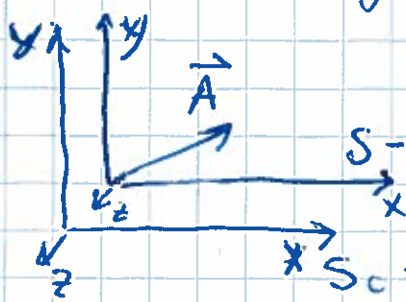
L06 Noninertial Frames

- accelerating and/or rotating frames.
  Examples: moving car, Earth, . . .

- accerating frames



$\vec{A}$ - acceleration of frame S wrt frame S_0

$\vec{A} = \dot{\vec{V}}$

$\vec{V}$ is relative velocity of S vs S_0

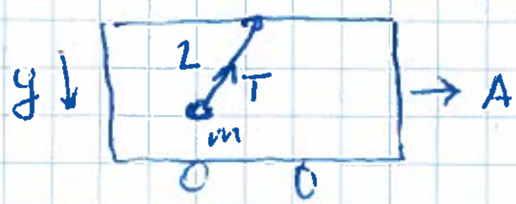
II Newton's law:

- frame S_0: $m\ddot{\vec{r}}_0 = \vec{F}$ - sum of all forces
- frame S: $m\ddot{\vec{r}} = m(\ddot{\vec{r}}_0 - A) = \vec{F} - m\vec{A}$

  we used that $\dot{\vec{r}}_0 = \dot{\vec{r}} + \vec{V}$

- $m A$ - inertial force $\quad F_{in} = -m\vec{A}$
- equivalence principle:
  a) one can not distinguish between inertial force and gravitational force
  b) inertial mass m_a and gravitational mass m_a are equal and independent on the nature of the body.

Example 9.1



$m\ddot{\vec{r}} = \vec{T} + m\vec{g} - m\vec{A}$

$m\ddot{\vec{r}} = T + m(\vec{g} - \vec{A}) = m\vec{g}_{ef}$

frequency of small oscillations:

- $\omega = \sqrt{g/L}$ ← all inertial frames $(\bar{A} = 0)$
- $\omega = \sqrt{g_{ef}/L}$ ← noninertial frames $(\bar{A} \neq 0)$

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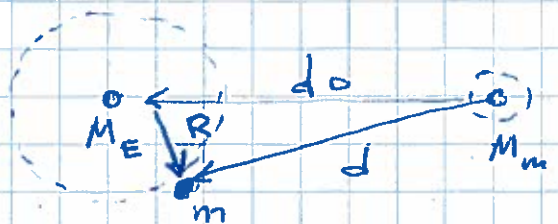
Example: Use Lagrangian $\mathcal{L} = \frac{1}{2} m \dot{\vec{r}}_0 - u(r_0)$

$S_0 \to S \quad \Rightarrow \quad \dot{\vec{r}}_0 = \dot{\vec{r}} + \vec{V}(t)$

$\mathcal{L} = \frac{1}{2} m \left[\dot{r} + V(t)\right]^2 - u(r_0)$ $\qquad m\ddot{r} = \vec{F} - m\vec{A}$

$$\frac{\partial \mathcal{L}}{\partial r} = -\frac{\partial u}{\partial r_0}\frac{\partial r_0}{\partial r} = -\frac{\partial u}{\partial r_0} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} = m\frac{d}{dt}\left(\dot{r} + \vec{V}(t)\right) = m\ddot{r} + m\vec{A}$$

Tides.



centripetal acceleration of the center of the Earth.

$\vec{A} = -GM_m \frac{\hat{d}_0}{d_0^2}$

Total balance of forces at particle m

$\vec{F} = m\vec{g} - GM_m m\frac{\hat{d}}{d^2} + \vec{F}_B$ $\qquad F_b$ – bouyant force

$m\ddot{r} = F - mA = m\vec{g} + F_b + F_{tid}$;

$F_{tid} = -GM_m m\left(\frac{\hat{d}}{d^2} - \frac{\hat{d}_0}{d_0^2}\right)$ $\qquad \vec{d} = \vec{d}_0 + \vec{R}$

a) 

$F_{tid} \sim -\left(\frac{1}{(d_0-R)^2} - \frac{1}{d_0^2}\right)\hat{d}_0$ $\qquad d = d_0 - R$

b) 

$F_{tid} \sim -\left[\frac{1}{(d_0+R)^2} - \frac{1}{d_0^2}\right]\hat{d}_0$ $\qquad d = d_0 + R$

c) 

$d = \sqrt{d_0^2 + R^2}$

$$F_\varphi \sim -\left(\frac{d_0}{d^3} - \frac{1}{d_0^2}\right) = -\frac{1}{d_0^2}\left[\left(1+\frac{R^2}{d_0^2}\right)^{-3/2} - 1\right] \sim \frac{R^2}{d_0^4}$$

$$F_r \sim -\left(\frac{R}{d^3}\right) \sim \frac{R}{d_0}$$

a, b → high tide

c → low tide.