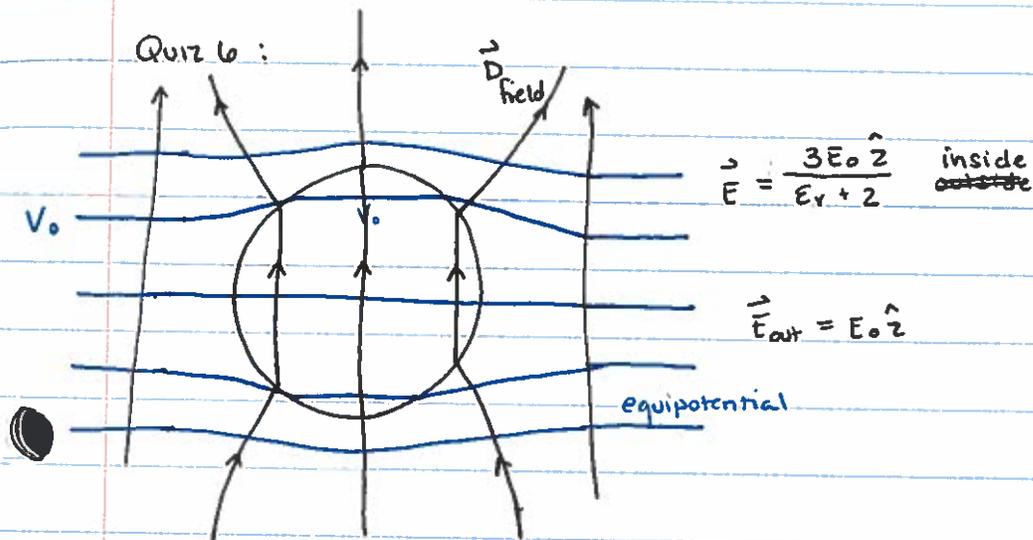
$$\epsilon^{\text{above}} E_{\perp}^{\text{above}} - \epsilon^{\text{below}} E_{\perp}^{\text{below}} = \sigma_f \quad (\text{only linear dielectrics})$$



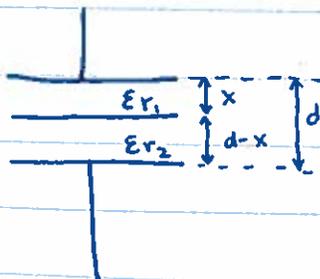
parallel plate capacitor

$$C = \epsilon_0 \frac{A}{d}$$

W/ dielectric,  $C = \epsilon_r \epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$       $\epsilon_r > 1$

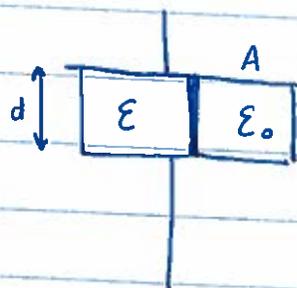


Know how to add capacitance in series



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \left( \epsilon_{r1} \epsilon_0 \frac{A}{x} \right)^{-1} + \left( \epsilon_{r2} \epsilon_0 \frac{A}{d-x} \right)^{-1}$$

This problem will be 1 pt! The answer will be given so that you can move on to the next part.



$$C_1 = \epsilon \frac{A/2}{d}$$

$$C_2 = \epsilon_0 \frac{A/2}{d}$$

$$C = \frac{\epsilon_0 A}{2d} (1 + \epsilon_r)$$



answer will be independent  
of x

$\vec{E}$  field points straight down  
equipotentials will be horizontal

Forces on dielectrics

Also,  $\frac{1}{2} CV^2$  Constant voltage (batteries connected)

$\frac{Q^2}{2C}$  Constant charge (batteries disconnected)

Give boundary conditions and implement correctly