PHY3513 (A. Hebard) **Exam 3** – Solutions **April 6, 2018**

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(Start each problem on a clean sheet with your name and problem # at top of page)

	True	**False**
1. The change in entropy of a reaction near absolute zero temperature is taken to be zero by convention but does not have experimental confirmation made at finite temperatures.		√
2. In a Joule expansion an ideal gas in a volume V irreversibly expands into an equivalent volume within a thermally isolated container. Since entropy increases and no work is done, the general statement can be made that pure forces derived solely from entropic considerations are not possible		√
3. Adiabatic demagnetization is a two-step process in which an isothermal increase of magnetic field is followed by an adiabatic decrease of magnetic field.	√	
4. The cohesive force between charge neutral atoms of a gas is due to attractive forces between magnetic dipoles.		√
5. A mug containing 0.2 kg of water is allowed to cool from 90°C to 18°C. The specific heat capacity is 4200 J K^{-1} kg^{-1}. Determine the change of entropy of the water to three place accuracy. Include sign and units. Write answer in either box.		**-185.7 J/K**

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2. Heat is supplied to an engine at the rate of 10^6 J/min and the engine has an output of 10 horsepower (hp) where 1 hp = 745.7 J/s.

(a) What is the efficiency (%) of this engine?

(b) How much heat per minute is rejected to the low temperature surroundings?

(c) If the engine is running at room temperature (20°C) what temperature would be needed in the high temperature reservoir supplying the heat to compete with a reversible Carnot engine running under the same conditions?

3. (25 pts) The Gibb's free energy of a system of N particles is given by,

$$G(T,p) = -Nk_BT\ln\left(\frac{aT^{5/2}}{p}\right)$$

(a) dG = ? (write in differential form similar to $dU = TdS - pdV$) **(5 pts)**

(b) Find expressions for S and V written as partial derivatives with respect to G. **(4 pts.)**

(c) Compute the constant pressure heat capacity C_p of the system: $C_P = T(dS/dT)_P$

Exam 3 solutions

#1) Question 5 0.2kg H_2O cooling from 90°C (363K) to 18°C (291K)

$$dS = \frac{dQ}{T} = \frac{C\,dT}{T} \Rightarrow \Delta S = C\int_{T_h}^{T_c}\frac{dT}{T} = C\ln\frac{T_{cold}}{T_{hot}} = -C\ln\frac{T_{hot}}{T_{cold}}$$

For specific heat capacity of 4200 $JK^{-1}kg^{-1}$, C (for 0.2kg) = 840 JK^{-1}

$$\therefore \Delta S = -840\ln\frac{363}{291}\ JK^{-1} = -185.7\ JK^{-1}$$

& heat leaving system gives decrease in entropy.

#2) (a) $10\ hp = 10\times 745.7\ \frac{J}{s} = 7457\frac{J}{s} \Rightarrow 7457\left[\frac{J}{s}\right]60\frac{s}{min} = 7457(60)\ J\,min^{-1}$

$$\eta = \frac{W_{out}}{Q_h} = \frac{7547(60)\ J/min}{10^6\ J/min} = 0.45 \Rightarrow 45\%\ \text{efficient}$$

(b) $Q_h - Q_\ell = W_{out} \Rightarrow Q_\ell = Q_h - W_{out} = 10^6 - 7547(60) = 0.547\times10^6\ \frac{J}{min}$

(c) If this was a Carnot reversible engine, we would have

$$\frac{T_\ell}{T_h} = \frac{Q_\ell}{Q_h} = \frac{0.547\times10^6}{1.0\times10^6} = 0.547 \text{ for Carnot efficiency}$$

and $\eta_c = 1 - \frac{T_\ell}{T_h} \geq \eta$

Less efficient for $\eta < \eta_c = 1 - \frac{T_\ell}{T_h}$ or $\frac{T_\ell}{T_h} > 0.547$ or $T_h < \frac{T_\ell}{0.547} = 536K$

$\therefore$ for $T_\ell = 20°C\ (293K) \Rightarrow T_h = \frac{T_\ell}{0.547} = \frac{293}{0.547} = 536\ K\ (263°C)$

Exam 3 solution – cont

#3 (a) $G(T,p) = -Nk_BT\ln\left(\frac{aT^{5/2}}{p}\right) = U + pV - TS = -Nk_BT\ln\left[aT^{5/2}/p\right]$

$\Rightarrow dG = \cancel{T dS} - \cancel{p dV} + \cancel{p dV} + V dp - \cancel{T dS} - S dT = \boxed{-S dT + V dp = dG(T,p)}$

(b) $dG = \left(\frac{\partial G}{\partial T}\right)_p dT + \left(\frac{\partial G}{\partial p}\right)_T dp \Rightarrow \boxed{S = -\left(\frac{\partial G}{\partial T}\right)_p \;\&\; V = \left(\frac{\partial G}{\partial p}\right)_T}$

(c) $C_p = T\left(\frac{\partial S}{\partial T}\right)_p = -T\left(\frac{\partial^2 G}{\partial T^2}\right)_p$

$= -T\frac{\partial^2}{\partial T^2}\left[-Nk_BT\ln\left[aT^{5/2}p^{-1}\right]\right]\Big|_p = Nk_BT\frac{\partial^2}{\partial T^2}\left[T\ln a + \frac{5}{2}T\ln T - T\ln p\right]\Big|_p$

$= Nk_BT\frac{\partial}{\partial T}\left[\ln a + \frac{5}{2} + \frac{5}{2}\ln T - \ln p\right]\Big|_p = \frac{5}{2}Nk_B = \frac{5}{2}R$ (ideal gas)

(d) $V = \left(\frac{\partial G}{\partial p}\right)_T = \frac{\partial}{\partial p}\left[-Nk_BT\ln\left(aT^{5/2}p^{-1}\right)\right]\Big|_T = -Nk_BT\frac{\partial}{\partial p}\left[\ln a + \frac{5}{2}\ln T - \ln p\right]\Big|_T$

$= Nk_BT/p \Rightarrow \boxed{pV = Nk_BT = RT}$ ideal gas

#4 Show $K_T - K_S = TV\beta_p^2/C_p$ where $K_{T,S} = \frac{-1}{V}\left(\frac{\partial V}{\partial p}\right)_{T,S}$, $\beta_p = \left(\partial V/\partial T\right)_p/V$
and $C_{V,p} = T\left(\partial S/\partial T\right)_{V,p}$

(a) $V(p,T) \Rightarrow dV = \left(\frac{\partial V}{\partial p}\right)_T dp + \left(\frac{\partial V}{\partial T}\right)_p dT \Rightarrow$ (b) $\boxed{\left(\frac{\partial V}{\partial p}\right)_S = \left(\frac{\partial V}{\partial p}\right)_T + \left(\frac{\partial V}{\partial T}\right)_p\left(\frac{\partial T}{\partial p}\right)_S}$

(c) $dG(T,p) = -S dT + V dp \Rightarrow -S = \left(\frac{\partial G}{\partial T}\right)_p \;\&\; V = \left(\frac{\partial G}{\partial p}\right)_T \Rightarrow \boxed{-\left(\frac{\partial S}{\partial p}\right)_T = \left(\frac{\partial V}{\partial T}\right)_p = \frac{\partial^2 G}{\partial T\partial p} = \frac{\partial^2 G}{\partial p\partial T}}$
which is the needed Maxwell relation.

(d) (b) $\Rightarrow -K_S = -K_T + \beta_p\left(\frac{\partial T}{\partial p}\right)_S \Rightarrow K_T - K_S = \beta_p\left(\frac{\partial T}{\partial p}\right)_S$

but $\left(\frac{\partial T}{\partial p}\right)_S = -\left(\frac{\partial T}{\partial S}\right)_p\left(\frac{\partial S}{\partial p}\right)_T = -\left(\partial S/\partial p\right)_T/\left(\partial S/\partial T\right)_p = -T\left(\partial S/\partial p\right)_T/C_p = T\left(\partial V/\partial T\right)_p/C_p$ by (c)

$\therefore K_T - K_S = \beta_p\left(\partial T/\partial p\right)_S = \frac{T\beta_p}{C_p}\left(\frac{\partial V}{\partial T}\right)_p = \frac{TV\beta_p^2}{C_p}$ ... QED.

(e) $\boxed{K_T - K_S \xrightarrow[\text{ideal}]{} TV\left(\frac{1}{T^2}\right)\frac{1}{\frac{5}{2}R} = \frac{2}{5}\frac{V}{RT} = \frac{2}{5p}}$ Agrees with $K_T = \frac{1}{p}$ & $K_S = \frac{1}{\gamma p} = \frac{3}{5p}$