Divergence theorem: A corollary.

$$\int (\nabla \cdot \vec{v})\, d\tau = \oint \vec{v} \cdot d\vec{a} \longrightarrow (1)$$

let $\vec{v} \Rightarrow \vec{v} \times \vec{c}$ which is of course a vector

and $\vec{c}$ is a constant vector.

$$\Rightarrow \int (\nabla \cdot \vec{v})\, d\tau \rightarrow \int \nabla \cdot (\vec{v} \times \vec{c})\, d\tau \longrightarrow (2)$$

$$\nabla \cdot (\vec{v} \times \vec{c}) = \vec{c} \cdot (\nabla \times \vec{v}) - \vec{v} \cdot \underbrace{(\nabla \times \vec{c})}_{=0 \ \because\ \vec{c} \text{ is constant.}} \longrightarrow (3)$$

$$(1), (2), (3) \Rightarrow \int \vec{c} \cdot (\nabla \times \vec{v})\, d\tau = \oint (\vec{v} \times \vec{c}) \cdot d\vec{a} \rightarrow (4)$$

$$\boxed{\text{Recall: } (\vec{A} \times \vec{B}) \cdot \vec{E} = (\vec{B} \times \vec{E}) \cdot \vec{A} = \vec{B} \cdot (\vec{E} \times \vec{A})}$$

$$(4) \Rightarrow \int \vec{c} \cdot (\nabla \times \vec{v})\, d\tau = \oint \vec{c} \cdot d\vec{a} \times \vec{v} \longrightarrow (5)$$

Taking $\vec{c}$ out of the integrals since $\vec{c}$ is constant

$$(5) \Rightarrow \vec{c} \cdot \int (\nabla \times \vec{v})\, d\tau = \vec{c} \cdot \oint d\vec{a} \times \vec{v}$$

$$\Rightarrow \int (\nabla \times \vec{v})\, d\tau = \oint d\vec{a} \times \vec{v}$$

$$\boxed{\Rightarrow \int (\nabla \times \vec{v})\, d\tau = - \oint \vec{v} \times d\vec{a}}$$