L3 The equivalent 1D problem

$$\dot\varphi = \frac{L_z}{\mu r^2} \rightarrow \mu\ddot r = -\frac{dU}{dr} + \mu r\dot\varphi^2 = -\frac{dU}{dr} + \frac{\ell^2}{\mu r^3}$$

$$F_{cf} = \mu r\dot\varphi^2 = \frac{\ell^2}{\mu r^3} = -\frac{d}{dr}\left(\frac{\ell^2}{2\mu r^2}\right) = -\frac{dU_{cf}}{dr}$$

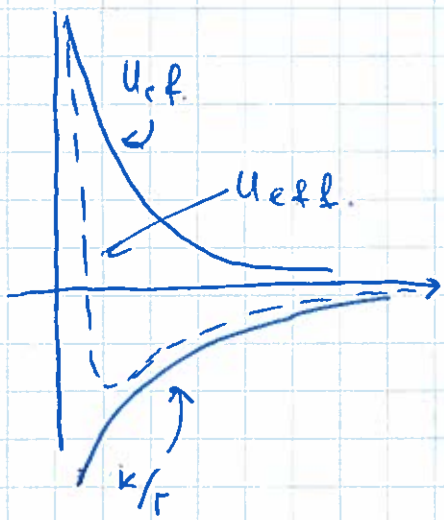
$$U_{cf} = \frac{\ell^2}{2\mu r^2}; \quad \mu\ddot r = -\frac{d}{dr}(U + U_{cf}) = -\frac{d}{dr}(U_{eff}(r))$$

$$U_{eff} = U(r) + \frac{\ell^2}{2\mu r^2}$$ – the radial motion is the same as for a particle in $U_{eff}(r)$

Solution of the Kepler's problem

- Example 1;

$$U_{eff} = -\frac{G m_1 m_2}{r} + \frac{\ell^2}{2\mu r^2}$$



$r \to 0$ U_{cf} – dominates

$r \to \infty$ $U \to 0$ $U_{cf} \to 0$

$\ell \neq 0$ – comet deflected by Sun at some small r

$\ell = 0$ – radial motion: fall to the Sun.

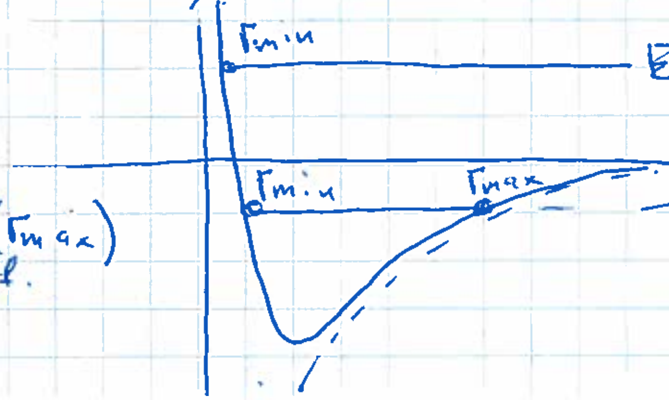
$$\mu\ddot r\,\dot r = -\frac{d}{dr}(U_{eff})\,\dot r$$

- Conservation of energy. $$\frac{d}{dt}\left(\frac{\mu\dot r^2}{2}\right) = -\frac{d}{dt}(U_{eff})$$

1D: $\frac{\mu\dot r^2}{2} + U_{eff} = const = E$ – energy.

2D: $E = \frac12\mu\dot r^2 + \frac12\mu r^2\dot\varphi^2 + U(r)$

- Example 8.2



$$U_{eff}(r_{min}) = E = U_{eff}(r_{max})$$