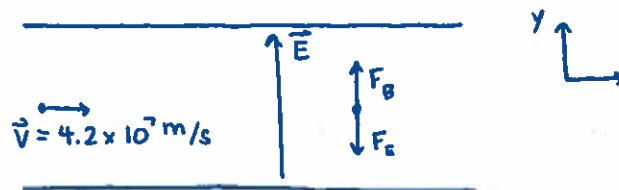
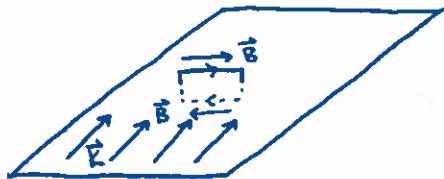


Electromagnetism Final Review

Ampere's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$

$$\int_{\text{above}} \vec{B} \cdot d\vec{l} + \int_{\text{vert}} \vec{B} \cdot d\vec{l} + \int_{\text{below}} \vec{B} \cdot d\vec{l} + \int_{\text{vert}} \vec{B} \cdot d\vec{l}$$



$$F_B = qvB \quad \rightarrow \quad v = \frac{E}{B}$$

$$F_E = qE$$

F_E and F_B must cancel out for the electron to travel in a straight line.

$$\vec{K} = \sigma \vec{V}$$

$$\vec{J} = \rho \vec{V}$$

$$\vec{I} = \lambda \vec{V}$$



$$\nabla \times \vec{E} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$



$$\vec{B}_{\text{outside}} = 0 \quad \text{for infinite solenoid.}$$

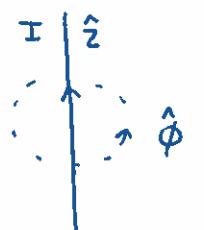
$$\vec{B} = \mu_0 n I \quad \text{for regular solenoid}$$

$n = \text{number of turns.}$

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

For P.S. 2b, cannot use $\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{\vec{j}(\vec{r}') d\tau}{r}$
 because it's infinite. (Does not converge at infinity.)



$$\vec{B} = \frac{\mu_0 I}{2\pi s} \hat{\phi}$$

$$\nabla \times \vec{A} = \vec{B}$$

B is a function of s and is in the $\hat{\phi}$ -direction.

Due to the definition of the curl in cylindrical components, A cannot be a function of ϕ (or else B would have some component in the \hat{s} -direction.)

$$\nabla \times A = \hat{\phi} \left[\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s} \right] = \vec{B}$$

→ \vec{A} vector is in the direction of the current so A must be a function of s .

$$\text{So } -\frac{\partial A_z}{\partial s} = \frac{\mu_0 I}{2\pi s}$$

Now, if the wire had some thickness (radius R)

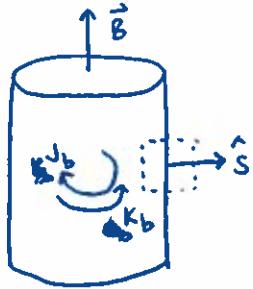


$$\oint \vec{B} \cdot d\vec{l} = B 2\pi s \\ = \frac{\mu_0 I}{\pi R^2} \cdot \pi s^2$$

$$\rightarrow B = \frac{\mu_0 I s}{2\pi R^2} \text{ a distance } s \text{ from the center}$$

Here, A will be proportional to s^2
 because $-\frac{\partial A_z}{\partial s} = \frac{\mu_0 I s}{2\pi R^2}$

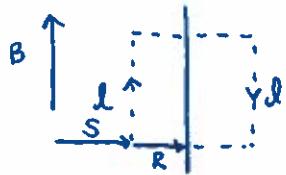
P 6.12



$$\vec{M} = Ks\hat{z}$$

$$\nabla \times \vec{M} = -K\hat{\phi} = \vec{J}_b \quad (\text{if } M \text{ is constant, } J_b = 0)$$

$$\vec{K}_b = \vec{M} \times \hat{n} = KR\hat{\phi}$$



(+) is into the page using RHR.

B was chosen to be in the positive \hat{z} -direction here.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

$$\rightarrow Bl = \mu_0 [KRl - Kl(R-s)] \\ = \mu_0 Kls$$

$$\rightarrow \vec{B} = \mu_0 Ks \hat{z} = \mu_0 \vec{M}$$

Auxiliary field: $\oint \vec{H} \cdot d\vec{l} = \mu_0 I_{f,\text{enc}}$

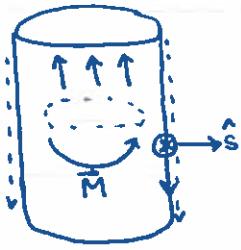
$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \text{ true for any magnetic material}$$

$$\text{Linear magnetic materials: } \vec{B} = \mu \vec{H} = \mu_0(1 + \chi_m) \vec{H}, \quad \vec{M} = \chi_m \vec{H}$$

$$\text{For this problem, } I_{f,\text{enc}} = 0 \text{ so } H_{\text{inside}} = 0$$

$$\text{Because of this } \vec{B} = \mu_0(0 + \vec{M}) = \mu_0 \vec{M}$$

Frozen-in magnetization means it is not linear



$$\vec{M} = ks^2 \hat{\phi}$$

$$\nabla \times \vec{M} = \frac{1}{s} \frac{\partial}{\partial s} (s M_\phi) \hat{z} = \frac{1}{s} k 3s^2 = 3ks \hat{z} = \vec{J}_b$$

$$\vec{K}_b = \vec{M} \times \hat{n} = -ks^2 \hat{z}$$

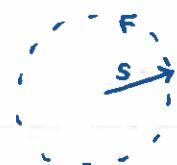
(at surface, so)

$$\vec{K}_b = -kR^2 \hat{z}$$

Same geometry as the wire.

Circular amperian loops:

Top view:



$$B 2\pi s = \mu_0 I_{enc} \quad \text{but } J_b \text{ is not constant}$$

Must integrate using infinitesimal rings

$$B 2\pi s = \mu_0 \int_0^s 3ks \cdot 2\pi s ds$$

$$= \mu_0 k \pi \int_0^s s^2 ds$$

area of infinitesimal ring

$$= 2\mu_0 k \pi s^3$$

$I_{enc} = 2k\pi s^3$

$$\rightarrow \vec{B} = \mu_0 ks^2 \hat{\phi} \quad (\hat{\phi} \text{ because that was the direction of integration})$$

So far, only volume current was used because rings were inside.

Next, we find \vec{B} outside the cylinder.

$$K_b = \frac{dI_\perp}{dl_\perp}$$

$$-KR^2 \cdot 2\pi R = -2k\pi R^3 \leftarrow \text{cancels out } I_{enc} \text{ contributed from volume current density}$$

Because they cancel out, $\vec{B}_{\text{outside}} = 0$.

What was the boundary condition for magnetic fields?

$$\vec{B}_{\text{above}} - \vec{B}_{\text{below}} = \mu_0 (\vec{k} \times \hat{n})$$

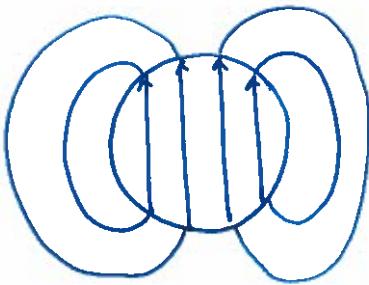
$$B_{\text{above}}^\perp = B_{\text{below}}^\perp \rightarrow \nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{H} = -\nabla \cdot \vec{M}$$

$$H_{\text{above}}^\perp - H_{\text{below}}^\perp = -(M_{\text{above}}^\perp - M_{\text{below}}^\perp)$$

$$\oint H \cdot dL = I_{f,\text{enc}}$$

$$\vec{H}_{\text{above}} - \vec{H}_{\text{below}} = \vec{k}_f \times \hat{n}$$



$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \quad (\vec{B} = \mu_0(\vec{H} + \vec{M}))$$

$$\vec{B} = \frac{2}{3}\mu_0 \vec{M} \quad \text{for uniformly magnetized sphere}$$

$$\vec{H} = \frac{2}{3}\vec{M} - \vec{M} = -\frac{1}{3}\vec{M}$$

\vec{H} field outside looks like \vec{B} field
(Not inside)

(see last year's final)